**Basic Electronics**
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**Lecture - 06**
**Phasors (continued)**

Welcome back to Basic Electronics. In the last lecture we have introduced phasors and seen how to interpret them in the sinusoidal steady state. In this lecture we want to apply phasors to RLC circuits, to figure out the steady state currents and voltages. In addition we will look at the maximum power transfer theorem for RLC circuits in the sinusoidal steady state. So, let us start.

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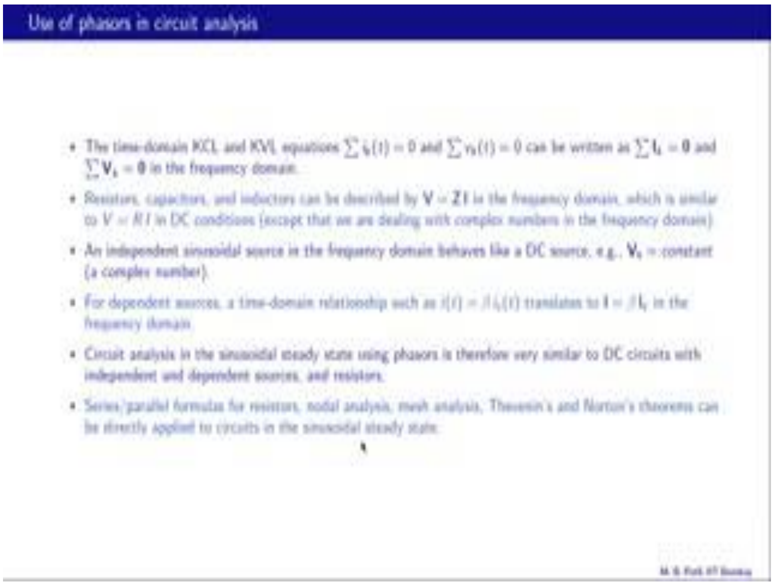


Let us now look at how we can use phasors in circuit analysis; the time domain KCL and KVL equations $\sum I_k = 0$ and $\sum V_k = 0$ can be written as phasor equations, $\sum I_k = 0$ where $I_k$ are current phasors now and $\sum V_k = 0$ where $V_k$ are voltage phasors in the frequency domain.

Resistors, capacitors and inductors can be described by V = Z I in the frequency domain, which is similar to V = R I in DC conditions. The only difference is that we are now dealing with complex numbers, when we talk about this equation. An independent sinusoidal source in the frequency domain behaves like a DC source for example, for a voltage source a sinusoidal voltage source we can say that the phasor $V_s$ is the constant, which is the complex number. For dependent sources a time domain relationship such as i (t) = β $I_c$ ( t) translates to the phasor relationship, phasor I = β $I_c$ in the frequency domain. So, the equation looks very similar except we have complex numbers here.

So, from all of these remarks we conclude that circuit analysis in the sinusoidal steady state using Phasors, is very similar to DC circuits with independent and dependent sources and resistors; and therefore, all the results that we derived for DC circuits are valid also for sinusoidal steady state analysis therefore, series parallel formulas for resistors, nodal analysis, mesh analysis, Thevenin and Norton's theorems can be directly applied to circuits in the sinusoidal steady state; totally difference of course, is we are now dealing with complex numbers.

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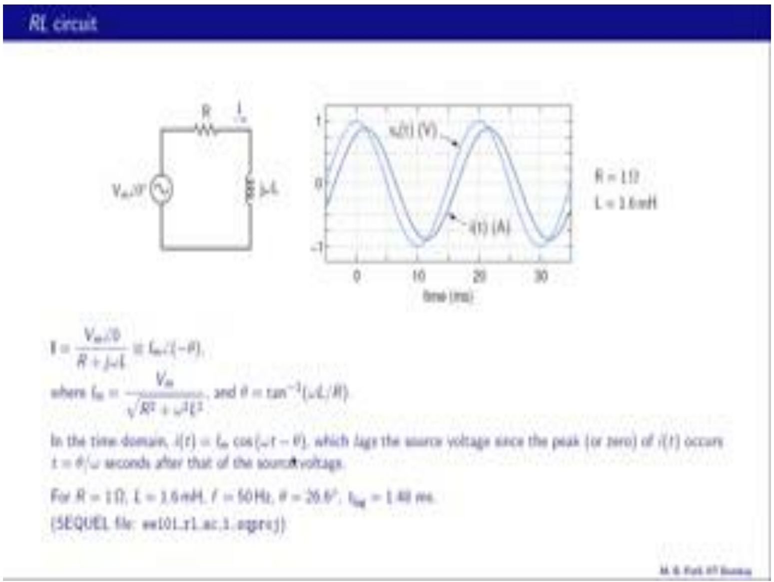


