**Power System Dynamics**

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**Lecture - 02**

**Introduction to Power System Stability Problem (contd…)**

Friends, today we continue our discussion on introduction of the power system stability problem.

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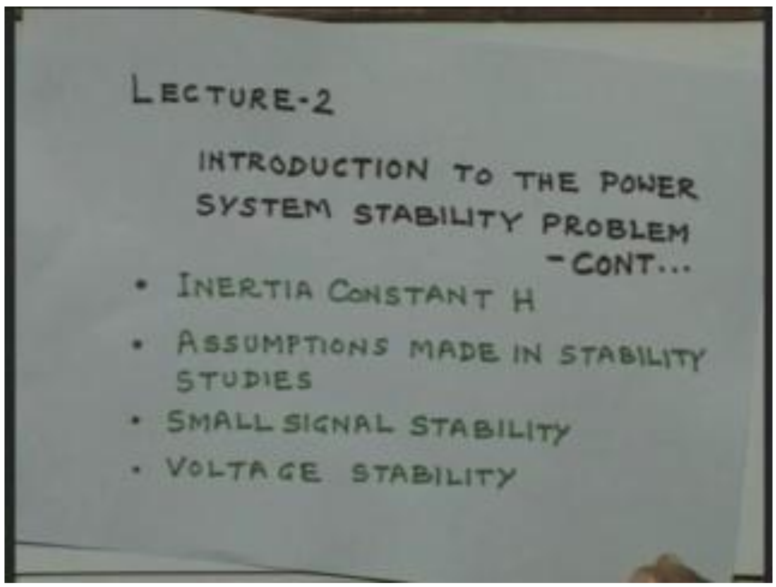


Today, I will discuss something more about inertia constant H, we introduced last time. I will also discuss certain assumptions that commonly made in stability studies. We will discuss small signal stability and voltage stability concepts; we shall be dealing with these topics in depth at later stage. Today our intention is just to give you basic concepts related to a small signal stability and voltage stability.

Last time, we have derived this swing equation, the swing equation is very important for analysis of the stability of a power system, this equation is used for transitivity analysis and this is the basis for small signal stability analysis also. This equation has the form the coefficient is 2H upon omega s ah d square delta by dt square equal to $P_m$ minus $P_e$. Initially when the equation was written, the coefficient was M, the inertia constant. However, we define a another constant known as the inertia constant H, let us see why we prefer inertia constant H for analyzing the stability of the system.

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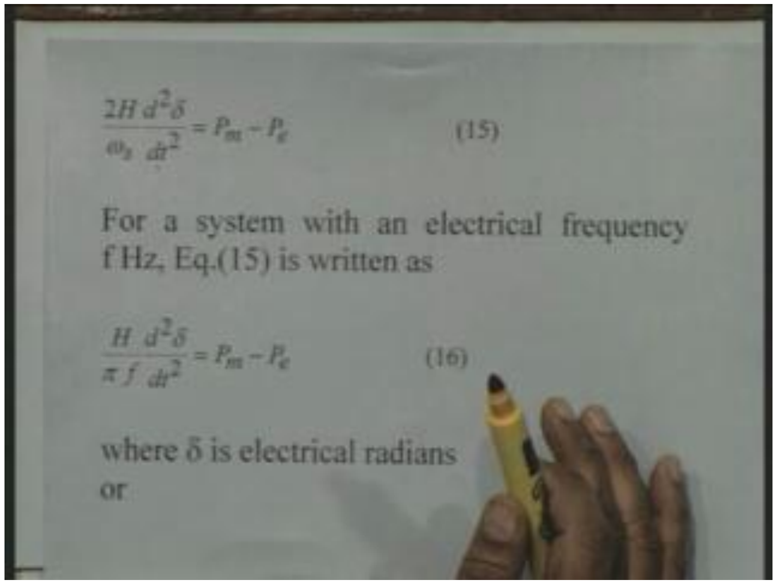


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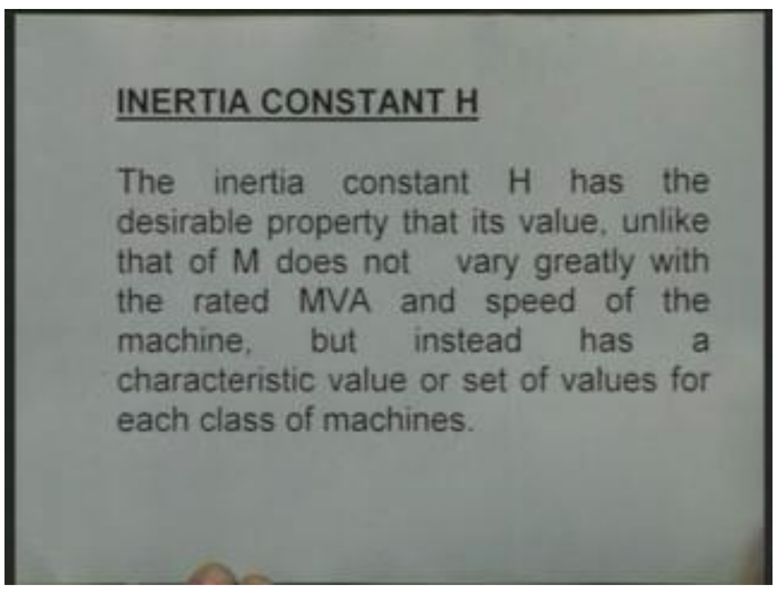


The inertia constant H has the desirable property, that its value unlike that of M does not vary greatly with the rated MVA and speed of the machine but instead has a characteristic value or set of values for each class of machines. I had very clearly, very in particularly told you last time that the coefficient M varies over a wide range depending upon the size of the machine, speed of the machine, type of the machine.

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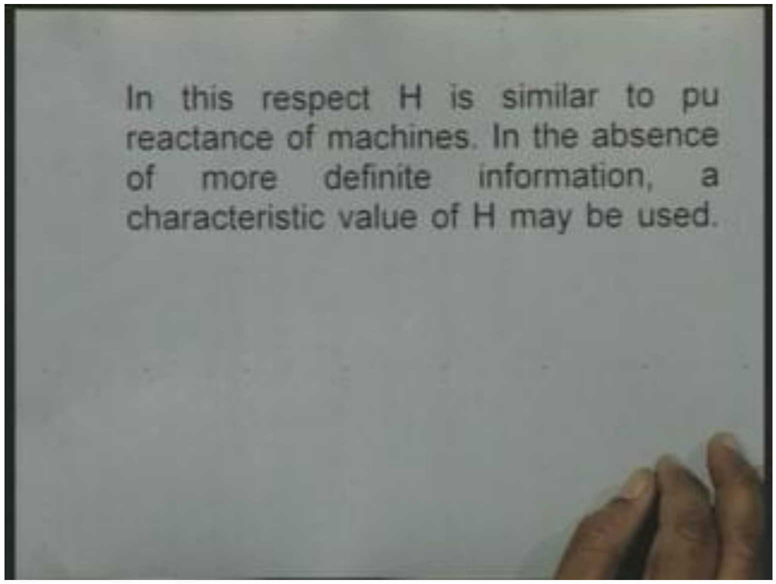


However, the inertia constant H has some characteristic values and in that respect H is similar to per unit reactance of machines as you all know that the per unit reactance of the synchronous machines have some characteristic values. Similarly, the inertia constant H also has certain characteristic values and sometimes in the absence of the specific knowledge about inertia constant H, we can assume some characteristic value and do our calculations, manufacturers specify or provide the value of H on the MVA rating of the machine.

