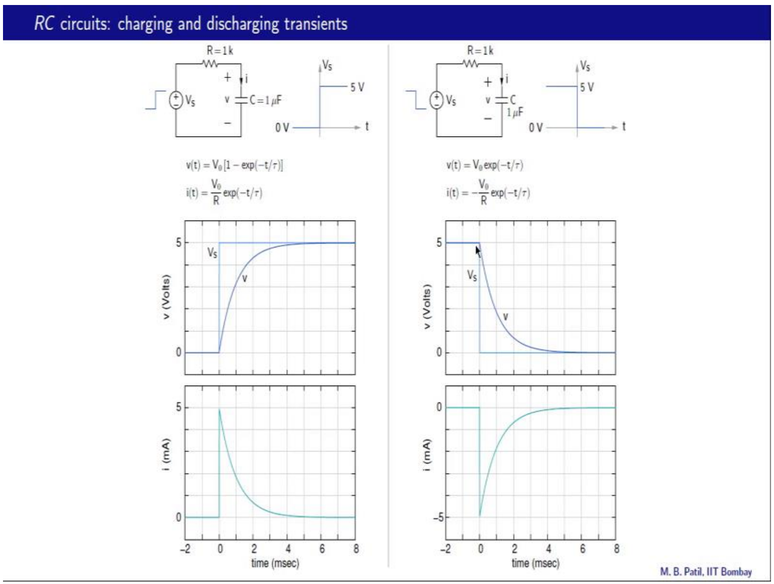
Basic Electronics
Prof. Mahesh Patil
Department of Electrical Engineering
India Institute of Technology, Bombay

Lecture - 09
RC/ RL circuits in time domain (continued)

Welcome back to Basic Electronics. In the last lecture we obtained analytic expressions for the capacitor, voltage and current, for a series RC circuit with the step input voltage; we considered both charging and discharging transients. In this lecture we will look at the graphs for these quantities, and learn a lot more about these transients. We will then look at how to handle an RC or RL circuit with the piece wise constant source and follow that up with an example. Let us start.

(Refer Slide Time: 00:56)



These are the plots the input voltage is given by this step V form the light blue curve, and the capacitor voltage is given by this curve here the dark blue curve; and let us check whether this does correspond to the expression for the capacitor voltage that we have over here; what is v (t) at t =0? At t = 0 , this term = 1. So, therefore, we have V_0 (1- 1) = 0. So, at t = 0 ,we expect v (t) = 0. What about t = ∞ ? At t = ∞ ,this term = 0 and therefore, we get v = V_0 and that is what we see over here and how long does the transient take? As we are seen earlier the transient takes 5 10 constant.

Let us check whether that is happening, we have R =1 k, C = 1 μf. So, the time constant = 1 k ***** 1 μ = msec . So, we expect the transient to last for 5 * the time constant that = 5 msec; 5 msec is over here and we observed that the transient does vanish at about that time and there after the circuit reaches steady state, and there is no perceptual variation in either the capacitor voltage or the capacitor current beyond that point.

What about the current? the current is given by this expression, at t = 0 + this term is 1 and therefore, the current =V_0 / R, our V_0 = 5 v, R = 1 k. So, this number = 5 v/ 1 k or 5 million and that is what we observe over here. At 0 -, the current = 0; at 0 +, it is 5 millions. As t → ∞ , this term goes to 0 and therefore, the current is expected to go to 0 and that is indeed what we observe over here.

Let us now make a 5 observation, first the capacitor of voltage is continuous and that is of course, expected as we explained earlier, because if there was a discontinuity in the capacitor voltage that would need to very large currents, which would not satisfy our circuit equations. On the other hand there is no subscription on the capacitor current, and we do observe a discontinuity at t = 0. Second observation the capacitor current is always positive as is also obvious from this equation, this term $e^{-t/\tau}$. e is always positive, and V_0 and R are positive constants and therefore, the capacitor current is always positive.

Now, since the capacitor current = c d V / dt, a positive capacitor current means a positive d V dt and therefore, we expect the capacitor voltage to always keep rising like that. Eventually of course, the capacitor current becomes 0 and therefore, the capacitor voltage becomes constant. If the current = 0 ; that means, d V dt = 0 ; that means, V becomes constant as you know over here in this graph. Since the current is large in the beginning, we have a larger slope d V dt here; now the current goes on decreasing so this slope also goes on decreasing and eventually of course, the current becomes 0 and the slope becomes 0 ; that means, V becomes constant.

