Basic Electronics
Prof. Mahesh Patil
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture – 07
RC/RL circuits in time domain

Welcome back to Basic Electronics. We will come across a few circuits in this course for which we will need to find the time taken by a capacitor to charge or discharge from an initial voltage V_1 to a final voltage V_2. In this lecture we will make a beginning in that direction, we will first look at the general solution for n RC or RL circuit see how to plot the solution as a function of time, in subsequent lectures we will consider some specific examples so let us begin.

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Let us now start our discussion of RC circuits with DC sources, and this situation as we will find out is very common in electronic circuits and once we have this background, we will be able to calculate for example, pulse width or oscillation frequency in a circuit. Here is the circuit a linear circuit which consists of resistors, independent voltage sources which are DC, independent DC current sources and then dependent sources such as current controlled voltage source, current controlled current source etc. To this circuit we connect capacitor and now let us see how this capacitor voltage or current evolves with time.

Since this circuit is a linear circuit, we can represent it with this thevenin equivalent consisting of a voltage source V_{Th} in series with a resistance R_{Th}, and this capacitor of course, appears as it was earlier over here. Let us now make a few important points if all sources are DC or constant, then V_{Th} = a constant and it is important to bear in mind this assumption if for example, we have a sinusoidal voltage source here or a triangular current source here then our analysis is not going to be valid.

Let us begin by writing the KVL equation for this loop, and what is that? That says that this V_{Th} must be equal to this voltage drop R_{Th} i + V like that. Now this current i that is flowing through this resistor is also the capacitor current and therefore, i = c, dv dt. So, we substitute for i C dv dt and then we get V_{Th} = R_{Th} C dv dt + v. We can rewrite this equation = dv /dt + v / R_{Th} C = V_{Th} / R_{Th} C, all we have done here is divided both sides by this vector here.

Notice that our right hand side here is a constant because our sources are DC sources and therefore, V_{Th} = a constant. Now how do we solve this ordinary differential equation? We first obtained the homogenous solution and we do that by replacing this right hand side the forcing function with 0, and then we get this equation here; we have defined τ which is called the time constant = R_{Th} C over here; this time constant τ has units of time, and let us quickly verify that what are the units of R_{Th}? It is a resistance. So, the units are Ω, Ω = V/A and A = Coul/sec. What about C? C has units of charge by voltage that is Coul / V. So, this volts cancels out, this coulombs cancels out leaving behind seconds that is the unit of time.

Let us now get the homogeneous solution; we rewrite this equation = dv / v = - dt / τ. Now we can integrate both sides, the integral of this left hand side will give us log (v) this log of course, = the base e and the integral of this part the right hand side will give = - t / τ and there is an integration constant here which = k_0. So, this equation can be rewritten as v = $e^{(-t/\tau) + k_0}$ and then we can define e^{k_0} = another constant k and therefore, we have V the homogenous part indicated here with this superscript h equal to $k(e^{-t/\tau})$

The second part of the solution is the particular solution, what is the particular solution? It is a specific function that satisfies the differential equation. In this case this is our differential equation, so we are looking for some solution, which satisfies this equation.

And in RC circuits also RL circuits it is very convenient to think of the steady state and that can be used as a particular solution and we do that as follows. We know that all time derivatives vanish as $t \rightarrow \infty$, that makes the current = 0 for a capacitor because the current is $i = C$ dv dt .if V = constant ,then $i = 0$ and then we get $v^{(p)} = V_{Th}$ for this specific case.
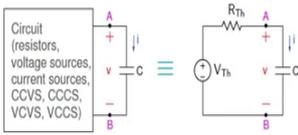
How do we get this? This current has become 0. So therefore, no voltage dropped here and therefore, we have $V = V_{Th}$. So, that is how we have got this particular solution and let us check that this solution does indeed satisfy the differential equation, let us substitute $V = V_{Th}$ in this equation here, since V_{Th} is a constant dv $_{Th}$ dt = 0 and then we have on the left hand side $(V_{Th} / R_{Th}) C$ and that is precisely the same as the right hand side. So, therefore, this particular solution does satisfy our ordinary differential equation.

Now finally, our net solution is given by the addition of the homogenous part which is here, and the particular part which is here; so we have $v = K e^{(-t/\tau)} + V_{Th}$, and in general $v(t) =$ some constant $* e^{-t/\tau} +$ some other constant where A and B can be obtained from known conditions on v and we will very soon see how that can be done.

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So, here is the summary of what we studied in the last slide, if all sources all independent sources these are DC or constant, then we have $v(t) = A e^{-t/\tau} + B$ where A and B are constants and τ = a time constant and its = $R_{thevenin} * C$, where $R_{thevenin}$ is the thevenin resistance as seen from the capacitor. What about the current? The current can be

computed once we know the voltage, because we know that i if ,C dv dt. So, all we need to do now is to differentiate this part and then multiply that by C. So, then that gives us C * A $^{-t/\tau}$ (-1/τ). So, that can be rewritten as some other constant A ' (e $^{-t/\tau}$).