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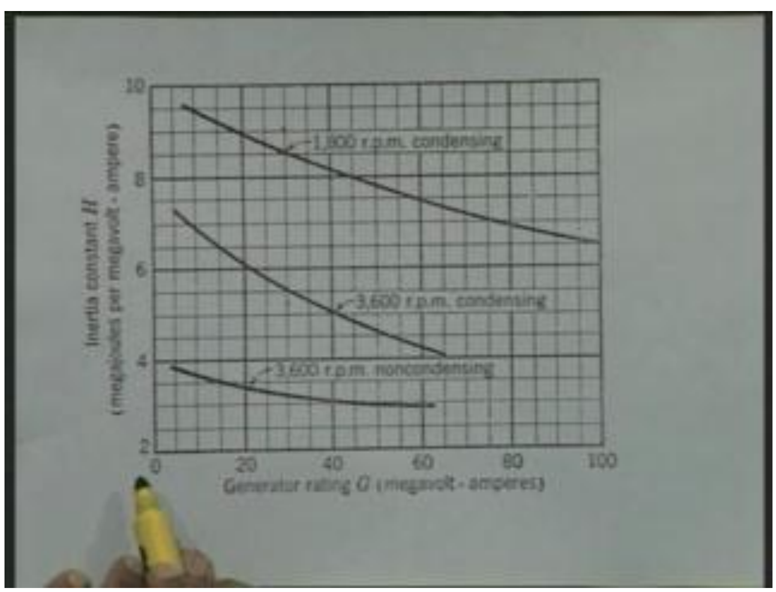

This diagram shows the typical value of the inertia constant H for turbo generators. On this axis, we have marked the rating of the machine in MVA and on this axis we have marked the inertia constant H in mega joules per MVA. Now machines, where turbo generators are of different type and different speeds, if you take a turbo generator whose lateral speed is 800 rpm a powerful machine then the variation of the inertia constant is shown by this graph, that is when the rating varies as something like about 10 mega, 10 MVA to say 100 MVA, the variation of H is in the range starting from about 9.5, it goes to 6.5.

Similarly, for a turbo generator whose speed is 3600 rpm , the variation is seen in this graph, it varies from 4 to as much as 7 or 7.5. Another graph which is shown here is the known condensing type of steam turbines, the inertia constant again varies but over a certain range.

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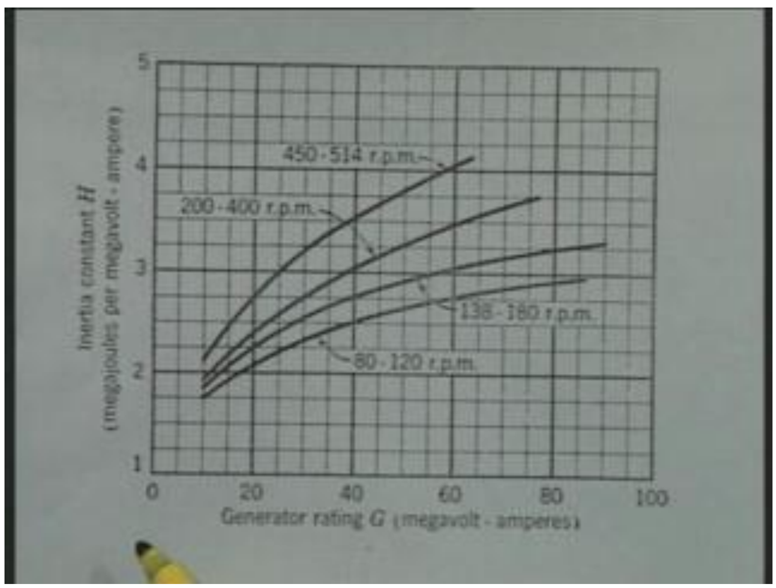


Now this graphs shows the characteristic value of inertia constant H for a hydro generator. Again on the x axis, we have marked the rating of the machine in MVA and y axis, the inertia constant H in mega joules per MVA. Here, you see actually that again you find that the machines having different speeds do have different value of inertia constant H, it increases with increase in the MVA rating of the machine but still you can see actually it lies in a certain range.

Now I would like to know from you what this inertia constant H is whether it is the inertia constant of the prime over or whether it is inertia constant of the rotor of the synchronous generator or it is inertia constant of the prime over and rotor grouped together, who will reply this? The question I imposed is, that I am talking of inertia constant H for the synchronous machine. This inertia constant whether it corresponds to the total inertia constant of the turbine plus generator rotor or only generator rotor or only turbine.

Yes you are correct, the inertia constant is the total inertia constant of the system that is for the rotating body okay. Now, here I will just give you a small example suppose in a power plant we have two machines okay of different ratings and the information which is provided to us is about the inertia constant on the MVA rating, if this two machines are tightly coupled. So that they form a coherent group, coherent group means the machines will swing together then we can the place this machine by equivalent machine.

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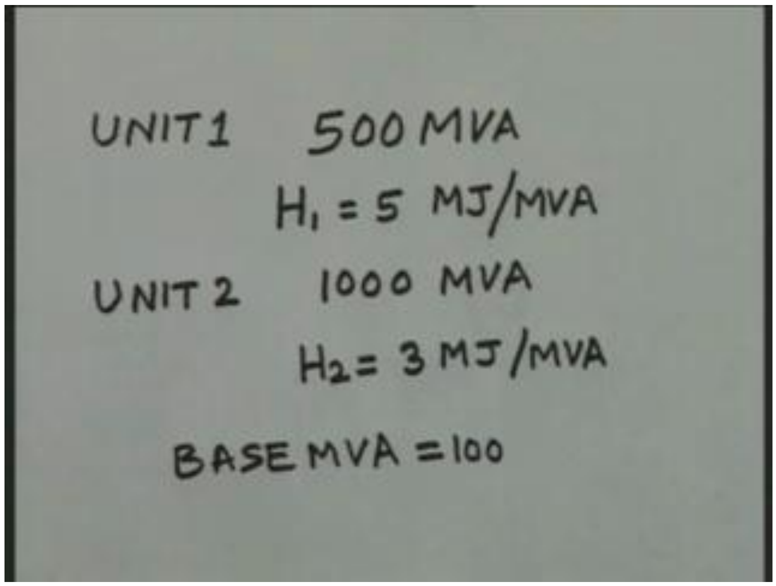


Now just take the example, let us say that unit 1, there are two units are there in the plant, unit 1 is 500 MVA and its is a constant $H_1$ is specified as 5 mega joules per MVA, unit 2 7000 MVA its inertia constant say $H_2$ is 3mega joules per MVA because the manufacturer provide the information about this H constant on its on MVA rating okay.

These two machines are in the same plant, we want to find out the equivalent inertia constant of the2 machines, how do we compute the equivalent inertia constant for that, we need one more information that on both MVA basis. We want to find out the equivalent inertia constant. Let us assume that we want to find out the equivalent inertia constant H on base MVA equal to 100 okay. Then the calculations proceed in a straight forward manner, first we find out the total kinetic energy stored in the two machines, that is total kinetic energy will be equal to, take the basic definition for the first machine it is going to be H that is 5, MVA rating is 500 plus second machine H is 3 and MVA rating is 1000. This comes out to be 5500 mega joules.