Let us now consider an R L circuit in the sinusoidal steady state; where R and L in series the impedance of the resistor = R as we have seen before and the impedance of the inductor = j ωL.

This is our sinusoidal source voltage= $V_m$ < 0, that = $V_m$ cos ωt in the time domain and we are interested in this current. When we use Phasors, this becomes an extremely simple calculation as we will see; let us imagine that we have a DC circuit with a DC source here $V_s$, a resistor $R_1$ here and a resistor $R_2$ here, what would the current be in that case? The DC current it would be = $V_s$ / $R_1$ + $R_2$; so we can use exactly the same equation except instead of $V_s$ we have a Phasors source now that = $V_m$ < 0 and instead of $R_1$ and $R_2$ they have R and j ωL, the impedances of the resistor and the inductor. So, then we can write $I = V_m < 0$, this source / R + j ωL as simple as that.

We can write this phasor current = $I_m < -\Theta$, where $I_m$ is the magnitude of this expression and that = $V_m$ / the magnitude of the denominator which $= \sqrt{R^2 + \omega^2 L^2}$. The angle of the denominator = $\tan^{-1}(\omega L / R)$ and the angle of the numerator = 0. So, the net angle of I = 0 - $\tan^{-1}(\omega L / R)$ that is all we get this minus sign over here. So, Θ here = $\tan^{-1}(\omega L / R)$.

In the time domain we have $i(t) = I_m \cos \omega t - \Theta$, $I_m$ is given by this expression and Θ is this one. Now let us take some component values say R = 1 **Ω**, L = 1.6 mH and f =50 Hz then, Θ = 26.6 º; and here are the plots for the source voltage and the current this is our source voltage , $V_m = 1$. So, it is = 1 ***** cos ωt, and this is our current the dark one and let us now try to understand this current plot in terms of this equation.

Our $I_m$ value turns out to be about 0.9 A and therefore, we see that the current goes from - 0.9 A to + 0.9 A. And as we have said earlier Θ = 26.6 º, now how do we relate this value to this plot? This is an angle and this is time over here. So, what we can do is to convert this angle to time; we know that 360 ºcorresponds to one period and that in this case is 20 msec, because 1 / f = 20 msec and therefore, we can use that fact to find the time that corresponds to this angle that turns out to be =1.48 msec.

Now let us get back to this equation $i(t) = I_m \cos \omega t - \Theta$, and ask the question when does this i (t) go through its peak? That is when ωt - Θ = 0 and the answer is that happens, when t = Θ/ω, and that turns out to be = 1.48 msec. So, what it means is that the current does not go through the peak here when the voltage goes through its peak, but a little later that is at 1.48 or ≈1.5 msec, and that is why we say that it lags the source voltage.

Notice how easy this entire calculation was, we did not write down any differential equation, we simply used this expression which is like 2 resistors in series and then we managed to get all the information that we required. So, that is how phasors really help in analyzing circuits in the sinusoidal steady state. So, here is the circuit file you can simulate the circuit and maybe change some component values and look at the results.

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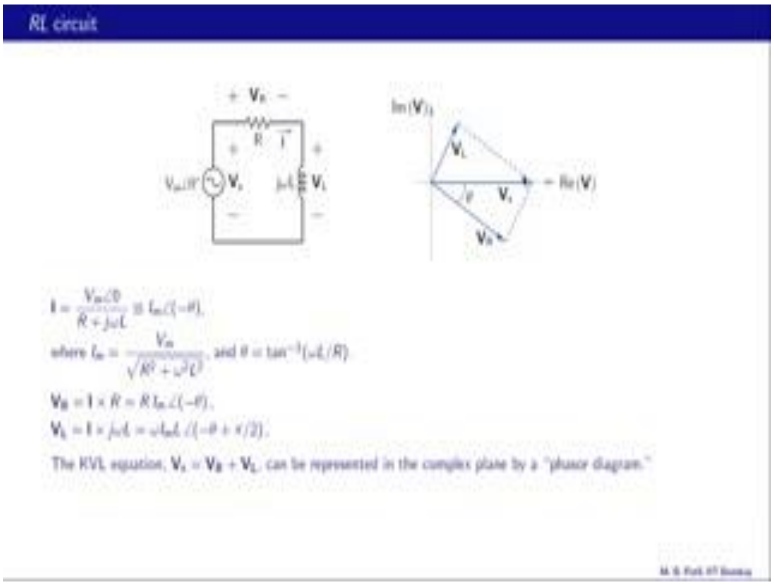


