Power System Dynamics
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Lecture - 04
Solution of Swing Equation
Method 1
Method 2

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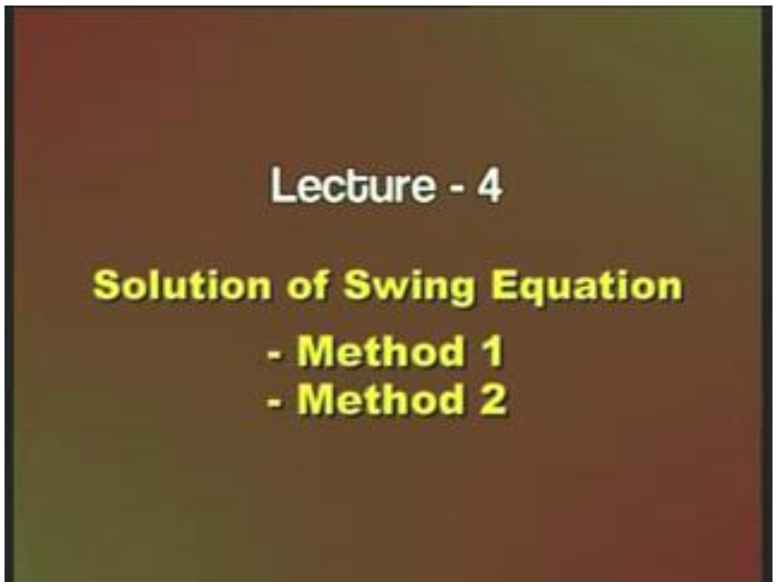

Friends we start today, the next topic that is on solution of swing equation. We have derived swing equation or a synchronous machine we have also derived swing equation for a multi-machine system. We have seen that the swing equation is function of or is a non-linear function of the power angles, the there is no formal solution available or possible because the swing equation is a non-linear differential equation and therefore numerical techniques have been developed to solve the swing equation. The 2 methods which is called method 1 and method 2 will discussed today. Before I tell you about the method 1 and method 2 for solving the swing equation, let us consider a simple case.

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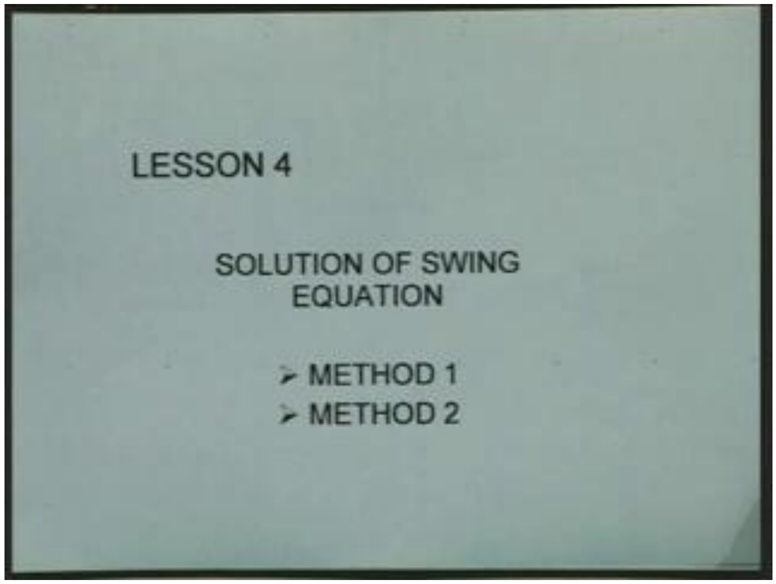

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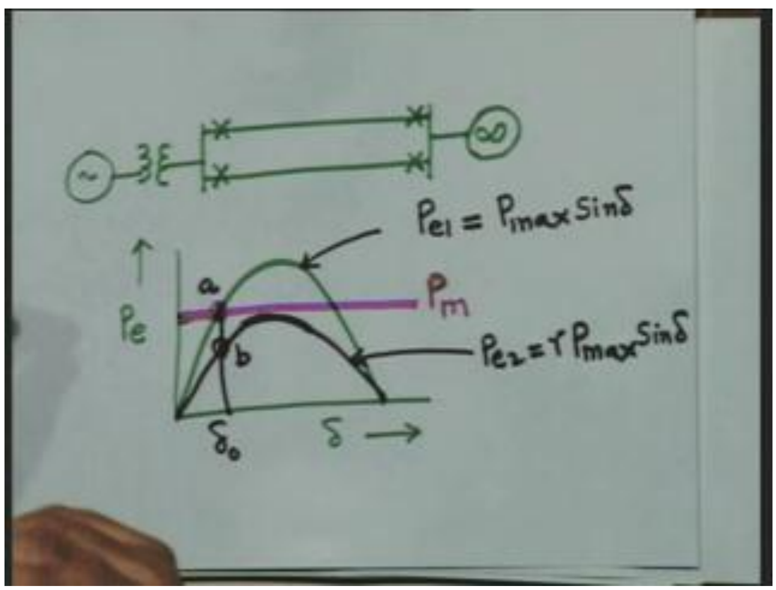

A synchronous machine connected through a transformer to a double circuit transmission line and hence and then connected to an infinite bus, a large system on this transmission lines we provide circuit breakers at these locations. The purpose of providing circuit breakers it is location is that as and when any fault occurs on the transmission line by operating the circuit breaker this faulting line can be taken out.

We will consider a disturbance where due to some unknown reason this line trips, okay. Now here before before the occurrence of this disturbance these 2 lines are in service. Now if I draw the power angle characteristic I will denote this as P_e on this axis, we put delta, okay for this case the power angle characteristic will come out to be a sin curve on this path okay.

Now in case the system is operating under steady state condition I will represent the mechanical input line by a straight line divided by P_m then this is our operating, now the moment a disturbance occurs where out of the 2 lines, 1 lines trips then the post fault system will have only 1 line in operation and the new power angle characteristic will be different from the power angle characteristic before the occurrence of fault or we call it pre-fault disturbance or pre-fault power angle characteristic or we can call it pre-disturbance power angle characteristic.

Now let us represent the power angle characteristic after the disturbance we will this characteristic can be represented as P_{e1} equal to P_{max} sin delta and this characteristic may be represented as P_{e2} equal to, let us say r into P max sin delta okay. Our initial operating point is here, this denoted by this angle delta naught. Now what we see here is that the moment fault has occurred or a disturbance has occurred the operating point will shift from this position a to position b because the moment disturbance occurs right the angle cannot change instantaneously, angle will remains same.

However, the power angle characteristic becomes P_{e2} equal to r P_{max} sin delta r is, r is a quantity or a fraction which is less than 1 and it depends upon what where the values of the transformer reactance, synchronous machine, transient reactance and transmission line reactance and so on. Now what we see here is that the moment this disturbance takes place there is difference in the mechanical input and electrical output, electrical output determined by this point, this difference become the acceleration power P_e, we call it accelerating power P_a.