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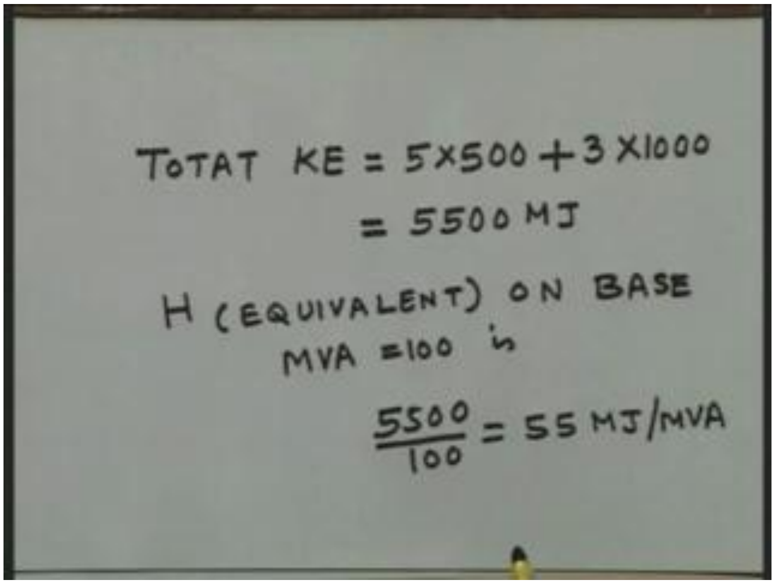


Therefore, "H" I can say equivalent on base MVA equal to 100 is 5500 divided by 100 because base MVA is 100, it comes out to be 55 mega joules per, this is the way one can find out the equivalent inertia constant of a number of machines which are running in the same plant and are coherent. Now before we talk further about the different types of stability, I would like to tell the the commonly made assumptions for stability analysis.

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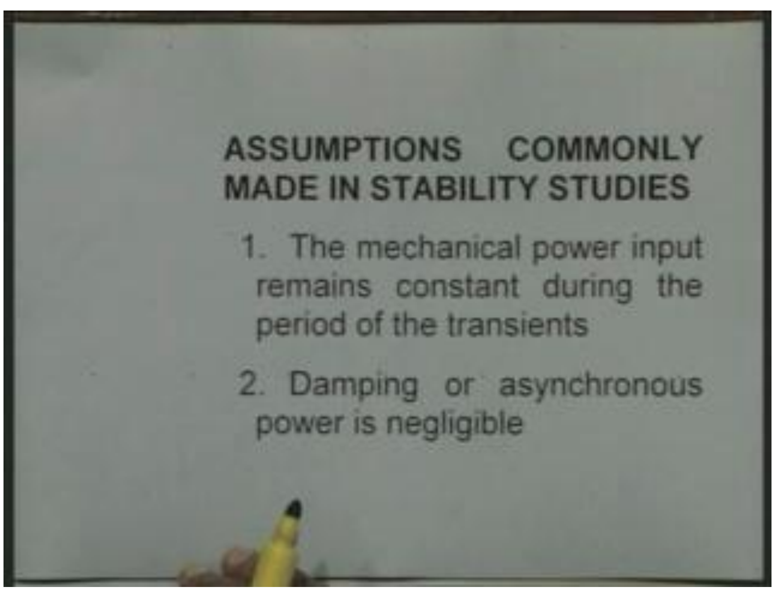

The first assumption which is very commonly made is the mechanical power input remains constant during the period of the transients, if you look into the swing equation, our swing equation is 2H divided by omega s d square delta by dt square equal to $P_m$ minus $P_e$ okay. But this assumption which I am telling you here is that this $P_m$ will be assumed to remain constant during the transient okay.

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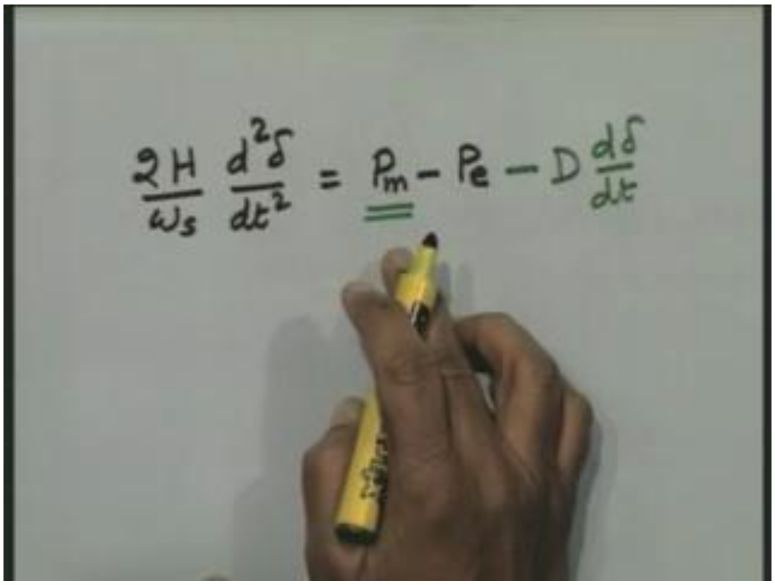

The logic for making this assumption is that in any any prime over whether is a stream turbine or is a hydro turbine right, the mechanical power input is governed by governor and by supplementary control action which you normally call a set point control. Now the governors sense the speed deviation and actuate the turbine walls. So that more steam enters when the speed drops and when speed increases the walls will be closed to decrease the speed decrease the speed by decreasing the steam input to the machine.

In practice, the speed deviation is very small during the transient stability analysis until unless the machine looses synchronism, the machine deviation is very the speed deviation is very very small and therefore governors do not act okay and therefore, this assumption is a very very valid assumption in all stability studies. We will assume this $P_m$ to remain constant during the transient period.

The second very important assumption is, we neglect damping or asynchronous power in the stability studies and you can say the damping or asynchronous power is negligible. In fact when the machine is in dynamic condition, it develops a synchronous power as well as asynchronous power. This asynchronous power results into damping, now the swing equation which I have derived, in this swing equation we have not accounted for damping.

Suppose, I want to modify this swing equation including damping aspect then I have to add one more term here and that becomes minus D times d delta by dt. The damping torque will always act in a direction opposite to the dragging torque, okay. So that is the accelerating torque at any instant of time will now becomes $P_m$ minus $P_e$ minus D times d delta by dt but to simplify the analysis we ignore this damping term and therefore, the second assumption is the damping or asynchronous power is negligible.

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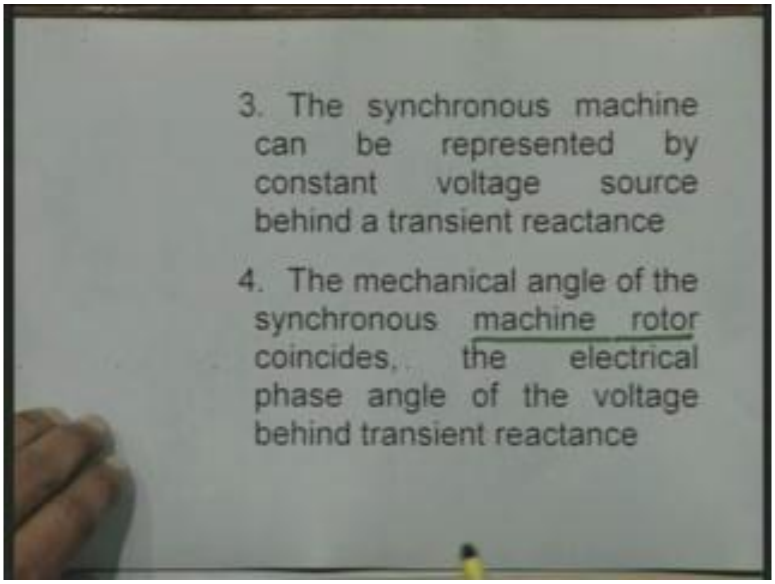


The third assumption which is made the synchronous machine can be represented by constant voltage source behind a transient reactance. This assumption is, is not very truly applicable however, for classical stability studies when you make this assumption the results obtained are not very far away from the actual results. As we will see when we talk about, the detail model of the synchronous generator right, we can represent this synchronous generator model in more detail while for performing classical stability analysis, this assumption can be made and this results into a very simplified model or simplified representation of synchronous generator.