Finally notice that the shape of v (t) or i (t) is one of the 2 shapes we saw earlier in the graph of F (t) = A $e^{-t/\tau}$ + B; if you do not remember that you can go back a few slides and make sure that these 2 shapes indeed follow what we predicted in that slide over there.

Let us now look at the discharging transient this is the circuit V s goes from 5 V to 0 V, at t = 0 and the expressions for v (t)and i (t) which we derived earlier are reproduced over here; $v(t) = V_0 e^{-t/\tau}$, $i(t) = -V_0 / R (e^{-t/\tau})$.Here are the plots this is our source voltage V s it starts at 5 V it has been 5 V for a long time, and then at t = 0, it abruptly changes to 0 and then remains 0 thereafter. This dark blue curve is the capacitor voltage as a function of time and this graph here is the capacitor current, notice that the capacitor current is negative.

Let us check whether these results the plots here correspond to the equations that we have derived. Let us look at the capacitor voltage first. At t = 0, this term is 1 and therefore, $v(t) = V_0$, $V_0 = 5$ V and therefore, the capacitor voltage = 5 V. As $t \to \infty$, this term becomes 0 and therefore, the capacitor voltage = 0 as in this graph. And how long does this take? We expect that to take 5 time constants R = 1 k here, C = 1 µf as in the previous case and therefore, the time constant = 1 msec. 5 τ = 5 msec. therefore, at 5 msec, we expect the transient to vanish and that is what we observe over here V becomes constant after about 5 msec.

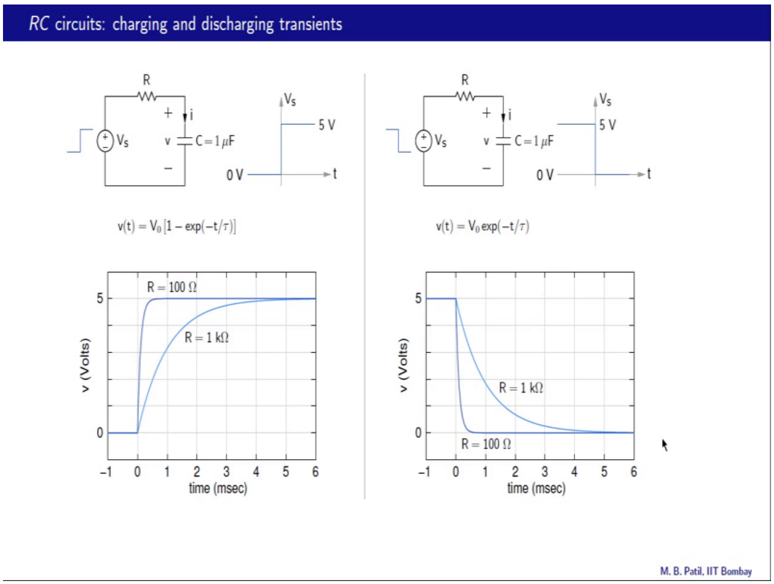
Let us look at the current plot now, here is the expression for the current and remember that this is valid for t > 0. Now at t = 0 + this exponential factor = 1 and we have $I(t) = -V_0 / R$, $V_0 = 5$ V, R = 1 k and therefore, we have $I(t) = -5$ V / 1 k = - 5 mA and that is indeed what we observe in this graph here. As t increases this exponential factor decreases and therefore, we expect the capacitor current to decrease in magnitude, it will of course, remain negative because of this minus sign.

Eventually as $t \to \infty$, this factor will become 0 and therefore, the current will become 0 and that is what we observe in this graph; as t increases the current decreases in magnitude and eventually it becomes 0. As we would expect the capacitor voltage is continuous, but the current is discontinuous, there is a discontinuity at t = 0 and this observation is similar to what we saw in the charging case.

Now, the capacitor current in this case is always negative eventually of course, it becomes 0, and what does that mean? Since the capacitor current i = c d V dt negative current means a negative d V dt; that means, the slope of the capacitor voltage versus time graph would always be negative and that is what we observed over here.