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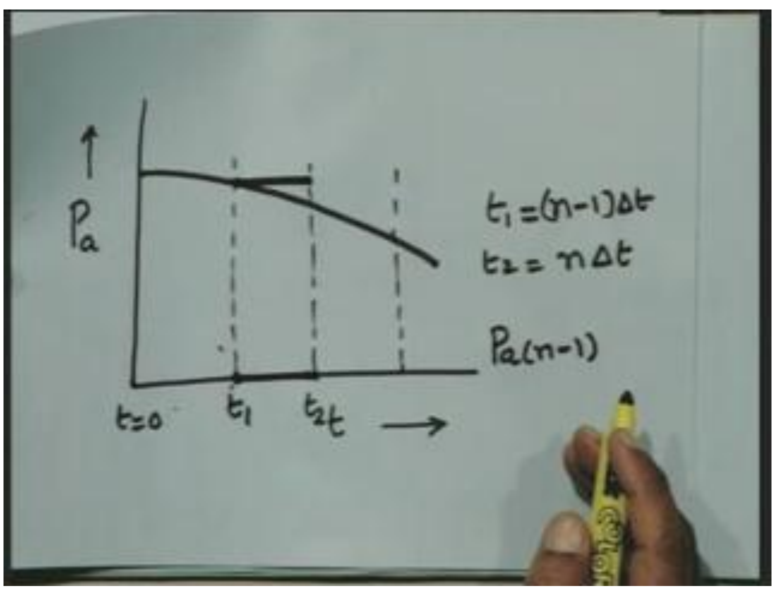

Now this as we will see actually that because of the accelerating power the rotor will accelerate okay and delta will increase, the increase of delta right as a the function of time right or we call it actually the delta versus time this plot is our swing curve and they are interested in plotting the swing curve of the system okay. Our primary objective is to obtain the solution of the swing equation and that solution is your swing curve. Now what I do here is that I plot the accelerating power P_a as a function of time.

We start with time t equal to say 0 at time t equal to 0, the accelerating power is equal to ab and it is a positive as time passes the accelerating power is going to decrease okay. Therefore, let us represent the variation of accelerating power by this graph I am not showing the complete one because as far as this problem is concerned in this problem the accelerating power is going to be positive all through and the resulting system will be unstable system.

Now what we do is that for is solving the swing equation, we will divide the time into small time steps. Let us consider 2 consecutive time intervals we will represent the nth time interval, the let us say this is nth time interval. Now this nth time interval at the beginning of this interval, let us call it time is t_1 at the end of the interval, let time is say ah t_2. Okay then if this is the nth interval right then t_1 is equal to n minus 1 delta t and t_2 will be equal to n delta t, right this is the notation which we will be adapting. Now in the step by step solution or we normally call point by point solution because these 2 terms are used interchangeably in the method 1.

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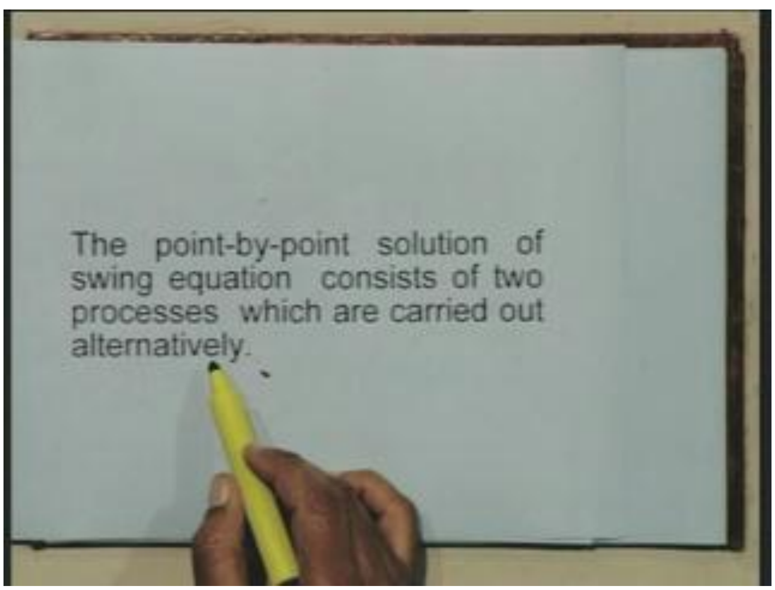

we assume that the accelerating power remains constant during the time interval and equal to equal to its value at the beginning of the interval that is we calculate the accelerating power at the time t1 which is the beginning of n minus 1th interval and we presume at, we that this accelerating power remains constant during this interval it means the accelerating power remains constant, okay and we shall denote this accelerating power by the symbol P_a n minus 1 okay, with this introduction we can proceed to discuss the method 1 for solving the swing equation.

As I stated in the beginning this, this method is called point-by-point solution therefore, the point-by-point solution of the swing equation consists of two processes which are carried out alternatively which are those two processes that is what we have to understand.

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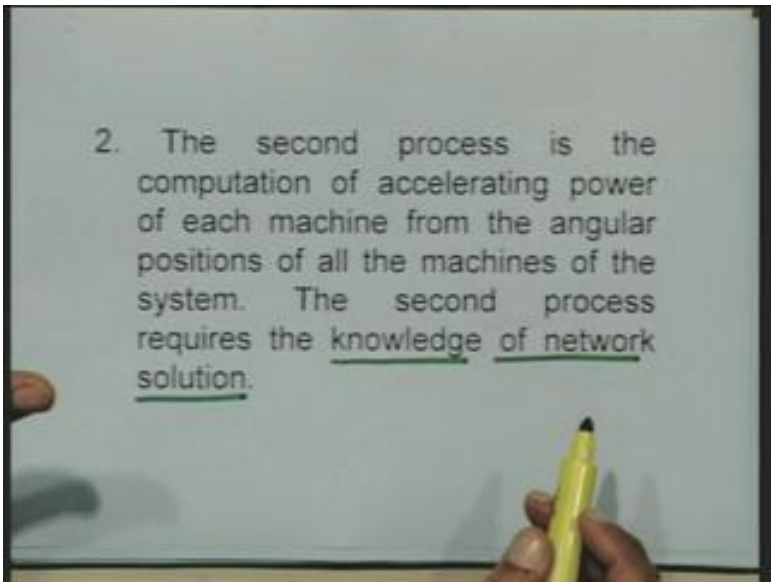

The first process is the computation of the angular positions and angular speeds, I am putting the word angular positions and angular speeds. At the end of each interval from the knowledge of from the knowledge of angular positions and speeds at the beginning of the time interval that is the, that is and the accelerating power is assumed for the interval.

Now this is general statement what will be the assumed value of the accelerating power during that interval right is a subject of concern and we will see actually that the method 1, there is one way of choosing the accelerating power during that interval method 2, we have another way of choosing the accelerating power. However, the accelerating power during the time interval remains constant right and therefore the fist step we understand is that we calculate the value of angular positions, angular speeds at the end of time interval with the knowledge of angular positions, angular speeds at the beginning of the time interval okay, this is the first step.

Now once we have obtained the angular positions at the end of the time interval then the next step we want to second step will be or the second process we call it the second process is the computation of accelerating power of each machine from the angular positions of all the machines of the system because when we go from one step to the next step right, we have to again compute the accelerating power of each machine, okay and this accelerating power can be computed from the knowledge of network solution.