One observation that we can make from this equation i(t) =A'(e $^{-t/\tau}$)is the following. As t → ∞ , this e $^{-t/\tau}$ = 0 and therefore, the current = 0 and this observation also agrees with our intuition because as t → ∞ , we reach a steady state and in steady state all quantities tend to be constant. So, therefore, this v =a constant and since the current here = C dv dt; if v = constant, that current is = 0.

So, therefore, the capacitor behaves like an open circuit in steady state as t → ∞ , what we have looked at so far is the voltage across the capacitor and the current through the capacitor, but because this circuit is a linear circuit, we can actually comment on all variables inside the circuit that is all voltages and all currents. Since the circuit in the black box is linear any variable, that is any voltage or any current in the circuit can be expressed as X (t) = K $_1$ e $^{-t/\tau}$ + K $_2$, where K $_1$ and K $_2$ can be obtained from suitable conditions on that particular variable X (t) and as we look at some examples this procedure will become very clear.

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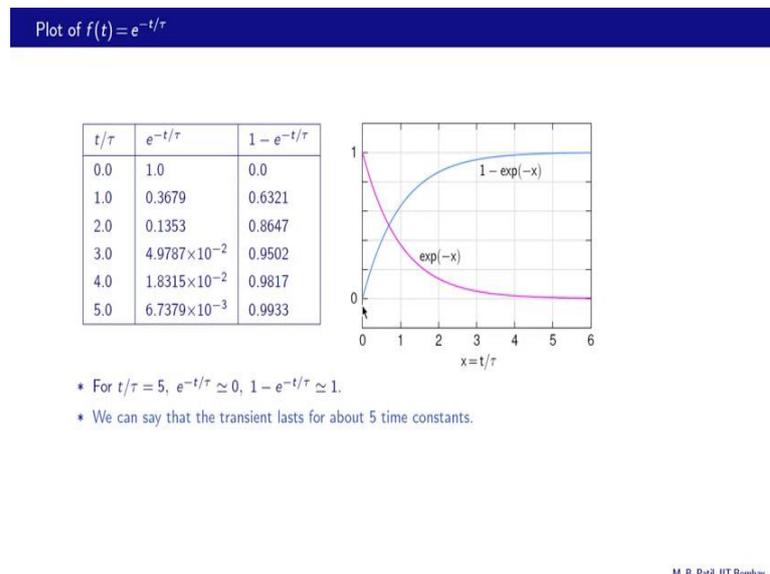
Let us now look at this function f (t) = e $^{-t/\tau}$, which appears in our solution as we saw in the last slide here is a table, which shows how this function varies with time, the first column here is t / τ. So, this is t = 0, this is t = 1 * τ, that is t = τ, this is t = 2 τ, this is t =

3τ and so on. This column here is our f (t) that $= e^{-t/\tau}$ and this column here $= 1 - f$ (t). For t $= 0$, $e^{-t/\tau} = 1$, for t $= \tau$, it is 0.37, for 2 $\tau =$ it is 0.13, for 3 τ, 0.05, 4 τ, 0.018 and 5 τ, it is 0.0067.

So, we seen that as t increases, this function f (t) gets closer and closer to 0; what about 1 - f (t)? To begin with, it is 1 - 1 $= 0$ .at t $= 0$, and then as f (t) goes on decreasing, 1 - f (t) goes on increasing and eventually of course, as $e^{-t/\tau} \rightarrow 0$, $1 - e^{-t/\tau} \rightarrow 1$.

So, by the time we come to 5 times the time constant this function has almost become $=$ 1. So, for t / $\tau = 5$, $e^{-t/\tau} \approx 0$, this number here and $1 - e^{-t/\tau} \approx 1$ this number here and therefore, we can say that the transient lasts for about 5 time constants after that things become constant. Here is the figure showing $e^{-t/\tau}$ as a function of t / τ, this curve here and $1 - e^{-t/\tau}$ as a function of t / τ, the blue curve here and as we have seen in this table at t $= 0$, we have $e^{-t/\tau} = 1$ and as t increases, this function goes to 0 and after about 5 time constants that is after t / $\tau = 5$, we see that this function becomes ≈ 0 and will stay at 0 thereafter.

Similarly, 1 - f (t) that $= 1 - e^{-t/\tau}$ starts off at 0 and eventually it becomes $= 1$, after about 5 time constants.