So, therefore, the capacitor voltage keeps decreasing eventually of course, the current becomes 0, d V dt becomes 0 therefore, the voltage becomes constant. So, there is a lot to learn from graphs and you would have heard that a picture is worth a thousand words, and that is also true about graphs. A graph is packed with information we only need to look for it, and there are several features that we should look for and we should make sure that they are compatible with the analytical understanding or the equations that we have.

(Refer Slide Time: 10:42)

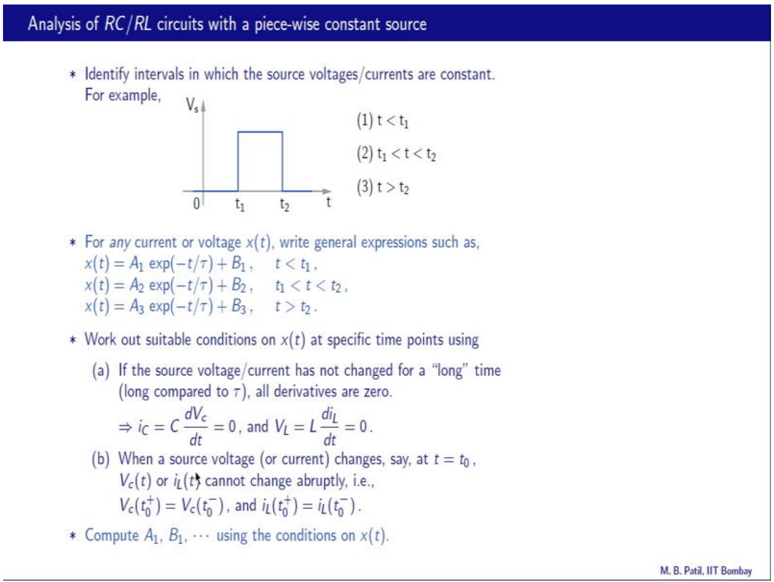


Let us now look at the effect of the time constant on the capacitor voltage transient; first let us consider the charging case in which the source voltage changes from 0 v to 5 v abruptly at t = 0, and as a result the capacitor voltage also changes it follows this equation which we are derived earlier. Now with R = 1 k, the time constant = 1 k * 1 µ = 1 msec and in about 5 time constants that is in 5 msec we expect the capacitor voltage transient to vanish and a steady state to be established; and that is what we observe in this plot, this sky blue curve corresponds to R =1 k, in the beginning the capacitor voltage =0 v for a long time. at t =0 ,it starts rising towards 5 v and in about 5 msec. this time point here it reaches almost 5 v and does not change after that.

Now, let us look at the second case namely R =100 Ω = 0.1 k, what is the time constant in this case? It is = 0.1 k * 1 µ = 0.1 msec. Now in this case we expect the transient to last for 5 * 0.1 msec, that = 0.5 msec and that is indeed what we observe in this plot. This is 1 msec 0.5 msec is somewhere here and we observe that for R =100 Ω, we reach the steady state in about 0.5 msec and after that point the capacitor voltage does not change.

Let us now consider the discharging case. So, V s changes from 5 v to 0 v at t = 0, and as a result the capacitor voltage also changes and it follows this equation which we have derived earlier. Now with R = 1 k ,the time constant = 1 msec and in about 5 time constants, the transient vanishes as seen in this graph this = 5 v, and at t = 5 msec, we reach steady state and the capacitor voltage does not change thereafter. For R =100 **Ω**, the time constant = 0.1 msec and in 5 τ = 0.5 msec, we reach steady state here and the capacitor voltage becomes constant.

(Refer Slide Time: 14:02)



So, far we have considered RC and RL circuits with the constant source; a voltage source or a current source. Let us now look at RC and RL circuits with a piece wise constant source, here is an example there is a voltage source here which varies with time like that, in this interval $t < t_1$ this voltage = 0, between t_1 and t_2 it is non 0 let us say 5 v, and then for $t > t_2$ it is 0 again. let us see how we can handle such a situation; clearly this is not a d c source or a constant source, but it is piece wise constant we have one piece here which is constant we have another piece here which is constant and we have a third piece here which is constant.