However actually in our studies, we will develop detail mathematical model of the synchronous generator and this assumption will need not be made at that time. Another very important assumption which is made is the mechanical angle of the synchronous machine rotor coincides with the electrical phase angle of the voltage behind transient reactance, okay as here we have assumed that a synchronous machine can be represented by a constant voltage behind transient reactance okay and further assumption which is made is the phase angle of this voltage coincides with the rotor angle

The last and the most important assumption here is the synchronous power may be calculated from a steady state solution of the network to which the machines are connected because in our swing equation, we have to compute the Pe , that is the electrical power output from the machine this is the synchronous power okay and for computing the synchronous power, we assume that the network is in steady state okay, although when the rotors of the synchronous machines are oscillating right, the network is not in steady state in the real sense that is the voltage and the frequency also is deviating slightly depending upon the but since this quantity is very small as compared to the power frequency and the computations can be conveniently done by assuming that the network is in steady state condition.

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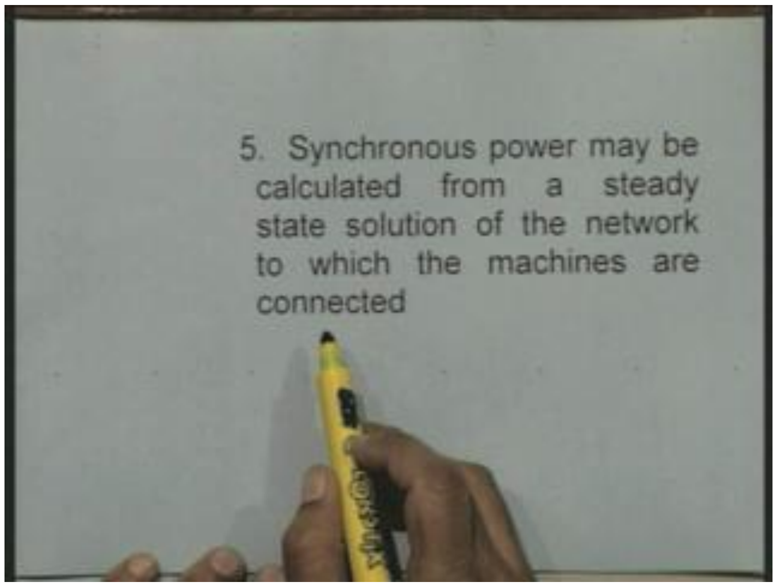


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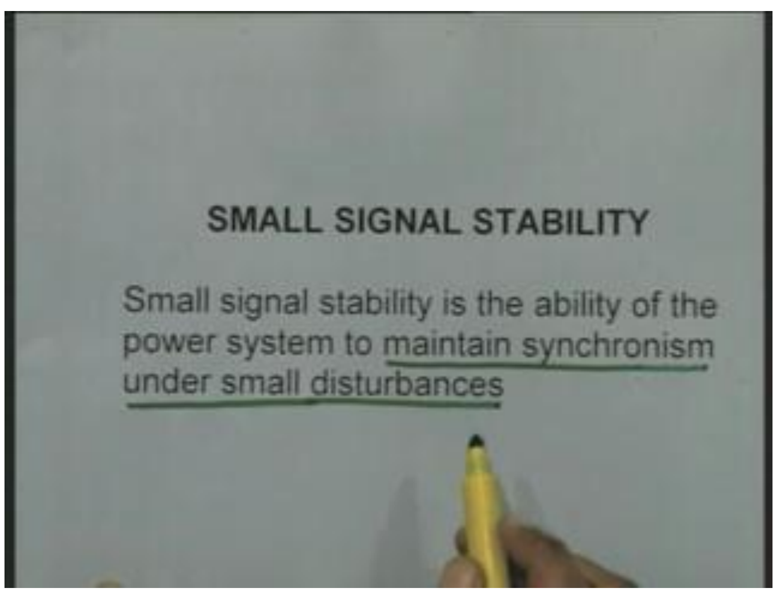

Now while classifying the power system stability we have classified into 2 categories angle stability and voltage stability, angle stability was further classified into different categories. Now, we will just look quickly, how we define the small signal stability the small signal stability is the ability of the power system to maintain synchronism under small perturbations, this is a important point, that is the machine or synchronous machine has the ability to maintain synchronism, this is primary requirement for stability under small perturbations or small disturbances.

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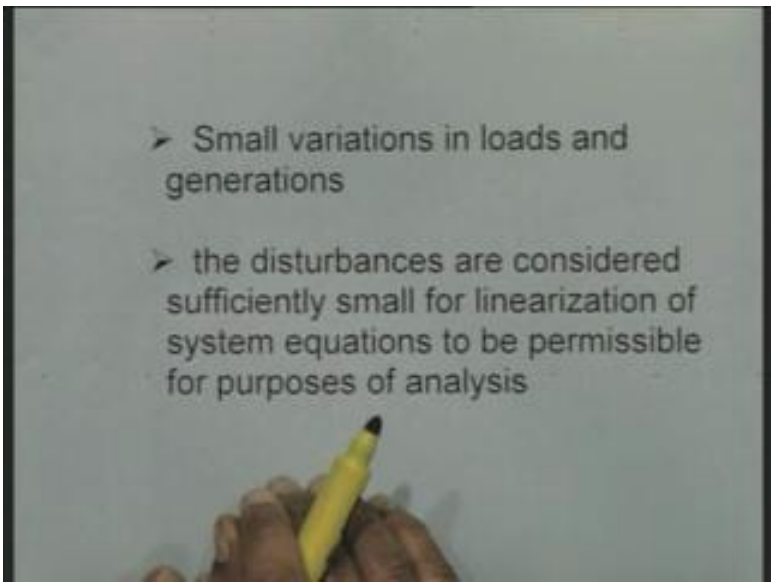


The small disturbances are small variation in loads and generations, any power system loads keep on changing continuously therefore this is one example of small disturbance, the second important aspect here is the disturbances are considered to be sufficiently small for linearization of the system equations to be permissible for purpose of analysis. As I have told you earlier the swing equation is a non-linear differential equation, okay but when we consider small perturbations this equation can be linearized around a nominal operating condition.

Once the equation is a linear equation, we can make use of linear control theory as well as we can get the closed form solution of the differential equation once it is a non-linear differential equation I have to solve by applying numerical technique however, if the differential equation is a linear differential equation, we know the closed form solution for the equation and once the differential equations are linear we can use a linear control theory to design controllers for the system. Now here, I would like to take one example how do we linearize a non-linear differential equation.