In general for a multi-machine system, one has to solve the network to find out the accelerating power of each machine at the beginning of interval or we can say at the beginning of next interval. Now this uh these 2 processes are carried out alternatively that you can understand now that suppose I know the angular positions, angular speeds at the beginning of time interval and I start with the knowledge of accelerating power at the beginning of the time intervals or accelerating power during that interval, with this knowledge we compute the angular positions, angular speeds at the end of the interval.

Then using this information we go to second step where we solve the network and we find out what will be the accelerating power to be used for the next interval right and this this these 2 steps are carried out alternatively that is I discuss in method 1.

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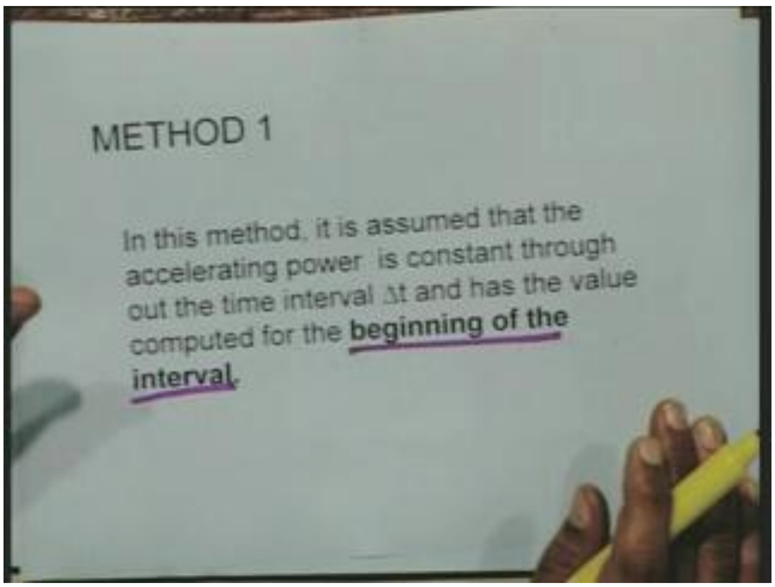

The accelerating power is constant throughout the time interval delta t and has the value computed for the beginning of the interval that value we have computed at the beginning of the interval to presume this to remain constant during that interval, okay.

Now here in this diagram, instead of showing the plot of accelerating power versus time what we are showing here is the acceleration that is d2 delta by dt2 which is equal to accelerating power Pa by that is M is constant you divide this by M. So that instead of plotting accelerating power if I plot acceleration versus the number of time steps. Now instead of plotting here ah time in seconds I plot this as t by delta t. Okay and this plot is similar to the plot for accelerating power except the units will be different because the accelerating power has been divided by M.

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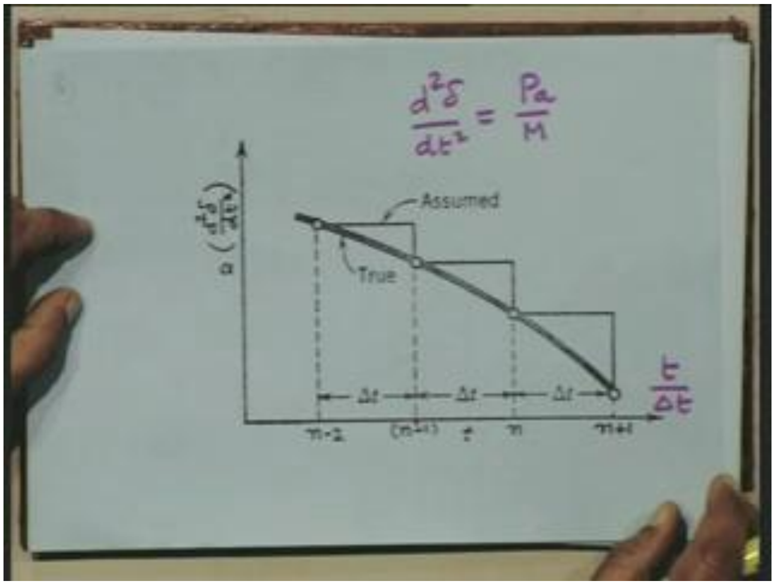

Now if you see here actually that suppose I compute for time interval n minus 1 then this is the assumed value of the accelerating power that is I compute the accelerating power at beginning of the time interval and this will remain constant. Then when I come here, we again compute what is the acceleration and this acceleration that assumed to remains constant here like this okay. Please note down actually this graph here what we observe in this graph. In this method 1, when the accelerating acceleration is decreasing the assumed acceleration is always more than the actual acceleration and the we will see that the error will depend upon what is the time step which we choose.

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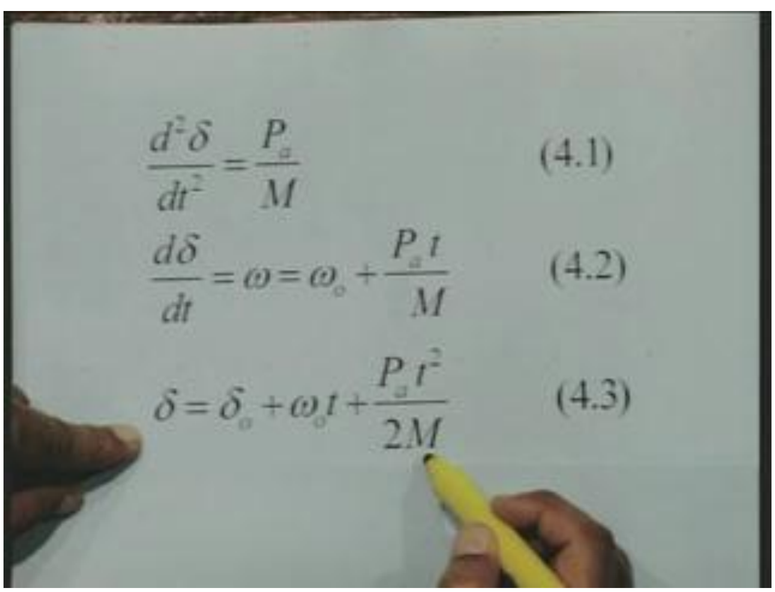

Suppose if I take very small time step then the assumed value of accelerating power will very close to the actual value of acceleration or accelerating power. We will derive now a simple algorithm to implement this method one. We start with our swing equation. Now to illustrate this problem I am considering a simple system where one machine connected to infinite bus and there is only one swing equation. Now this is our swing equation dt delta by dt^2 equal to P_a by M, okay.

Instead of using this coefficient here H by phi here, we prefer to use this M because it is easy to what I told the time then you integrate this equation assuming that P_a is constant accelerating power is constant then this integration will give you d delta by dt equal to omega, this is the definition of dt delta by d, dt represented as omega this is going to be equal to omega O plus P_a into t by M. Omega O is the initial value of angular speed.

Now here d delta by dt is not the actual speed of the rotor it is, it is excess speed of the rotor over synchronous speed. Initially when the system is operating under steady state condition then what happens P_a will be 0 and d delta d delta by dt is also 0 because there is no acceleration, there is no difference in the in the rotor actual speed and the synchronous speed, the rotor rotates at synchronous speed.