And in each of these intervals, we can use the expressions that we have derived that is for any current or voltage x (t) in the circuit, we can write a general expression such as $x(t) = A_1 e^{-t/\tau} + B_1$ for the first piece, that is $t < t_1$. Similarly for the second interval from t_1 to t_2, we can write $x(t) = A_2 e^{-t/\tau} + B_2$. Now this A_2 and B 2 are also constants and generally they would be different from A_1 and B_1; and finally, in this third interval that is $t > t_2$ we can write $x(t) = A_3 e^{-t/\tau} + B_3$. In order to get a complete picture, what we need to do next is to find this τ, the circuit time constant and also these constants A_1 B_1, A_2 B_2 etc.

Now, the time constant is given by R times c for an RC circuit, where R is the Thevenin resistance seen by the capacitor; for an RL circuit, $\tau = L / R$,where R is the Thevenin resistance seen by the inductor. We have already seen these formulas earlier; how do we obtain these constants? A_1 B_1 A_2 B_2 and so, on what we need to do is to work out suitable conditions on x of , which could be a current or a voltage at specific time points. For example, in this case we might look at $x(t)$ at t_1, t_2 $-\infty$ and $+\infty$.

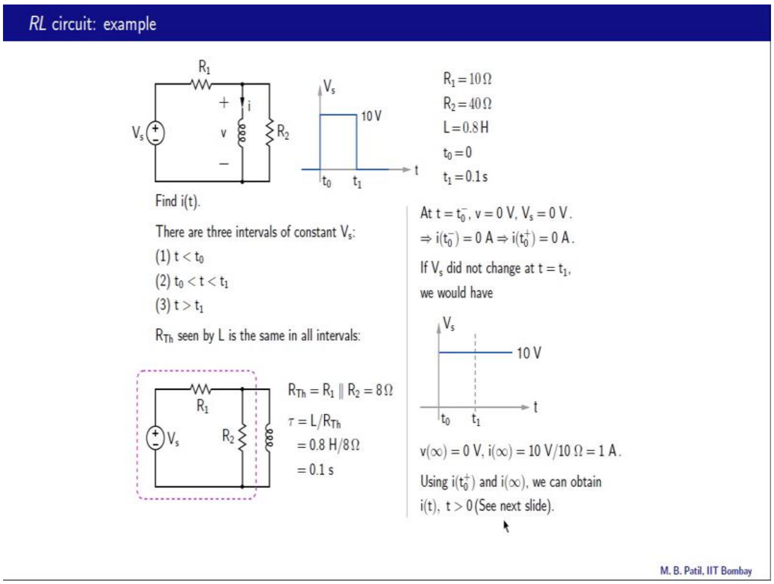
And in order to do that in order to find these conditions, we can use the following considerations; a, if the source voltage or current has not changed for a long time long compared to the circuit time constant τ, then we know that all derivatives must be 0 because all quantities in the circuit, all voltages and currents must have become constant, the circuit must be in a steady state. For example, take this piece if V_s has been 0 for a long time, then we know that when we come to t_1 minus that is just a little bit before t_1, then the circuit is in steady state and all voltages and currents must be constant; and once we know that, we know that the capacitor current which is given by $C \, dV_c / dt$ must be = 0.

So, the capacitor can be treated as an open circuit and the inductor voltage which is given by $L \, di_L / dt$ must also be= 0 because i_L would have become constant; that means, the inductor can be replaced with a short circuit since the voltage across the inductor = 0; b, when a source voltage or current changes say at $t = t_0$, in this example $t_0 = t_1$ or t_2 then the capacitor voltage or the inductor current cannot change abruptly as we have seen earlier; that means, we see at t_0^+ the capacitor voltage at t_0^+, must be = V_c at t_0^- and in an RL circuit the inductor current i_L at t_0^+ must be = the inductor current at t_0^-.

So, using these considerations, we can work out suitable conditions on the current or voltage at specific time points and then use those to compute these constants A_1, B_1, A_2, B_2 and A_3, B_3 in this example.

So, once we have tau and once we have these constants, we have a complete description for the quantity of interest.