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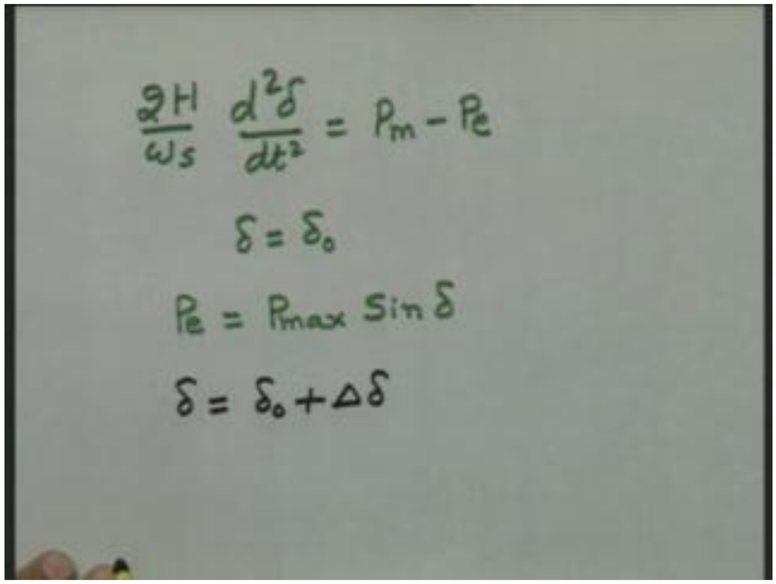


Our non-linear differential equations which we will consider is, same as the swing equation derived okay. Now, when I say that we linearize this equation around a nominal operating condition or point. The nominal operating condition here can be characterized by the initial value of the angle delta. Let us say, machine is operating where delta is equal to delta naught and we will assume here that $P_e$ is the electrical power is given by this equation $P_{max}$ sin delta okay.

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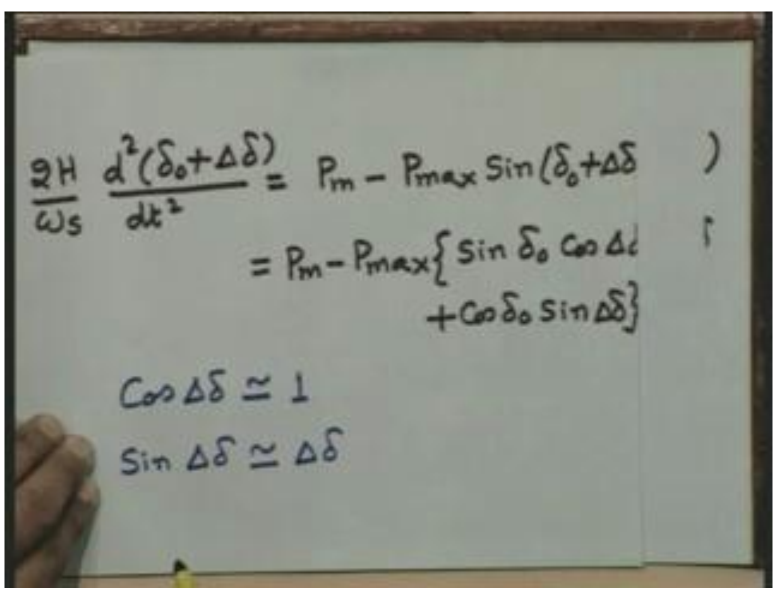

Now under steady state operating condition $P_m$ is equal to $P_e$ that is $P_{max}$ sin delta O, that is I substitute the value of delta equal to delta O, that will give me the electrical power output and under steady state condition these 2 powers are equal. Now what we do is that we give an incremental change or we make an incremental change in delta, let us say that delta is equal to delta naught plus delta okay.

Let us substitute, this in our equation, swing equation it becomes 2H upon omega s d square delta naught plus delta divided by dt square, $P_m$ we assume to be constant it remains $P_m$, $P_{max}$ sin I put here delta naught plus delta, okay. Now you expand this equation and write down $P_m$ minus $P_{max}$ sin delta O cos delta, okay plus plus cos delta O, sin delta okay. Now let us look the at this equation and since we know that, delta delta is a incremental change, small one.

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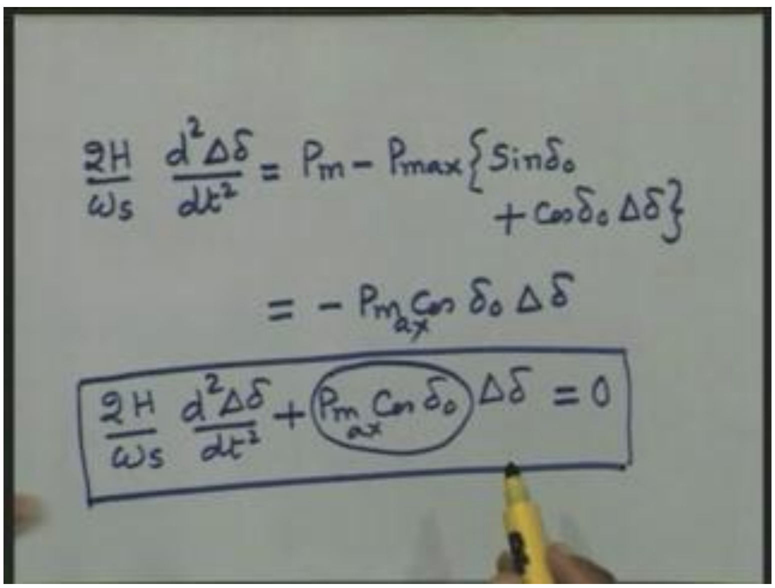


So that cos delta delta is very nearly equal to 1, sin delta delta is very nearly equal to delta delta good okay. We will make this substitution in this equation now let us write down what is the value of this derivative. The derivative of delta O is 0 and therefore this derivative term becomes d square delta delta divided by dt two equal to $P_m$ minus $P_{max}$ into sin delta O cos delta delta we have made it one, okay and I put plus here plus cos delta O delta delta

Now $P_m$ minus sine $P_{max}$ sin delta O, $P_{max}$ sin delta O is equal to $P_m$ therefore this becomes now equal to minus $P_m$ cos delta O delta delta okay or now I can write down my differential equation as 2H upon omega s, d square delta delta by dt square plus $P_m$. $P_{max}$ right, we will write in $P_{max}$ here cos delta O, this is also $P_{max}$ delta delta. This differential equation is a second order linear differential equation, do you agree, why it is second order linear differential equation?

Because, here we do not have delta delta or any term, it is trigonometrical term or non-linear term. So far this term is concerned, this coefficient is concerned this is a constant because $P_{max}$ is known, delta O is known $P_{max}$ cos delta O is known. $P_{max}$ cos delta naught is known as the synchronizing power coefficient, this term is known as synchronizing power coefficient and may be denoted by the symbol Sp, that is Sp is equal to P max cos delta naught. So that we can write down the swing equation as 2H divided by omega s $d^2$ delta delta by $dt^2$ equal plus Sp delta delta.