Let us now represent the KVL equation that is $V_s = V_R + V_L$ in a graphical form in the complex plane, using a phasor diagram; and to do that we require $V_s$ which = $V_m < 0$, and $V_R$, $V_L$. Let us look at $V_R$ first; what is $V_R$? $V_R$ = I * the impedance of the resistor which is R. So, that = R $I_m < -\Theta$, because our $I = I_m < -\Theta$. What about $V_L$? $V_L$ = I * the impedance of the inductor, which = j $\omega$L. Now this j is nothing, but $< \pi/2$ and $I = I_m$ $<-\Theta$. So, therefore, this quantity = $\omega I_m L <(-\Theta+\pi/2)$.

So, this is our Phasor diagram, this vector = $V_s$ and $V_s = < 0$ therefore, it is a longer X axis that is the real part of V. This is our $V_R$ and $V_R =<-\Theta$; that means, $\Theta$ in the clockwise direction and this is our $V_L$, what is the angle of $V_L$? It is = $- \Theta + \pi/2$; that means, we go clockwise by $\Theta$ and then we go anti clockwise by $\pi/2$, that brings us to this angle all right and now this equation $V_s = V_R + V_L$ is essentially a vector equation, if we add $V_L$ and $V_R$ then = $V_s$.

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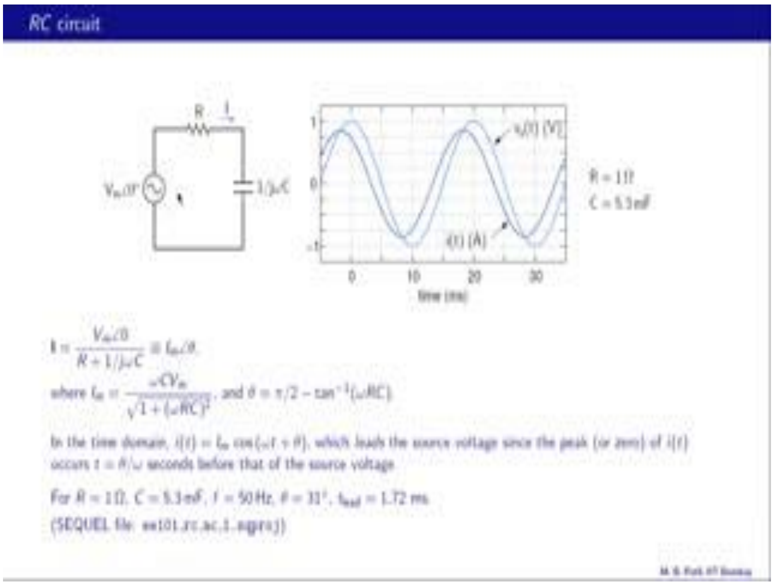


Next let us take an R c circuit a series R c circuit, and this can be analyzed in exactly the same manner as the R L circuit which we just saw; what do we do in this case? We replace the resistor with its impedance which = R, the capacitor with its impedance which =1/ j ωC and then we can obtain this current = $V_m < 0$ / (R + (1 / j ωC)), that we write as $I_m < \Theta$, where $I_m$ its given by this quantity you should really verify this; and Θ = π/ 2 - $\tan^{-1}$ (ωRC). Now this $\tan^{-1}$ (ωR C) can vary from 0 to π/ 2, and therefore this angle is basically a positive angle.