Now here with finite value of P_a the accelerating power right d delta by dt is omega and this is the excess speed over the synchronous speed, now if you further integrate this equation 4.2 with respect to time, we will get delta equal to delta naught omega O into t P_a into t square by 2M. Now you have to very clearly understand the meaning of all the terms which are involved in the these 3 equations 4.1, 4.2 and 4.3. Is there any doubt, this is state forward okay. Now what we do is we will use these equations to write down the speed and angular positions at the end of interval in terms of information at the beginning of the interval.

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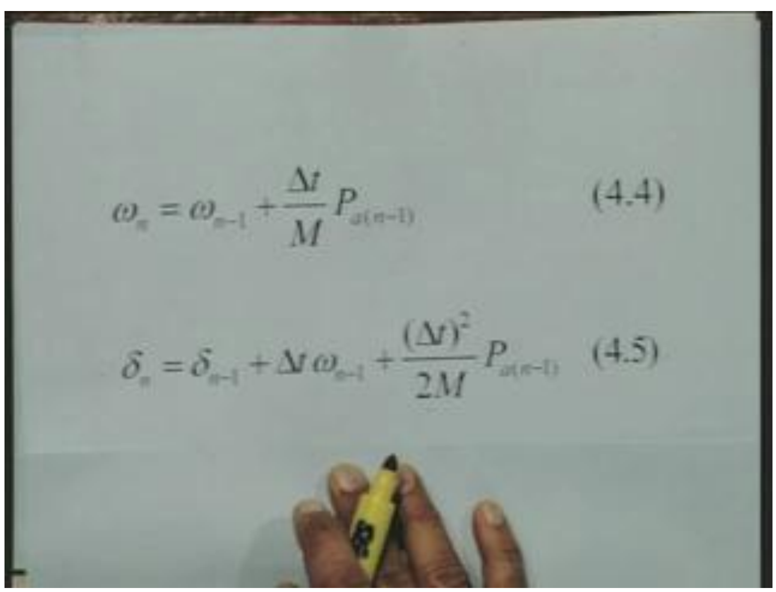

In the equation 4.2, I will substitute the value of the speed at the end of the nth interval. The speed at the end of nth interval is omega n, the speed of the rotor at the beginning of the nth interval is omega n minus 1, t delta t is the time, time state and P_a n minus 1 is the accelerating power at the beginning of the interval. Okay, therefore from the equations which we have obtained from swing equation by integrating.

Now what we are trying to do is that we are implementing or applying this equations to a particular time interval. Now if we substitute the values of angular positions at the beginning and at the end of time interval, we get the equation 4.5, delta n equal to delta n minus 1,omega n minus 1 into delta t delta t square by 2M P_a n minus 1 okay and these equations are valid for any value of n, they can start with n equal to 0 that is time t equal to 0 okay and then we go from n equal to 1, 2, 3 like this, that is from one step to the next step next step to next step and so on.

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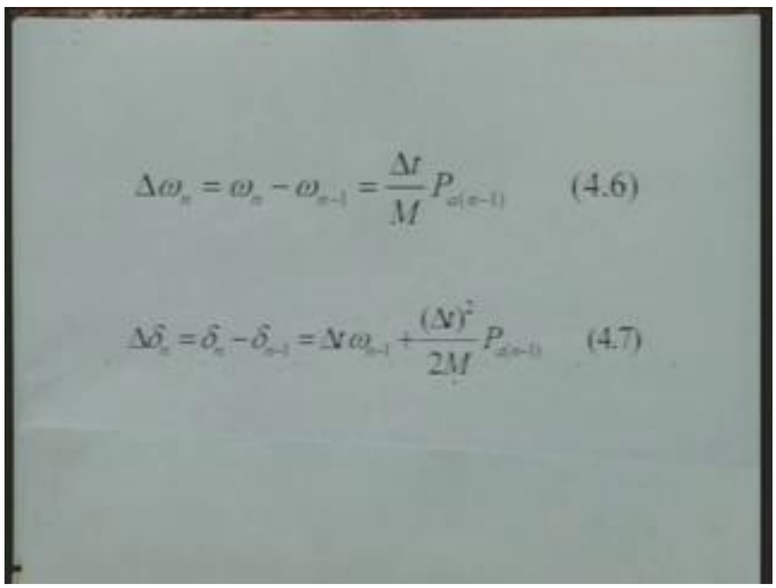

From the equation 4.5, we can write down the change in speed during the nth interval as omega n minus omega n minus 1 as delta omega n which is equal to delta t by M P_a n minus 1.This can be easily understood because this is the accelerating power accelerating power by M is the acceleration and we are assuming this acceleration to remain constant during the time interval therefore deviation in the speed during the time interval is called delta omega n equal to the assumed value of acceleration multiplied by time period.

Similarly, we can write down the change in angular position during the nth time interval as omega n minus 1 delta t equal plus delta t square by 2M P_a n minus 1. Now with the information of the quantity at the beginning of the interval what are the quantity at the beginning of the interval omega n minus 1, delta n minus 1 and P_a n minus 1.

We can solve this equation 4.6 and 4.7 okay and obtain this swing curve okay what we have to we have to do is that we change, this angular change during the interval and we can find out the actual angular position by adding this change to the angular position at the n minus 1th at the beginning of n minus 1th interval okay and you can find out the swing curve. Now when I said that there are 2 steps involved, this is the first step, the second step will be that you obtain the value of this angle delta and substitute in the power angle characteristic applicable during that transient period to find out the new value of accelerating power that is your second step.

Once you obtain a new value of accelerating power we will substitute here in this equation and obtain the new values of angle deviation and speed deviation. Now do we require the information about speed deviation as function of time for assessing the stability of the system, the answer is no. If I know the plot of delta as function of time then by examining the swing curve I can say whether system is stable or not as we have seen in last time and therefore if I am interested only in plotting the swing curve then what we do is that we eliminate this speed term and obtain an expression which is independent of or which does not contain speed term.

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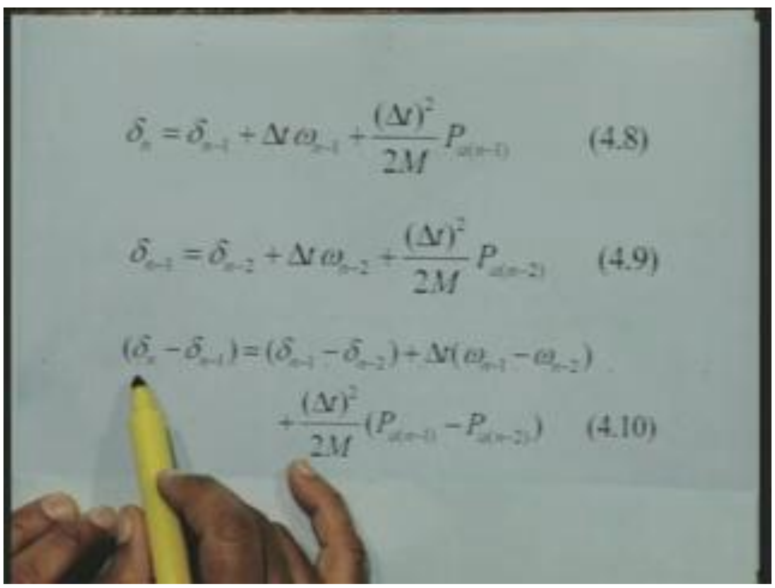

Now this can be easily done easily done, here equation 4.8 is same as equation 4.5, I will just written for the safe of convince. Now what we do is that we write a similar equation for the preceding time interval that is instead of n we put it n minus 1, n minus instead of n minus 1 you put n minus 2.