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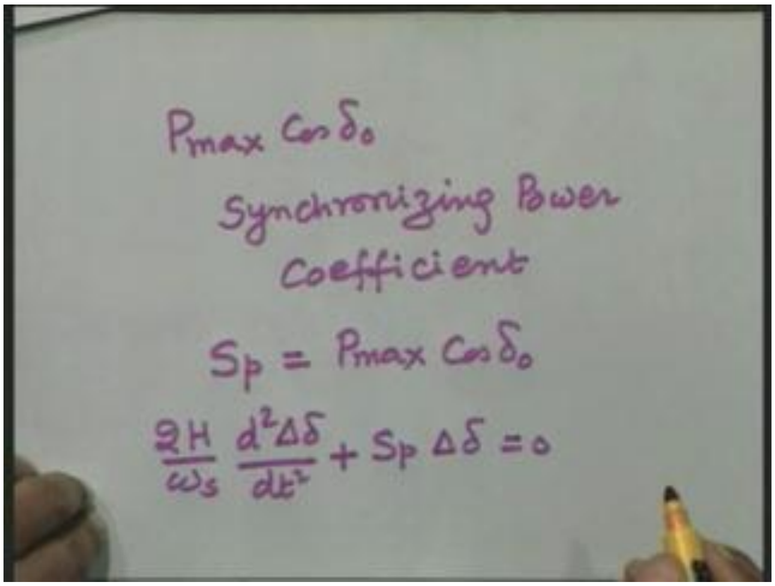


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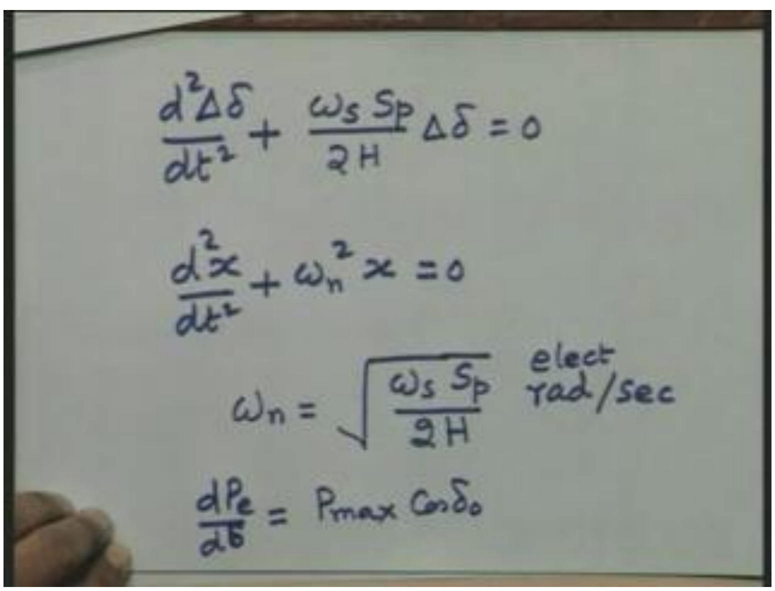

Further this is generally written in the form, sorry is written in the form d square delta delta by dt square plus omega s, $S_p$ divided by 2H delta delta equal to 0, in case $S_p$ is positive the solution of this equation will be similar to, similar to the solution of a simple harmonic motion. It is going to be a sinusoidal, in case $S_p$ is negative then it gives you a exponentially increasing value of delta as a function of time. Therefore primary requirement for the system to be stable here stable under small perturbations is that this synchronizing power coefficient should be positive.

Now, when we write the equation for simple harmonic motion the equation is written as d square x by dt square plus omega n square x equal to 0, where omega n is the natural frequency of oscillation. Now here by comparing the coefficients, we can write down that omega n is equal to square root of omega s, $S_p$ divided by 2H. This will be in radians electrical radians, we call it electrical radians per second.

Now further, you can see that the synchronizing power coefficient is the slope of the power angle characteristic at the operating condition, that is if you find out $dP_e$ divided by d delta it comes out to be $P_{max}$ cos delta O and therefore dP by d delta is the slope of the power angle characteristic and our primary requirement is that the synchronizing power coefficient should be positive for the system to be stable under small disturbances, and that is why when I talk to you about the stable and non-stable operating points, where this was our power angle characteristic P versus delta.

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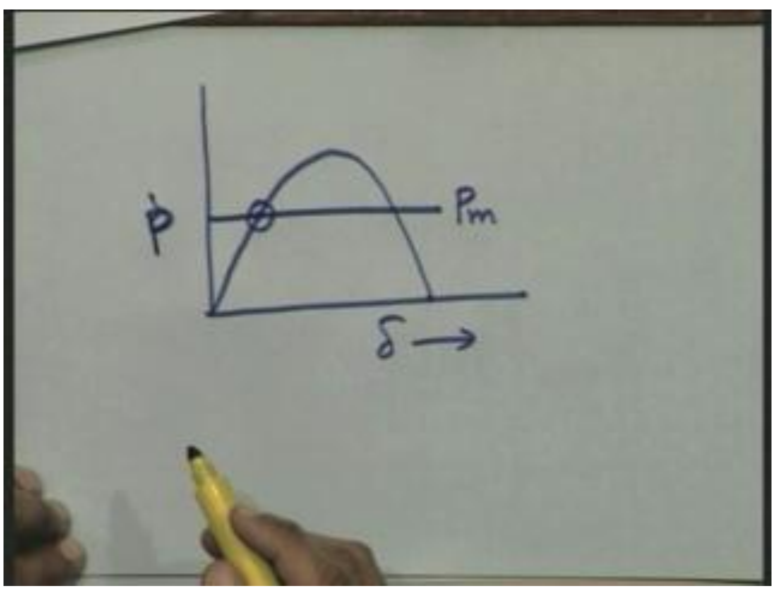


This was your mechanical input line, these are the operating condition and this is the stable operating condition because at this point the slope of the power angle curve is positive. I shall take one small example to find out what is the frequency of oscillations, which we normally come across.

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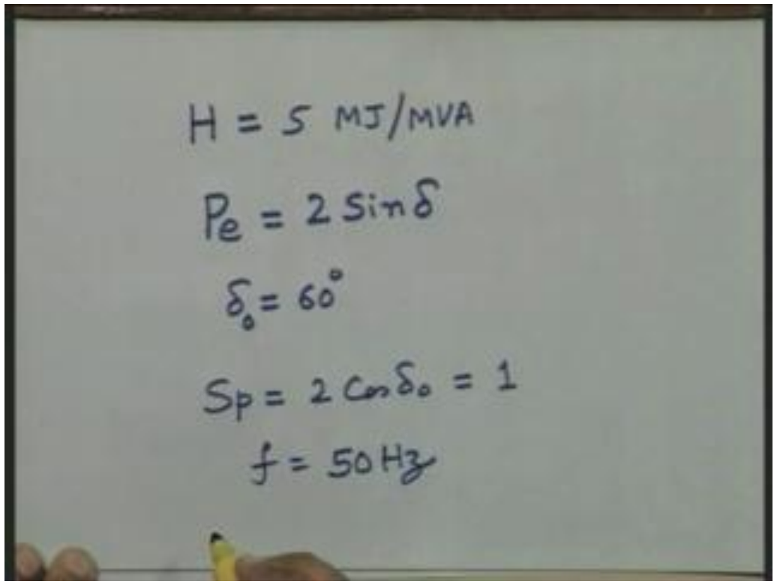