In the time domain we write the current = $I_m$ cos (ωt + Θ), and now let us calculate Θ for some component values say R =1 **Ω**, C = 5.3 mf and f =50 Hz. For this combination, Θ =31 $^o$ and that corresponds to a time = 1.72 msec. Now let us ask this question when does i ( t) go through its peak and that happens when ωt + Θ = 0; that means, t = -Θ/ω and that time =1.72 msec.

Let us look at the plots law; this light curve is the source voltage= 1 < 0 and it goes through its peak at t = 0. Because there is the cos function, this dark curve is the current and that goes through its peak when t = - 1.72 msec as we just discussed. In other words the current goes through its peak before the source voltage goes through its peak, and that is why we say that the current leads the source voltage since the peak of i (t) occurs Θ/ω seconds before that of the source voltage.

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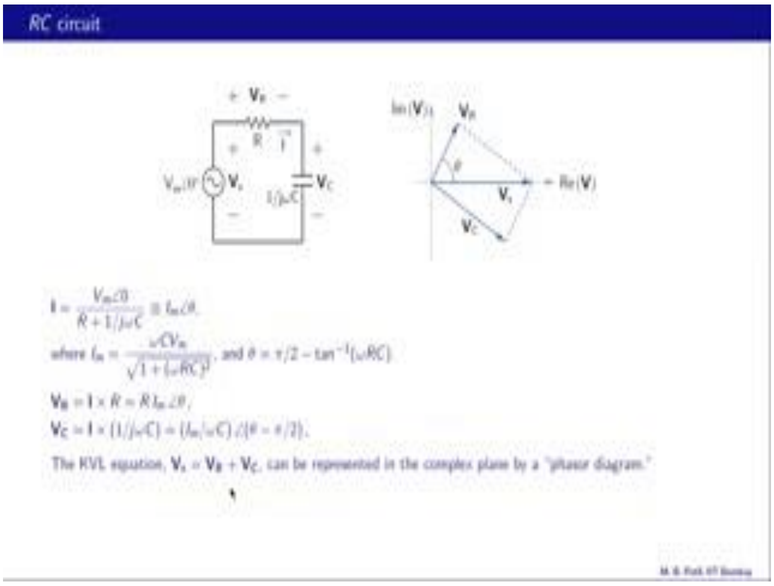


Let us draw a phasor diagram to represent the KVL equation in this case, that is Vs = V $_R$ + V c. What is V $_R$? $V_R$ =I R where I = I $_m$ < Θ. So, therefore, V $_R$ = R I m < Θ. What about V $_c$? V $_c$ = I * the impedance of the capacitor, which = 1 / j ωC. Now 1 / j = - j that is an angle of – π/2 and therefore, we get V $_C$ = I $_m$ / ωC that is the magnitude, and for the angle we get Θ,which comes from I and then we have this –π/ 2 coming from this 1 / j.

And as we mentioned earlier our Θ = a positive angle between 0 and π/ 2 all right; with that information we can now draw the phasor diagram, this is our V $_s$ and that is along the X axis, because it has an angle of 0. What about V $_R$? V $_R$ has got an angle Θ, which is positive, and between 0 and π/ 2. So, that is what V $_R$ looks like; what about V $_C$? V $_C$ has an angle of Θ-π/2. So, we go anti clockwise by Θ and then become clockwise by π/2 that bring us to this angle. So, that is our V $_C$ and now we can see that the vector equation V $_s$ = V $_R$ + V $_C$; each satisfied this is our V s. So, V $_R$ + V $_C$ =V $_S$. So, this is the phasor diagram corresponding to this KVL equation in this case.

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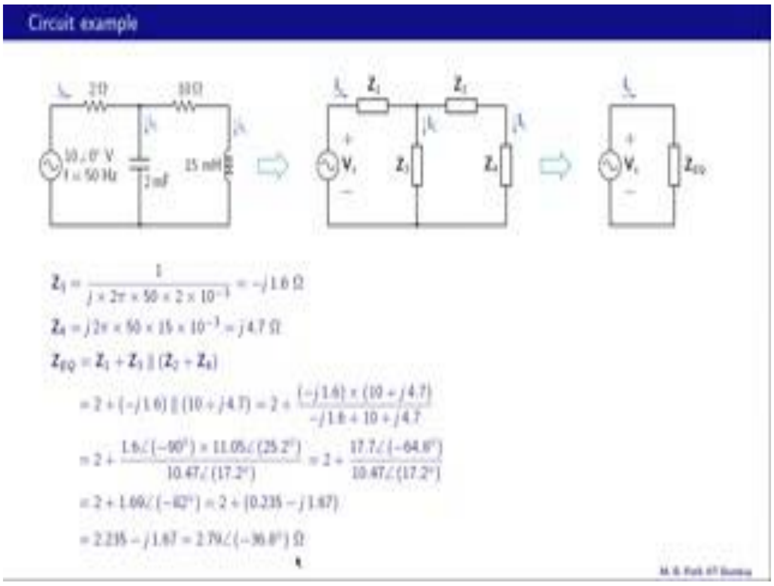