So that we can write down the equation of this form delta n minus 1 equal to delta n minus 2 delta t omega n minus 2 delta t square by 2 M P_a n minus 2. This equation is for nth interval this equation is for n minus 1th interval, okay our next step will be that you subtract these 2 equations, you will get delta n minus delta n minus 1 equal to delta n minus 1 minus delta n minus 2 plus delta t into omega n minus 1 minus omega n minus 2 plus delta t square by 2 M P_a n minus 1 minus P_a n minus 2.

Now what we do is that we make some substitutions, we denote the speed change I am sorry, correction we denote the angle change or angle deviation during nth interval as delta delta n. Similarly, for n minus 1th interval delta delta n minus 1 and so on now if we make these substitutions here in this equation we can write down expression in the form delta delta n equal to delta delta n minus 1.

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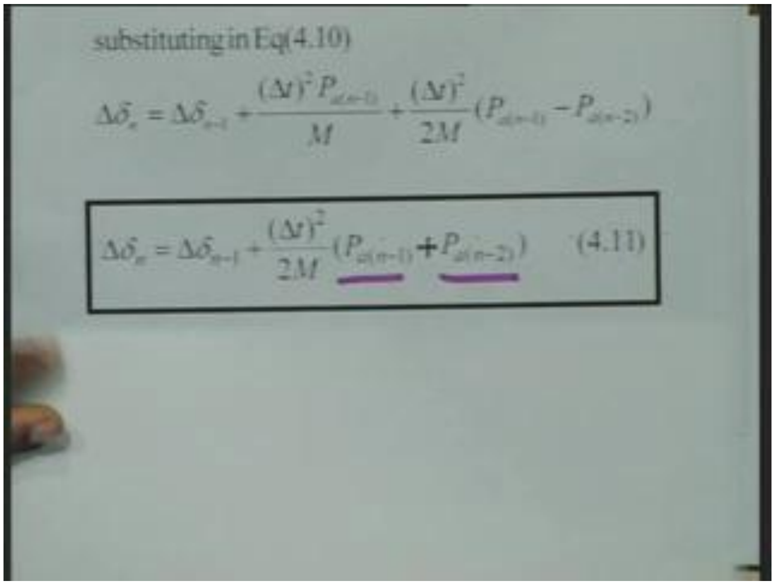

Now we had the term delta t into omega n minus 1 minus omega n minus 2, now this term is nothing but delta omega n minus 1 this is the speed deviation in n minus 1th interval and using this equation 4.4, we can write down that the speed deviation during the n minus 1th interval can be written as or nth interval can be written as delta t by M P_a n minus 1 that is equation 4.4 okay. Now you make that substitution here.

So that now what we find that in this equation we do not have speed term speed has been eliminated okay. This equation these 2 terms can be combined and you get the resulting expression for delta delta n equal to delta delta n minus 1e plus delta t square by 2 M P_a n minus 1 plus P_a n minus 2. Now this is the algorithm for obtaining the swing curve or a machine connected to infinite machine.

Similar equations can be derived if you have a multi machine system. A process goes like this when I want to find out the change in total angle position during nth interval I know what was the change in angle rotor angle position during the n minus 1th interval but that was that computation has already been completed, we know this information, we also know the uh assumed value of accelerating power at the beginning of n minus 1th interval and at the beginning of nth interval, this is the accelerating power at the beginning of nth interval.

The accelerating power at the beginning of n minus 1th interval is also known because we have already done the calculation for that interval and therefore this algorithm goes in a iterative manner you know the complete information on the which is required to compute the expression on the right hand side of this equation, you get the value of delta n.

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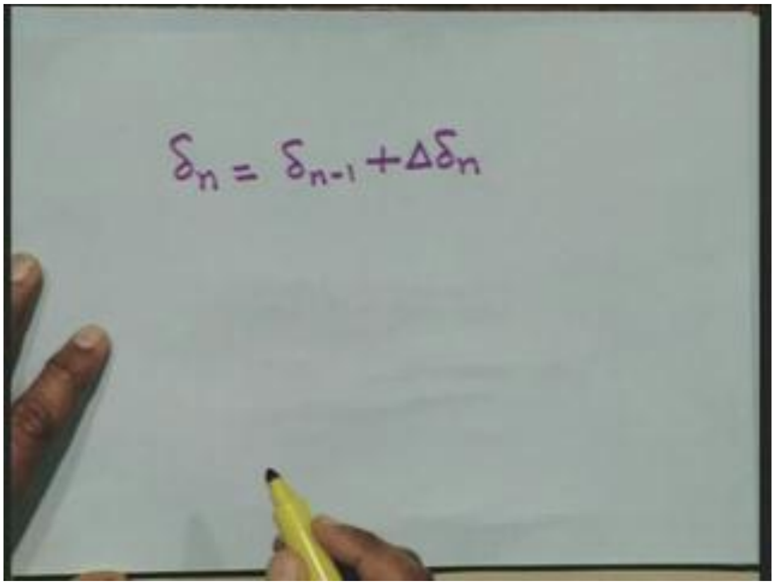

Then the moment you get the value of delta delta n, you obtain the new value of delta n as delta n minus 1plus delta right and you continue to do it because as I told you that the process has 2 steps, the step one is to compute the new angular positions and new angular speeds. However in the method one if you are interested only in swing equation then we do not require the information about the angular speeds right.

Then once we get this new value of angle we refer back to the power angle characteristic compute the electrical power output, mechanical power input is assumed to be constant we compute what is acceleration power and use that accelerating power for the next interval okay and therefore this process is to continue alternatively.

We have discussed actually this technique now how good this technique is in giving you the correct solution and as we will see actually they are numerical techniques are not the exact solutions they will give you always approximate solutions because we make some assumptions. Only this is that you would like to get the solution which is very close to the exact solution and to evaluate the method 1 step by step method 1 there we assume swing equation of this form.

Let us presume that swing equation is given by this equation, now this is the swing equation which we want to solve. Let me know whether this swing equation is a linear differential equation or non-linear differential equation. This is a linear differential equation, I have chosen the linear differential equation deliberately to illustrate the we will say illustrate the effect of, effect of time step on the accuracy of the solution.