(Refer Slide Time: 19:56)



Let us now apply the ideas that we have just learnt in the last slide to this particular example; we have an RL circuit here with 2 resistors R_1 and R_2 and a single inductor. $R_1 = 10\ \Omega$, $R_2 = 40\ \Omega$and the inductance = 0.8 henry. The input voltage is a pulse given by this graph here; $t_0 = 0$, and $t_1 = 0.1$ sec. Up to t_0 the source voltage $V_s = 0$ V. between t_0 and t_1 it is = 10 v, and after t_1, it is again 0 v and we are interested in finding this current through the inductor as a function of time.

Step number one is to identify intervals in which the input voltage is constant and from this graph you can see that there are 3 such intervals, interval 1, $t < t_0$ where $V_s = 0$ V; interval 2, $t_0 < t < t_1$ in which $V_s = 10$ V and interval 3, that is $t > t_1$, in which $V_s = 0$ V. Like we mentioned in the last slide what we need to do next is to consider each of these intervals identify the circuit time constant in each interval, write a general expression for i (t) in each of these intervals and then find suitable conditions on i (t) at suitable time points, which will enable us to calculate the coefficients, the constants involved in those expressions.

So, let us begin with the circuit time constant; now we figure that in all of these intervals the time constant does not change, because the circuit topology is the same in all 3 cases. So, let us proceed with that calculation now. What is the circuit time constant τ ? It is L / R_{Th}, where R_{Th} is the Thevenin resistance as seen by the inductor. To make this calculation a little easier let us redraw the circuit as shown here, what we have done is we have taken this inductor on the other side that of course, does not change the circuit and now we look at the rest of the circuit this circuit here from the inductor; and note that the circuit topology is the same in all 3 intervals, the only thing that is changing is the value of V_s; in the first interval, $V_s = 0$, in the second interval, it is = 10, and in the third interval, it is = 0 .

So, the Thevenin resistance seen by the inductor is the same in all 3 cases and how do we find that? We deactivate the voltage source; that means, we short this voltage source and then what happens is R_1 and R_2 come in parallel so therefore, $R_{Th} = R_1 // R_2$. So, that turns out to be = 10 // 40 or 8 Ω, then the time constant is given by L / R_{Th} / L = 0.8 H, R_{Th} = 8 Ω. So, therefore, the time constant = 0.1 sec. Next let us find suitable conditions on i (t), which will eventually enable us to find the constants involved in the expressions for i (t) in these 3 intervals. Let us consider t_0 -; that means, just before t_0, what is the situation at $t = t_0$ - ? The source voltage has been 0 for a long time, we have a steady state all currents and voltages have become constants and therefore, this V which = L di / dt = 0.

If V = 0, the voltage across R_2 = 0 that means no current can flow through R_2. So, the current path that we have is like this, and since this voltage drop = 0 ,the current $I = V_s / R_1$. Now $V_s = 0$ in this interval up to t_0 - and therefore, the current i at t_0 - = 0 like that. Now since this i is an inductor current, we know that it must be continuous; that means, i at t_0 + = i at t_0 - and therefore, we get i at t_0 + = 0 A.

Let us now obtain another condition on the current, and we will do that by assuming that this V_s is did not change; in real life of course, V_s has changed at t_1, but let us pretend that it has remained constant.

The question is does this make sense, how can we assume that V_s = 10 V in this interval when it is actually 0 V? The answer to that question is that it does make sense, as long as we are in this region up to t_1. So, what we will do is we will assume this condition, but we will consider the solution to be valid only up to this point t_1. To put it a bit informally, the inductor or the circuit does not really know that a change is going to take place at this point, and therefore the response of the circuit would be the same in this interval.

Whether or not V_s state constant or it went back to 0 at this point; with this situation let us now find this current = $t \to \infty$, what do we have at $t = \infty$? The circuit would have reached steady state all currents and voltages would have become constant therefore, this V which = L di dt would have become 0; so we have a short circuit here, and the current i would then be = V_s / R_1.

The current through R_2 of course, would be = 0 because this voltage drop = 0, at $t = \infty$. V_s would be= 10 V. So, therefore, the current I would be= $10 / R_1$ that = 10 V/ 10 Ω that = 1 A. Let us now use these 2 conditions namely i at t_0 + and i at ∞ to obtain the solution for i (t) in this interval, and we will do that in the next slide.

In conclusion we have started looking at RC and RL circuits with a piece wise constant source, in the next lecture we will continue with the RL circuit example and obtain the complete solution until then goodbye.