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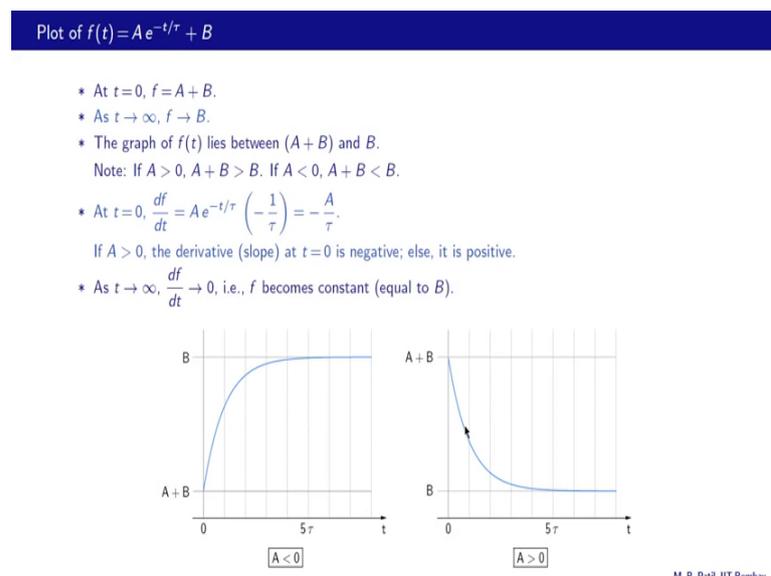
Let us now take a look at the plot of f (t) $= A e^{-t/\tau} + B$; where A and B are constants and why do you want to do that? That is because as we have seen in the earlier slides this

is the general form for our solution, and this will apply to any quantity, any current or any voltage in an RC circuit or RL circuit with DC sources. So, let us first figure out some constraints on this function and then we will look at the plot. What about t = 0? If we substitute t = 0 ,here e $^{-t/\tau}$ = 1 and therefore, we have f = A + B. What about t =∞? As t → ∞ , e $^{-t/\tau}$ = 0 and therefore, we have f → B.

From these 2 observations we conclude that the graph of f (t) will always lie between A + B and B. If A is positive then ,A + B is going to be > B and otherwise it is going to be < B. Let us now look at the derivative of f, what is df dt? We need to differentiate this part that is A e $^{-t/\tau}$ (- 1 / τ) and the derivative of B of course, = 0 because that is a constant. So, therefore, we get df dt = A e $^{-t/\tau}$ (- 1 / τ). At t = 0 e $^{-t/\tau}$ = 1 and therefore, we get df dt = - A /τ.

Now, if A is positive then this derivative at t = 0 is going to be negative, because of this minus sign here; and what is the derivative? This derivative is nothing, but the slope of the function at t = 0. If A < 0 , then this derivative is going to be positive; what about t → ∞ ? As t → ∞, this term = 0 and therefore, df dt will → 0; that means, f becomes constant and what is that constant? We have already seen that that constant is B. So, as t → ∞, d of dt → 0 irrespective of the sign of A, because A simply does not come into the picture it gets multiplied by this 0 here.

Let us now look at these 2 cases namely A > 0, A + and A -. Let us start with A < 0 in this case, (A + B) < B. So, our initial value which = A + B is going to be smaller than our final value, which = B. What about the slope? If A < 0, then the slope at t = 0 is positive like that and as t → ∞ , the slope anyway goes to 0 irrespective of the sign of A and we notice that also over here and notice also that things stop wearing after 5 τ as we have already seen in the last slide. After 5 τ, this function f (t) ≈ constant, does not change thereafter.

Now, let us look at the second case that is A positive; if A is positive , (A + B) > B; that means, our initial value is greater than our final value and that is what we see over here, this is the initial value that is the final value, what about the slope at t = 0? If A is positive then the derivative at t = 0 is negative and that is indeed what we observe over here, there is a negative slope here at t = 0.

And after 5 time constants the function becomes constant and that = b as we expect and of course, df dt = 0. So, these are the only 2 possibilities we have if A is negative; then f (t) starts off with a positive slope at t = 0 there is some variation in f (t) up to 5 τ and then it approaches the constant that is B and the slope of course = 0. In the other case when A is positive, f (t) starts with a negative slope at t = 0, then there is some variation up to 5 τ and after that f (t) becomes constant; that means, the derivative = 0.

Let us notice also that these functions are continuous, the derivatives are also continuous and therefore, we do not see any abrupt changes anywhere either in this case or in this case and all of these are very important points and they will help us when we want to plot a current as a function of time or a voltage as a function of time in RC or RL circuits with B C sources. Now very soon we are going to work out some examples, and the plots that we have going to obtain let us say for a current or a voltage would be either of this type or of this type. So, whenever you see a plot you should really come back to this slide and confirm that what we see over there is one of these forms.