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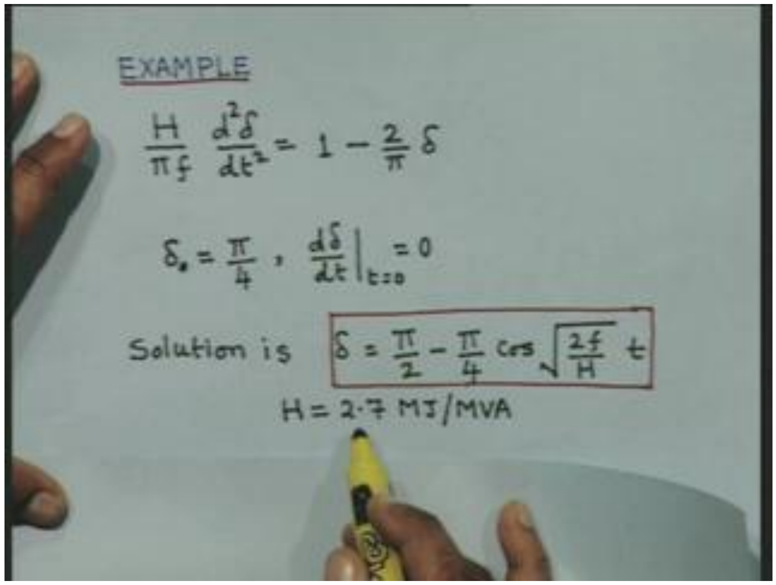

Now so far this swing equation is concern the initial conditions are given as delta naught equal to phi by 4 and the initial speed is 0 d delta by dt initial is 0, with these initial condition the formal solution of the swing equation is delta equal to phi by 2 minus phi by 4 cosine square root of 2 f by H, t. This you can obtain yourself please do the it is an exercise find out the formal solution of this linear second order equation. Now to compare the accuracy of step by step method one we solve this equation by step by step method because step by step method which is applicable are suitable for solving non-linear equation can also be applied for solving a linear differential equation, for solving this H is equal to 2.7 mega joules per MVA is assumed.

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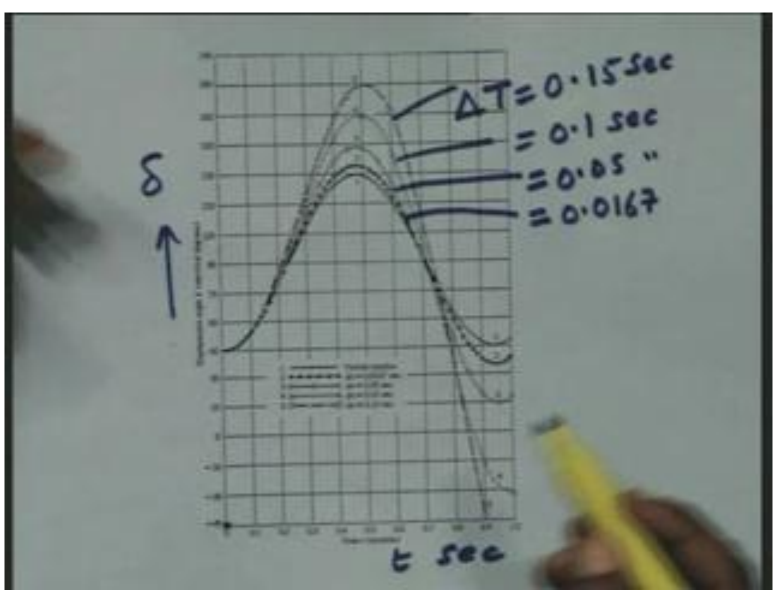

This graph shows ah swing curve obtained using the formal solution and by using this method 1, this firm curve shows this graph 1, a firm curve it shows the solution obtained using the formal solution. The graphs which are shown here in this diagram are obtained for different values of time step delta T equal to 0.15 second. Then next graph is for delta T 0.1 second then the third graph is for 0.05 second and the last one which is obtained using the method 1 point by point solution is 0.0167 delta T equal to 0.0167 is one third of the delta T equal to .05, originally a delta T equal to 0.05 second is chosen and in order to understand how much improvement one gets in the solution delta T is reduce by a factor of 3 and graph is obtained for delta T equal to 0.0 second.

This axis shows the time in seconds and y axis shows delta in electrical degrees, it can be very clearly seen that as the time step increases the accuracy of the solution of the swing equation decreases or deteriorates and hence one concludes that for getting solution closer to the accurate solution one has to use very short time step. However, if we use a short time step it will require more time for computation and this is the major drawback of method 1.

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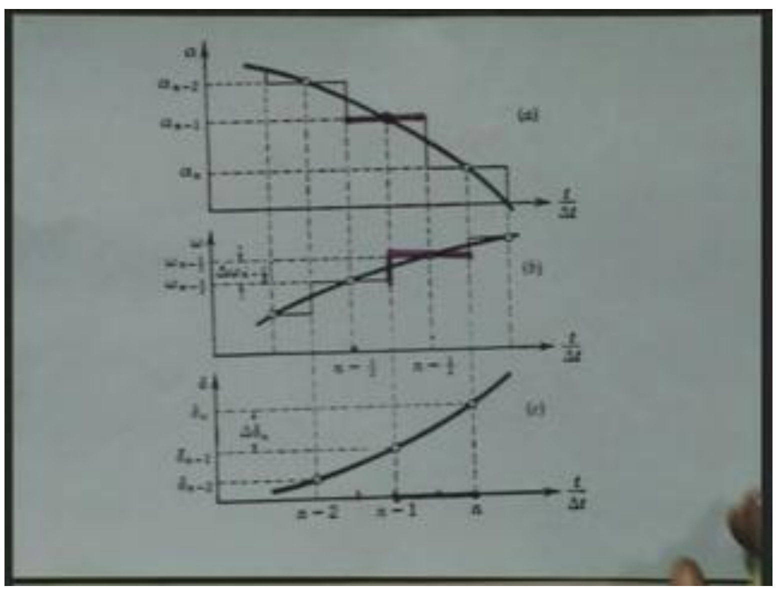

In order to overcome this shortcoming of this method 1, a method 2 has been developed and this method two is different from method 1 in terms of the accelerating power which is assumed to remain constant during the time step. Now to understand this method 2 let us look at this graph which shows the acceleration versus t by delta T, now here on this x axis we have put t by delta T.

So that it shows the u the time step count, now this graph shows the variation of acceleration with t by delta T or with t, now in this method two instead of assuming the accelerating power remaining constant at the value equal to the 1 computed at the beginning of the time interval.

We compute the value of the accelerating power or acceleration at the beginning of the time interval for example, say time interval starts at n minus one and ends at n that is the nth time interval. Now at this time step n minus 1, we compute the acceleration alpha n minus 1 and assume that this acceleration remains constant from half the preceding interval to the next half interval that we if we see in this diagram the accelerating power computed at the beginning of nth interval remains constant from n minus 3 by 2 to n minus half it can be very clearly seen actually that by assuming this accelerating power in this fashion, the assumed value of the accelerating power is equal to the average value over the time step.