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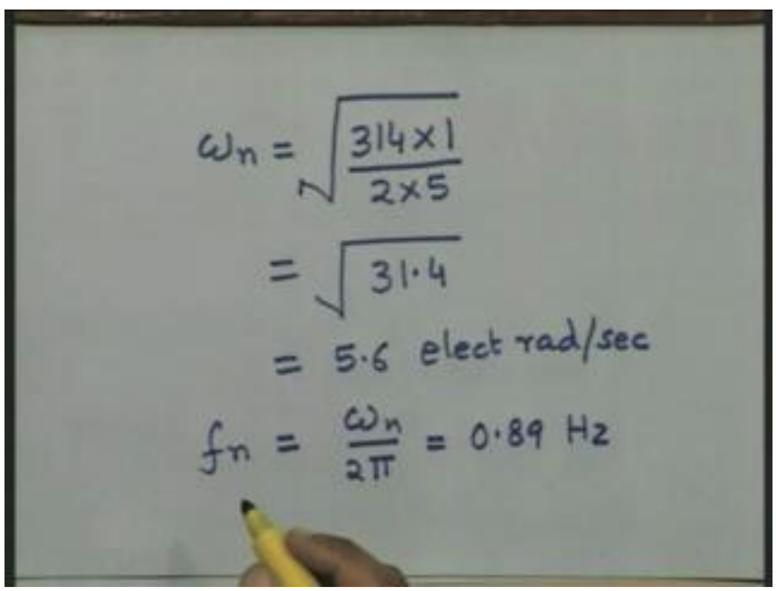


Let us take a case where a synchronous machine as H equal to 5 mega joules per MVA. Further we assume that Pe equal to 2 sin delta expressed in per unit 2 is your Pmax now. Let delta equal to delta naught equal to 60 degree, okay. Then Sp comes out to be equal to 2 cos delta naught which is equal to 1 okay. Let us take system frequency as 50 hertz, f equal to 50 hertz, omega n natural frequency of oscillation of the rotor can be computed by substituting the values of small s, omega s is 314 for 50 hertz system, $S_p$ has come out to be equal to one synchronizing torque coefficient 2 into 5H is 5.

This quantity is square root of 31.4, if you calculate it comes out to be equal to 5.6 electrical radians per second. Frequency of oscillation in hertz can be obtained as omega n by 2 phi which comes out to be in this case 0.89 hertz. This example is given here to give you the frequency of oscillations we come across, system frequency is 50 hertz but when the rotors oscillate the frequency of oscillation depends upon initial operating condition which determines the synchronizing power coefficient, it depends upon the initial constant H of the machine and it also depends upon the system frequency okay and it is of the order of 1 hertz in this case but it does vary depending upon the value of this parameters but the variation is in the range of point 5 to 2 hertz normally.

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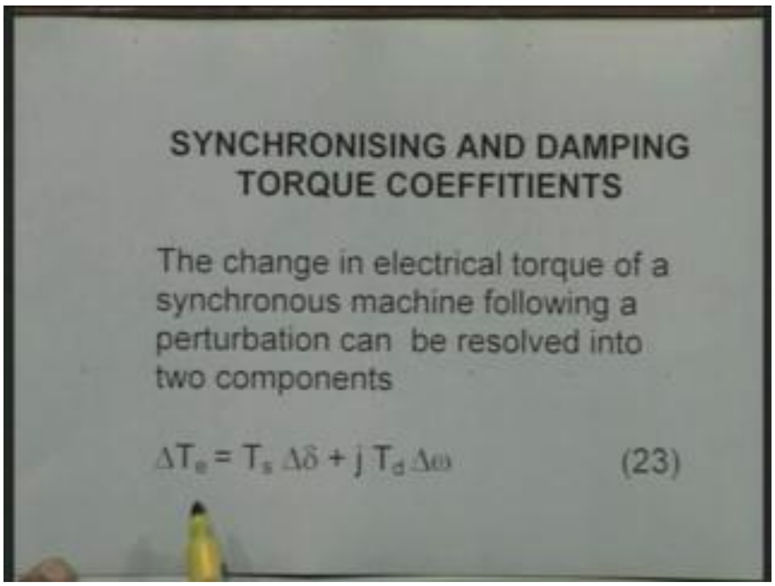


Now we have defined the small signal stability and small signal stability is the ability of the synchronous machines to remain in synchronism under small perturbations. Now when the synchronous machines rotors are oscillating it develops an electrical power delta T, when the rotors are oscillating it produces a electrical torque we call delta T. This torque can be decomposed into two components, one is called synchronizing torque, another is called damping torque.

The synchronizing torque is in phase with speed deviation, I am sorry there is a correction it is in phase with phase with angle deviation delta delta and the damping torque is produced in phase with speed deviation. when the rotor is oscillating and if the oscillations are sinusoidal in nature then we can represent this torque as a phaser in a plane where, delta delta is the x axis and delta omega is the y axis

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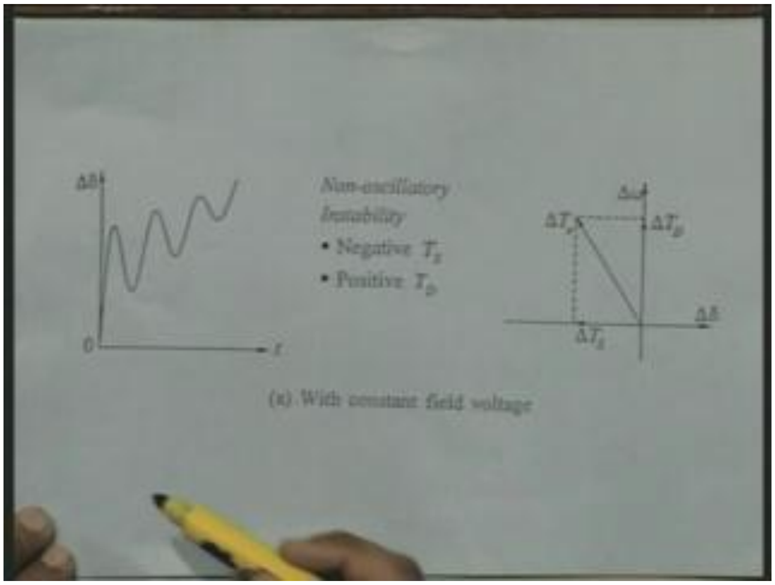


Look here, actually in this diagram the plane is x axis is marked as delta delta y axis is delta omega and in case, the delta T the change in electrical torque lies in the second quadrant then you have 2 components, 1 component is damping torque component, another is the synchronized torque component. Damping torque component is positive while synchronizing torque component comes out to be negative

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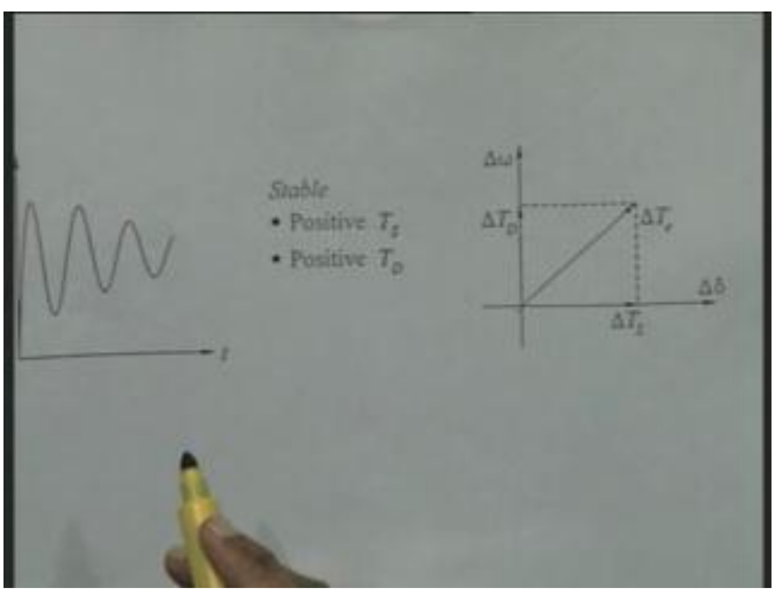

In case delta T lies in the first quadrant of the delta delta delta omega plane, then both synchronizing torque and damping torques are positive for stability of the system both synchronizing and damping torque should be positive. Okay I will discuss these aspects slight in more detail when we talk about the small signal stability problem.