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RL circuits with DC sources

* If all sources are DC (constant), $V_{Th} = $ constant.
* KVL: $V_{Th} = R_{Th}\, i + L\dfrac{di}{dt}$.
* Homogeneous solution:

$\dfrac{di}{dt} + \dfrac{1}{\tau} i = 0$, where $\tau = L/R_{Th}$

$\rightarrow i^{(h)} = K\,\exp(-t/\tau)$.
* Particular solution is a specific function that satisfies the differential equation. We know that all time derivatives will vanish as $t \rightarrow \infty$, making $v = 0$, and we get $i^{(p)} = V_{Th}/R_{Th}$ as a particular solution (which happens to be simply a constant).
* $i = i^{(h)} + i^{(p)} = K\,\exp(-t/\tau) + V_{Th}/R_{Th}$.
* In general, $i(t) = A\,\exp(-t/\tau) + B$, where A and B can be obtained from known conditions on i.

M. B. Patil, IIT Bombay

Let us now look at RL circuits with DC sources. So, here is a linear circuit once again consisting of resistors independent voltage sources independent current sources and dependent sources. To this circuit we connect an inductor and our interest is to find the voltage across the inductor as a function of time, and the current through the inductor as a function of time step number one. We replace this original circuit with its thevenin

equivalent consisting of a voltage source V_{Th}, in series with the resistance R_{Th} and the inductor of course, appears as it is. As in the RC circuit case we will assume that all our sources these independent voltage sources and independent current sources are DC; that means, they are constant and therefore, this V_{Th} is going to be a constant.

Let us now write the KVL equation for this loop and what is that? This V_{Th} = this voltage drop + this voltage drop. Now this voltage drop across the resistance = R_{Th} i and the voltage drop across the inductor = L di by dt. To solve this differential equation we first obtain the homogenous solution and we do that by replacing this forcing function with 0 like that and then we divide both sides by L. So, we get di dt + 1 /τ (i = 0), where $\tau = L / R_{Th}$; this τ has units of time and that is the circuit time constant.

The homogenous solution indicated here with the superscript h is then = $K(e^{-t/\tau})$, where this K is a constant. To obtain a particular solution once again we will use information from the steady state condition like we did in the RC circuit case, that is as t $\to \infty$, we expect all quantities to become constant therefore, this current will also become constant and v = L di dt well then = 0. When V = 0 , this current = V_{Th} / R_{Th}. So, that is a particular solution that we can use and finally, we can add the homogenous and particular solutions to obtain the net current as a function of time. So, $i = i^h + i^p$, $i^h = K e^{-t/\tau}$, $i^p = V_{Th} / R_{Th}$. So, therefore, our net or total current = $K e^{-t/\tau} + V_{Th} / R_{Th}$.

And we do not need to really go through this derivation every time, the moment we see a linear circuit with a single inductor connected to it, and when the sources are DC sources we can write the current = $A e^{-t/\tau} + B$ where A and B are constants to be obtained from known conditions on the current.

To summarize the discussion from our last slide, if we have a linear circuit with DC sources then we can write this inductor current $= A\,e^{-t/\tau} + B$ where A and B are constants, and τ, the time constant is $= L / R_{Th}$. What about the voltage across the inductor? Once we know the current we can obtain the voltage $= L\,di\,dt$. So, all we need to do now is to differentiate this current and then multiply that by L and that $= LA\,e^{-t/\tau}(-1/\tau)$.

Now, all of this $L\,A\,(-1/\tau)$ can be clubbed together into another constant called A'. So therefore, the voltage $= A'\,e^{-t/\tau}$. What happens as $t \to \infty$? As $t \to \infty$, this $e^{-t/\tau} \to 0$ and therefore, the voltage across the inductor $= 0$ and what is the meaning of that? When the voltage here $= 0$; that means, it is just like a wire with no voltage drop across it, so the inductor behaves like a short circuit.

Now, we have already looked at the voltage across the inductor and the current through the inductor, but we can make a much more general comment, because this circuit is a linear circuit and that is since the circuit in the black box is linear any variable that is any current or any voltage inside the circuit, can be expressed as $x(t) = K_1\,e^{-t/\tau} + K_2$; where K_1 and K_2 are constants and they can be obtained using suitable conditions on $x(t)$. To summarize if we have a linear circuit with DC voltage sources or DC current sources connected to a capacitor or an inductor, then any quantity in the circuit that is any current or any voltage is given by this expression here, $x(t) = K_1\,e^{-t/\tau} + K_2$.

If we have a capacitor connected to this linear circuit then this time constant τ is given by $R_{Th} C$, where R_{Th} is the thevenin resistance seen by the capacitor if we have an inductor then this $\tau = L / R_{Th}$; where R_{Th} is the thevenin resistance seen by the inductor.

In summary we have looked at the general form of the solution in RC or RL circuit with DC sources, we have also seen how to plot the solution as a function of time. These observations will be very useful when we look at some examples in the next lecture that is all for now.

See you next time.