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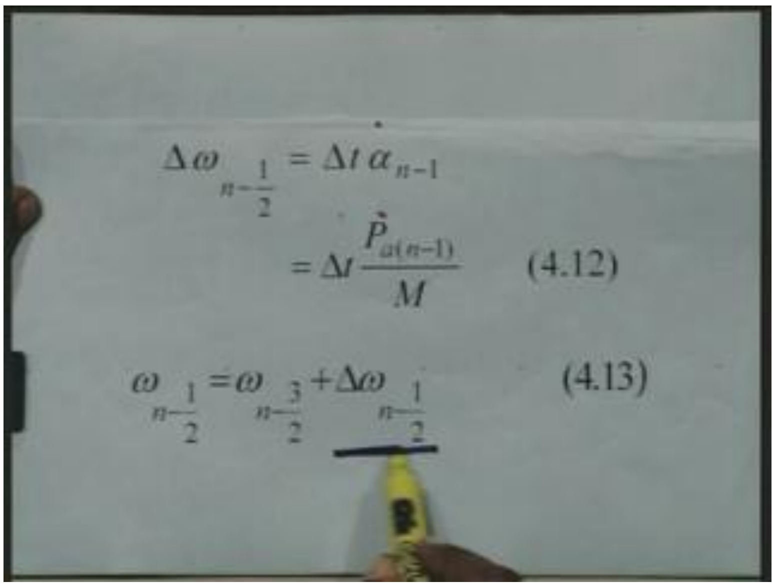

Now we can write down with this assumption the change in speed during the time step n minus half this can be written as delta t into alpha n minus 1 where alpha n minus 1 is the acceleration computed at at step n minus 1. Now alpha n minus 1can be replaced by this P_a n minus 1 by M. So that we can say that speed deviation delta omega n minus 1 by 2 is equal to delta t P_a n minus 1 by M.

Now we can write down the rotor speed rotor speed at the end of n minus 1 by 2 interval as the speed at the beginning of this interval that is omega n minus 3 by 2 plus delta omega n minus 1 by 2. This quantity is the change in speed during the time step delta t.
Further, since the acceleration assumed to remain constant during this time step speed is obviously going to vary during this time interval.

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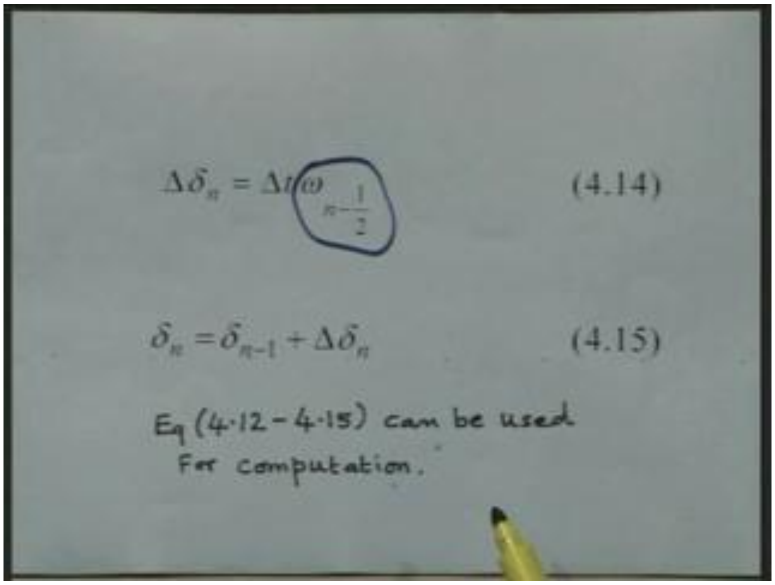

However, the total change in the time and total change in the speed will be accounted by considering a step change in the speed that is the total change in the speed which is equal to delta omega n minus half which is equal to delta t into alpha n minus 1, this changes in the speed is assume to take place in a step manner at the instant at which we compute the accelerations of the rotor, with this assumption with this assumption the speed remains constant during the nth interval and it is its value is equal to omega n minus half, with this speed, with this speed during this interval the change in rotor angle is equal to delta t into omega n minus half.

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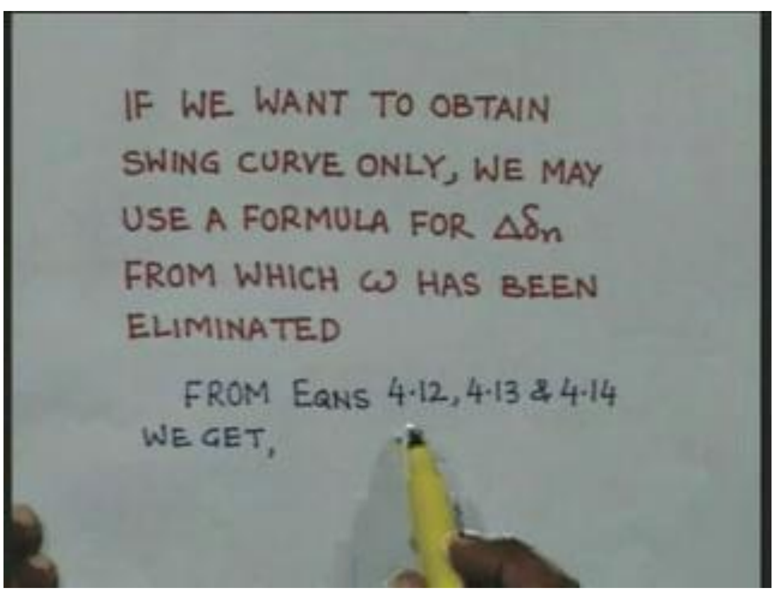

Since omega n ,omega n minus half is constant during this interval therefore change in angular position is obtained as the constant speed a omega n minus half into delta t therefore, angular position of the rotor at the end of nth interval is equal to angular position at the beginning of the nth interval plus change during the nth interval therefore, we can write down delta n equal to delta n minus 1plus delta delta n.

Now when we solve this equations four point 1, 2 to 4.14 we can get the swing curve as well as we can get the speed of the rotor as a function of time. However if we want to obtain swing curve only we may use a formula for delta delta n from which omega has been eliminated from the equations 4.12, 4.13 and 4.14.

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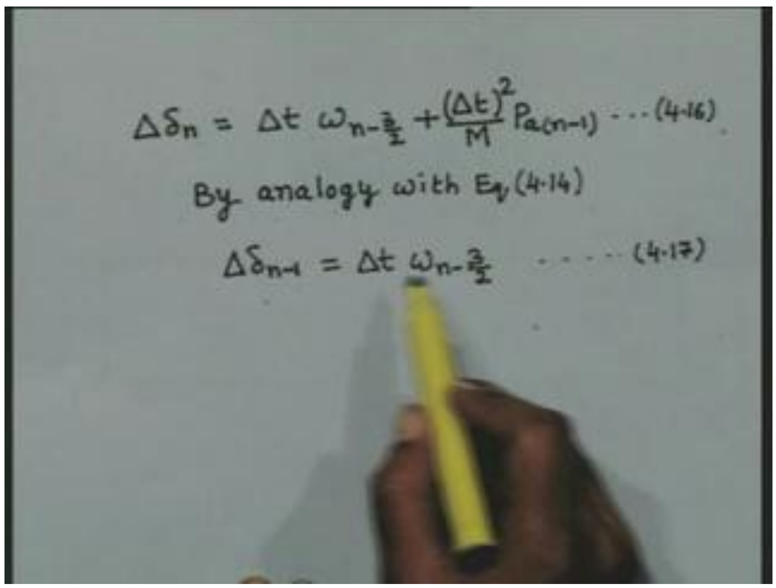

We get an expression for delta delta n as delta t omega n minus 3 by 2 plus delta t square by M P_a n minus 1. By analogy with equation 4.14, one can write down the change in angular position during the n minus 1th interval as delta delta n minus 1 equal delta t into omega n minus 3 by 2. So, that we can substitute the value of delta delta n minus 1 that is the change in the rotor angular position in the equation 4.16 using equation 4.17. We get the expression for delta delta n equal to delta delta n minus 1 plus delta t square by M P_a n minus 1.