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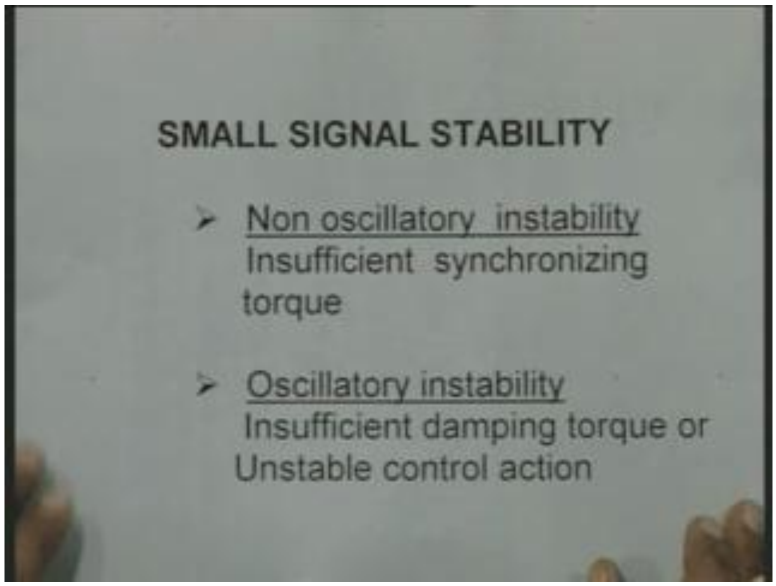


The small signal stability can be classified into 2 categories, one is called non oscillatory instability not a stability instability, it can be non oscillatory instability this occurs because of insufficient synchronizing torque, another is oscillatory instability this is due to insufficient damping torque or by due to unstable control action.

Now we can see the response of the system under small perturbations, under 2 conditions, one is that synchronous machine has constant field voltage there is no voltage regulator acting, another case is where the synchronous machine is provided with a voltage regulator where we call this as one as a constant field voltage another is with excitation control . Now suppose a situation is like this where you have a negative synchronizing torque while positive damping torque the response of the rotor to small perturbation will be of this form, starts from 0 and these oscillations are oscillations are like damp because damping is positive but since synchronizing torque coefficient is negative it increases monotonically. Therefore, this represents a case of insufficient synchronizing torque.

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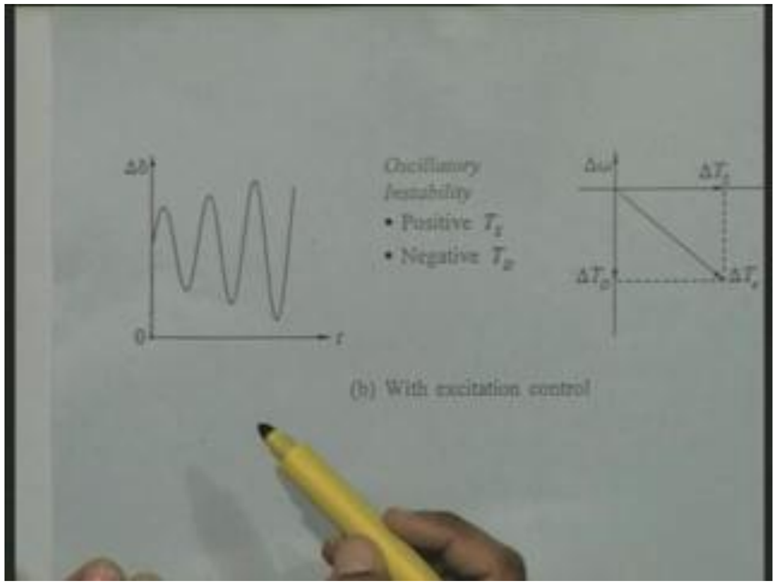


Here is another case where synchronizing torque coefficient is positive but damping torque coefficient is negative, the response of the rotor or del to perturbation starts from 0, you can see actually that the oscillations are growing. Okay, therefore this is the case of oscillatory instability and this is primarily due to lack of damping torque or here I can say that the system has negative damping. In any power system, we will come across different modes of oscillations, a power system is a large interconnected system okay.

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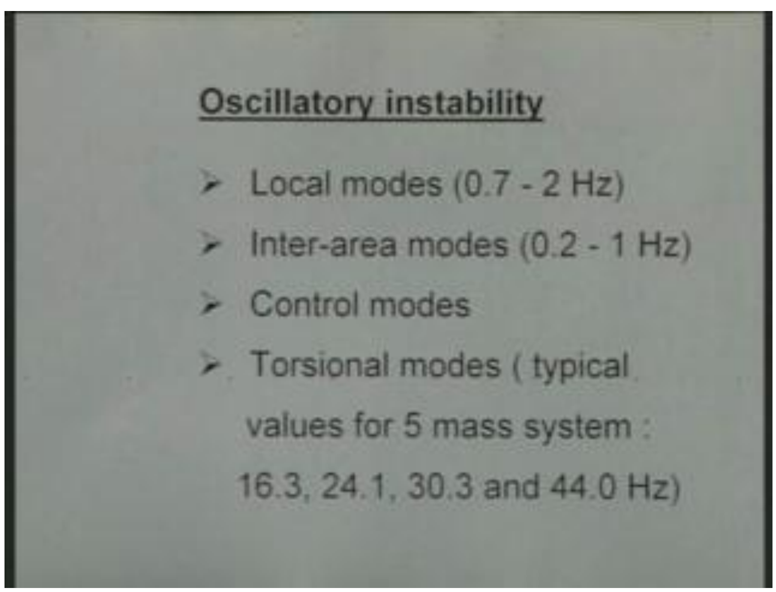

We come across a local modes, inter area modes, control modes and torsional modes. The local modes are the one where a synchronous machines located in a power plant oscillate with respect to rest of the system and the frequency is in the range of .7 to 2 hertz. We have been taken the example we found that frequency came out to .89 okay. Similarly, another case comes where a group of machines of a power system oscillate with another group of machines connected by a weak tie line and these group of machines will form one area another group forms, another area and therefore we say this is inter area oscillation and when the group of machines are considered the inertia constant H is large and the frequency of oscillation is in the range of point 2 to 1 hertz.

So for control modes are concerned the frequency of the different modes depend upon the actually control phenomena therefore, no specific frequency is specified. The torsional modes is another phenomena which occurs in the steam turbine and the different frequencies which we come across are for typicals mass system it comes out be 16.3, 24.1, 30.3 and 44 hertz.

We will devote about two lectures to deal with about the torsional modes in detail at that time it will be clear, how the different masses which are mounted on the same shaft will result into different modes of sub synchronous oscillations and I will conclude here, my today is presentation by summarizing what we have done.

We have addressed to 3pecific aspects, one is inertia constant H its importance, second aspect we have considered here is the basic assumptions which are made instability analysis and the third aspect we have studied is the small signal, stability, its definitions ,we have also talked about how to linearize a non-linear differential equations and how to compute the natural frequency of oscillations through an example. We will cover rest of the aspects of the introduction to power system stability in the next lecture.