Let us consider a little more complex circuit now the one shown here. So, we have a sinusoidal voltage source= 10 < 0, frequency =50 hertz, and we want to find these currents $I_S$, $I_C$ and $I_L$. Step number one we convert all the components to their impedances, so this 2 Ω of course remains 2 Ω; 10 Ω remains 10 Ω ,2 mf = $Z_3$, where $Z_3$ = 1 / j ω = - j 1.6 Ω. 15 mH= $Z_4$ where $Z_4$ = j ωL , ω = 2 π * 50 that = l. So, that turns out to be = j 4.07 Ω. And now next step is we can calculate the equivalent impedance of this combination and this is exactly like series parallel resistor combination, we have $Z_2$ and $Z_4$ in series that combination in parallel with $Z_3$ and the whole thing in series with $Z_1$. So, that is what $Z_{EQ}$ = ($Z_1$ + $Z_3$ ) // ($Z_2$ + $Z_4$).

You are definitely encouraged to go through all of these steps and arrive at this final result for $Z_{EQ}$, but also look up your calculator and it is possible that you will be able to do this calculation in a smaller number of steps, depending on what your calculator allows.

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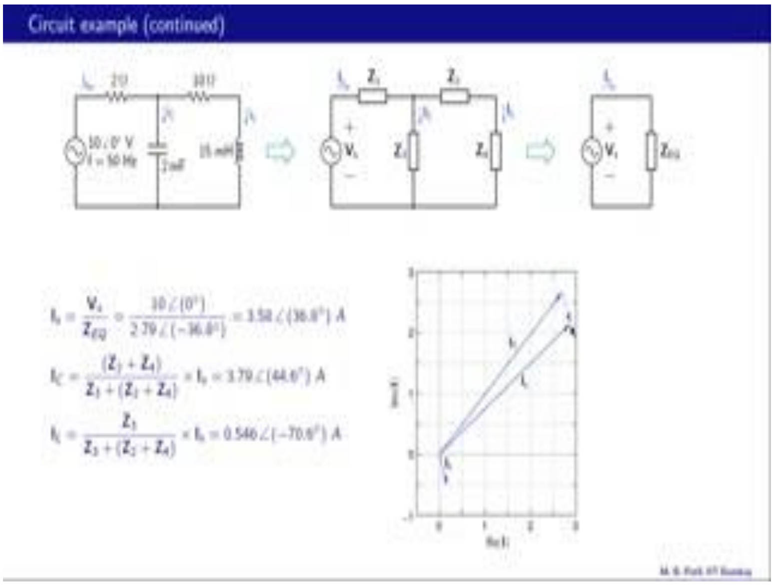


Now, let us go ahead. So, we have come up to this step, you have found $Z_{EQ}$ we already have $V_s$. So, now, we can calculate $I_s$. So, $I_s = V_s / Z_{EQ}$, $V_s = 10 < 0$ and $Z_{EQ}$ from the last slide is this number here, so that turns out to be $= 3.58 < 36.8\,^o$ A.

Once we get $I_s$ we can now get $I_c$ by using the current division formula, that is $I_c = (Z_2 + Z_4 / (Z_2 + Z_4 + Z_3))\ I_s$ like that and that turns out to be this number, what about $I_L$? You can get $I_L$ in 2 ways: one $I_s - I_c = I_L$ by KCL or we can use the current division formula $I_L = (Z_3 / (Z_3 + Z_2 + Z_4))\ I_s$ either way you should get this number for $I_L$. And now let us draw the phasor diagram, which describes the KCL equation at this node. Here is the phasor diagram and notice that we have used the same scale for the X and Y axis that is this distance which represents 1 unit on the X axis also represents 1 unit on the Y axis. And we do that so as to represent angles correctly; that means, a $45\,^o$angle would you indeed look like a $45\,^o$angle if we follow this practice.