This is a equation which can be used for computing the swing curve the left hand side term delta delta n, shows the change in rotor position rate rotor angular position during the nth interval, this is expressed in terms of the change in angular position during n minus 1th interval plus delta t square by M P_a n minus 1 and this can be easily programmed and using this expression the problem which was solved using method 1 is again solved here.

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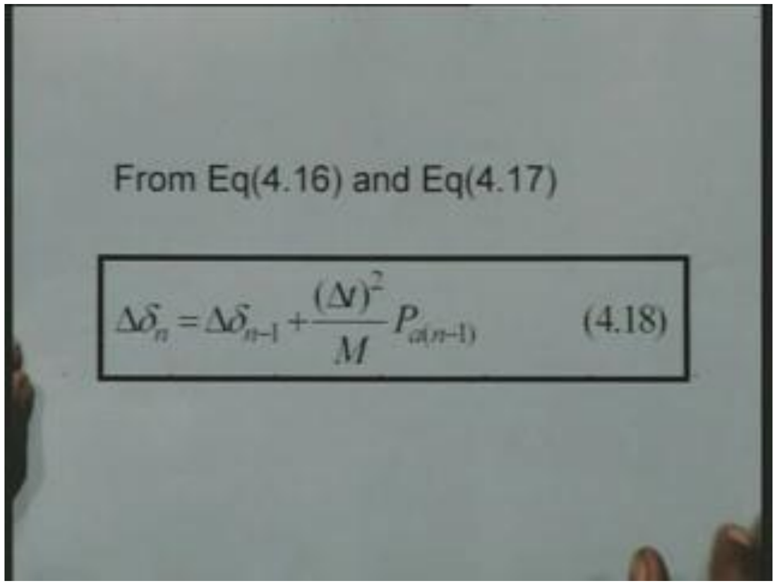

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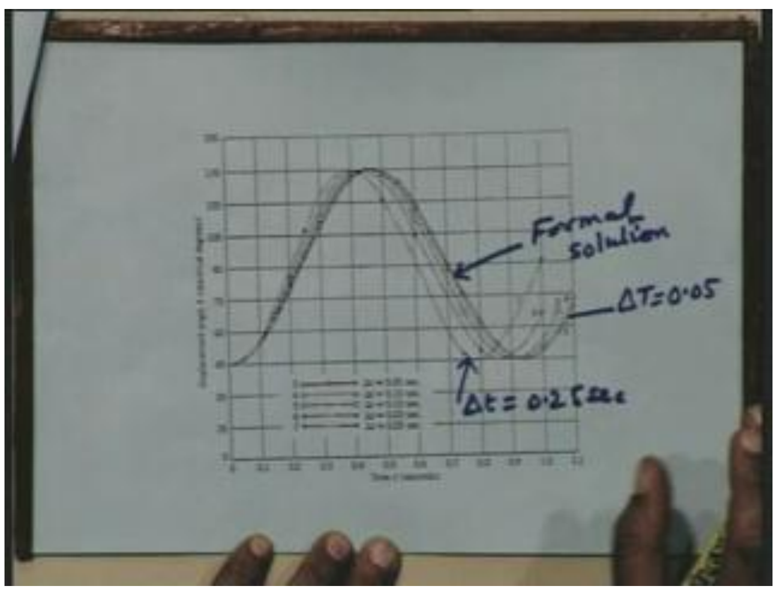

Now this graph shows the swing curve obtained using formal solution of the second order differential equation and using method 2 considering the several values of time step. This graph shows the solution obtained using the this graph shows the swing curve obtained from the formal solution, you can say formal solution. Now I say formal solution means it is the closed form solution of the second order differential equation. Now here this swing curve is for delta t equal to 0.25 second, well this swing curve is for delta T equal to 0.05 second.

Now if you, if you carry if we carefully examine this swing curves we can notice that, that the amplitude of the swing curve or the maximum deviation of delta obtained using the formal solution and that obtained using method 2, have hardly any difference, only difference which we observe is the time, time period of the solution obtained within the formal solution and that obtained using the method 2 have some difference and therefore, I can conclude here that the method 2 provides the accurate swing curve and we can use reasonable value of time step as we see that for time step equal to delta T equal to 0.05, the solution obtained by method 2 that is the step by step method 2 is very close to the formal solution.

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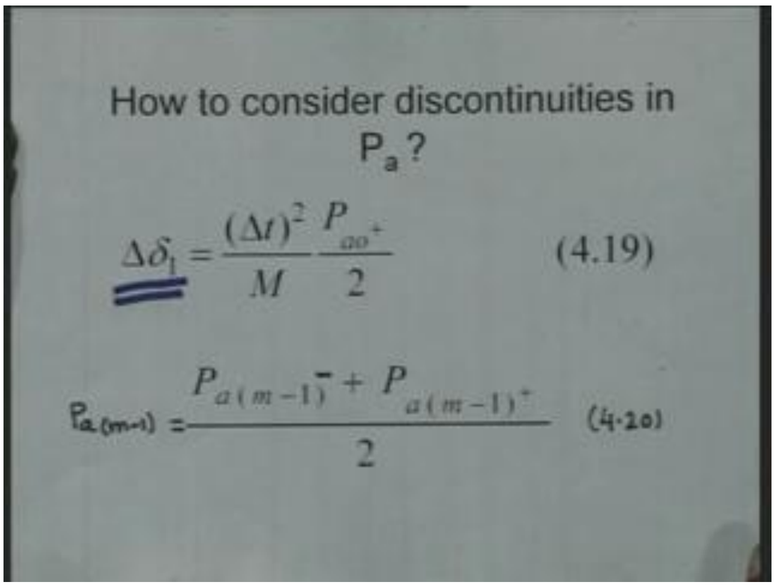

Now we consider how do we account for the discontinuities in the accelerating power, discontinuities in the accelerating powers occurs at the occurrence of disturbance or at the instance of switching. In case the discontinuities at the beginning of the time interval that is the first time interval then this can be computed delta delta 1 can be computed by using this expression delta t square by M, P_a O plus by 2, where P_a O is the accelerating power just after the disturbance and by putting 2 we are taking the average value of the accelerating power half the accelerating power.

In case the discontinuity occurs at the beginning of mth interval then this can be the accelerating power during the m th interval will be computed by this formula P_a m minus 1 minus plus P_a m minus 1 plus by 2, this minus indicates the accelerating power just before the occurrence of disturbance and P_a m minus 1 plus indicates the plus sign indicates just after the occurrence of incidence.

With this, we conclude the computation of the swing curve using 2 different methods method one and method 2, method 2 is more accurate and this can be used. Thank you!