All right let us now verify whether the KCL equation at this node is satisfied. What is the equation we have $I_s = I_c + I_L$. Our $I_s = 3.58 < 36.08\,^o$, that is this vector this magnitude is 3.58, and this angle is $36.8\,^o$. $I_c$ has a magnitude of 3.79. So, little bit larger than $I_s$ and an angle of $44.6\,^o$, that is $I_c$ this angle is $44.6\,^o$. $I_L$ it is much smaller in magnitude 0.546 and it has a negative angle, $- 70.6\,^o$. So, that is our $I_L$; and now we see that $I_c + I_L = I_s$ so; that means, KCL is verified.

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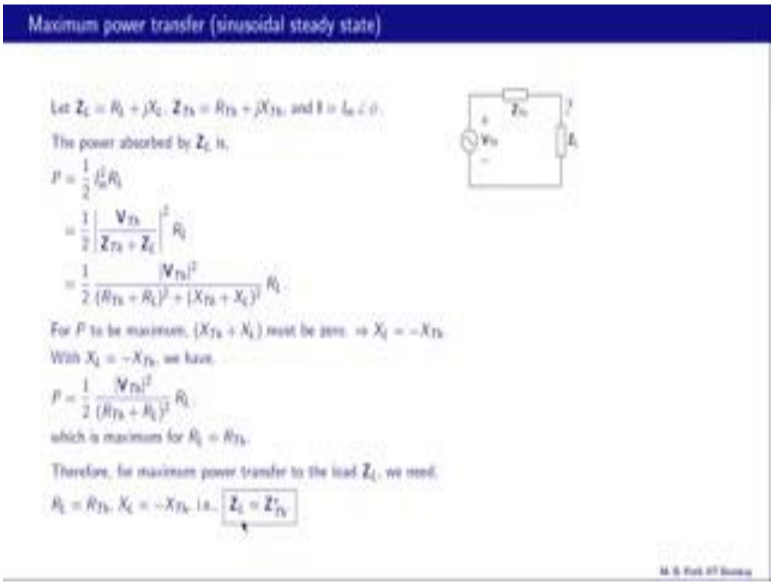


We have looked at the maximum power transfer theorem for linear DC circuits; now let us look at maximum power transfer in the sinusoidal steady state. So, let us consider circuit whose Thevenin equivalent is given by this circuit here namely a voltage source $V_{th}$ in series with an impedance $Z_{th}$; both of these of course, are complex numbers. Now we connect a load impedance to the circuit and as a result a current is going to flow and that current is denoted by this spacer I.

We want to find the condition for which the power transfer from this circuit 2 $Z_L$ is maximum. Let us begin with $Z_L = R_L + j x L$, where this is the real part of $Z_L$ and that is the imaginary part of $Z_L$ and similarly let $Z_{th} = R_{th} + j x_{th}$. Let $I = I_m < \phi$, $I_m$ is the magnitude of this phasor I here and $\phi$ is its angle.

Now, the power absorbed by $Z_L$ is $p = 1/2\ I_m^2\ R_L$, where $I_m$ is the magnitude of this phasor I and $R_L$ is the real part of $Z_L$. What is $I_m$? It is the magnitude of I, and what is I? it is $= V_{th} / Z_{th} + Z_l$. So, we get p equal to this expression over here. Let us rewrite this expression $= ½[\ |V_{th|}\ ^2 /(\ (R_{th} + R_L)^2 + (X_{th} + X_L)^2\ )]\ R_L$. Now this $V_{th}^2$ is independent of $Z_L$ and as far as we are concerned that is just a constant. So, what we need to do now is to find conditions on $Z_L$ for which this whole expression is maximum.

Now, for P to be maximum, clearly this denominator must be minimum and that will happen if $(X_{th} + X_L)^2 = 0$, because there is a square here the smallest value that this term can take is 0 and therefore, that gives us $X_L = - X_{th}$ and with $X_L = - X\ t\ h$ this second term disappears, and we get $P = ½ [ | V_{th} |^2 / (R_{th} + R_L)^2 ] R_l$. So, for maximum power transfer we need to maximize this expression now.

How do we do it? We differentiate P with respect to $R_L$ and we equate d p d $R_L$ = 0 and then find that P is going to be maximum when $R_L = R_{th}$. So, we have 2 conditions: one the imaginary part of $Z_L$ that is $X_L$ = negative of the imaginary part of $Z_{th}$ that = $X_{th}$ and the real part of $Z_L$ = the real part of $Z_{th}$.

To summarize for maximum power transfer to the load $Z_L$, we need $R_L = R_{th}$ and $X_L = - X_{th}$; that means, our load impedance must be the complex conjugate of the Thevenin equivalent impedance that is $Z_{th}$. So, $Z_L = Z_{th}^*$.

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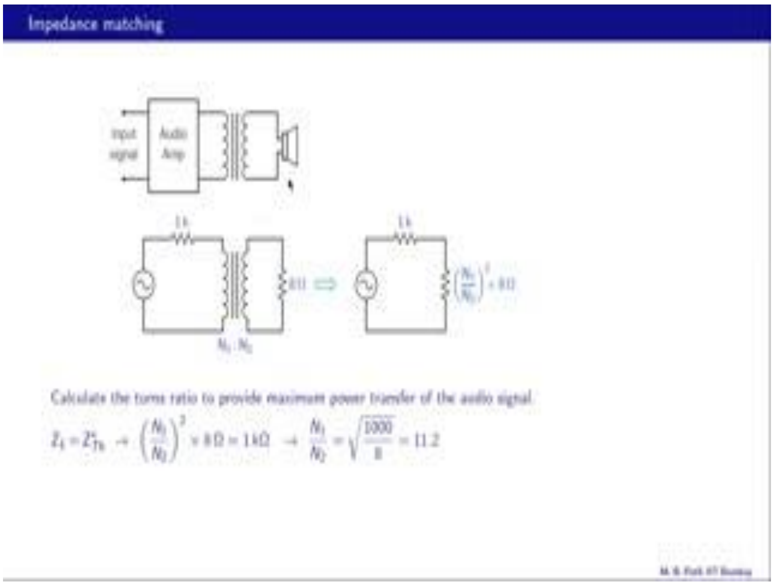


Let us now look at an application of the maximum power transfer theorem for sinusoidal steady state here it is an audio amplifier driven by an audio input signal. So, the frequencies here would be in the range 20 Hz to say 16 KHz or so. This audio amplifier is followed by transformer and we will soon comment on why this transformer is required and then finally, we have this speaker. This speaker has a complex impedance which varies with frequency, but in the audio range its resistance is more or less constant typically 8 Ω, and its imaginary part can be ignored. So, the equivalent circuit represent take this entire situation is given by this circuit here, this source here represents the input signal amplified by the gain of the audio amplifier and then this 1 k resistance here is the output resistance of the audio amplifier, and then we have the transformer with a turns ratio $N_{1\,2}$, $N_2$ and finally, the speaker which is represented by this 8 Ω resistance here.

Our objective of course, is to maximize the power transfer from this circuit to the speaker that is how we will hear the loudest sound that is possible with the given input signal. So, let us simplify the circuit, we can transfer this resistance to the other side of the transformer and then it becomes $(N_1 / N_2)^2 * 8\ \Omega$.

Here is our actual problem statement, we look at this circuit and calculate the turns ratio to provide maximum power transfer of the audio signal; what is the maximum power transfer theorem? It says that $Z_L = Z_{th}^*$, and in this case since the imaginary parts of $Z_{th}$ and $Z_L$ are 0 all it means is that the real parts must be equal; that means, we must have $(N_1/ N_2)^2 * 8\ \Omega = 1$ k like that.

And we can now solve this equation for $N_1 / N_2 = 11$. So, if we pick a transformer with this translation then it is current it that the audio signal will deliver maximum power to the speaker, and we will hear the loudest possible sound that is possible with this input signal.

In conclusion we have seen how to use phasors to analyze RLC circuits in the sinusoidal steady state; this background is going to be very useful when look at filter circuits be a little later. We also looked at the maximum power transfer theorem for RLC circuits in the sinusoidal steady state. We considered an example which is very important in practice namely how to obtain maximum audio power from a speaker, that is all for now.

See you next time.