Basic Electronics

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Lecture - 10
RC/RL circuits in time domain (continued)

Welcome back to Basic Electronics. In this lecture we will continue with the RL circuit example from the previous lecture; we will then look at an RC circuit with a switch and learn how to obtain currents and voltages in the circuit, before and after the switch position changes. We will then look at an RC circuit in the periodic steady state with a square wave input let us get started.

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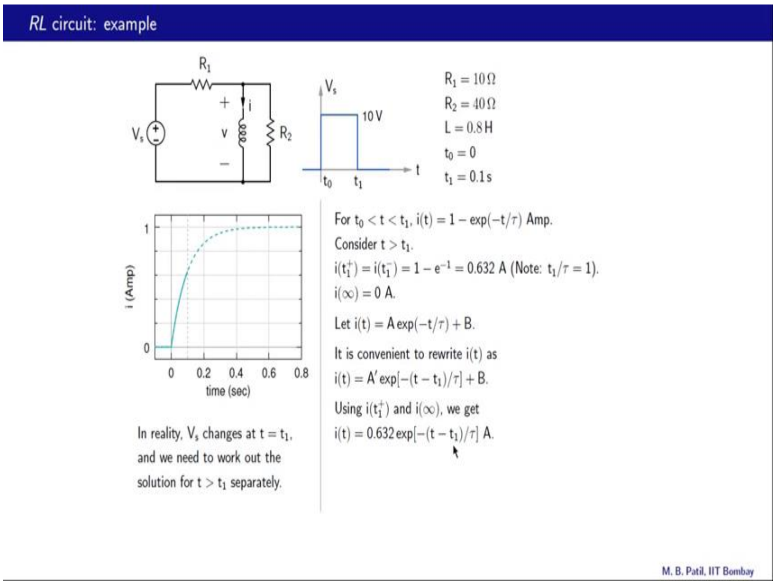

So, this is the solution that we obtained using the conditions that we found for i (t) at 0 + and ∞, at 0 +, i = 0 and as $t \to \infty$, i = 1 A. Now this of course, is only part of the story why? Because in reality the source voltage V_s changes at $t = t_1$, so therefore, this solution is really valid up to t_1 that is up to 0.1 sec, and we need to work out the rest of the solution that is the solution for $t > t_1$ this interval here and how do we go about that? We know that as we go from this interval t_0 to t_1 to this next interval t_1 to ∞, the inductor current must remain continuous; that means, the value of the inductor current i at t_1 - that is just before t_1 must be the same as the value of i at t_1 + that is just after t_1.

For this interval $t_0 < t < t_1$ this interval here, $i(t) = 1 - e^{-t/\tau}$ A; and this expression describes this part of the solution and you are of course, encouraged to derive this expression, how do you do that? Let $i(t) = k_1 e^{-t/\tau} + k_2$, where k_1 and k_2 are constants to be determined and now use the conditions that we derived in the last slide namely i at 0 + = 0, and i at ∞ = 1 A.

In those conditions you can find k_1 and k_2 and then you will end up with this expression here and once we have this expression we can find i at t_1 - ; that means, i at 0.1 sec, all we need to do is replace this t with 0.1 sec, this τ as we saw in the last slide is = 0.1 sec. So, we end up with $i(t) = 1 - e^{-0.1/0.1} = e^{-1}$.

So, that turns out to be = 0.632 A, and i at t_1 + , as we just argued is the same as i at t_1 -. So, therefore, i at t_1 + = 0.632 A. So, that is one condition that we have for i (t) in this last interval here. What is the second condition? The second condition is i at ∞. So, let us see what that should be; what is the situation at t = ∞? V s = 0 and it has been 0 for a long time, the circuit is in steady state all quantities voltages and currents would have become constant, this current would have become constant so therefore, this V = l di dt would be = 0 this voltage is therefore, 0 no current flows through R_2, and $i = V_s / R_1$. Since V s = 0 at t = ∞, the current = 0 A like that.

So, now we have 2 conditions i at t_1 + and i at ∞, and now we can get the complete solution for this last interval here. Let $i(t) = A e^{-t/\tau} + B$ in this interval, that is $t > t_1$ and now let us find A and B using these 2 conditions. It is convenient to rewrite $i(t) = A' e^{-(t-t_1)/\tau} + B$, we are not really changing this equation all we are doing is shifting the origin from t = 0 to $t = t_1$

Let us now use these 2 conditions i at t_1 + = 0.632 A, and i at infinity = 0 A. At t_1 +, we have i = 0.632 A. So, we put 0.632 here, t =t_1 + here. So, this argument becomes 0 and $e^0 = 1$. So, therefore, we get 0.632 = A '+ B. At ∞ ,i = 0 so we put 0 here, and $e^{-\infty} = 0$ therefore, this first term goes away and we get 0 = B; and putting these together we get our final expression for i (t) in this last interval, $t > t_1$, and that = $i(t) = 0.632 e^{-(t-t_1)/\tau}$ A.

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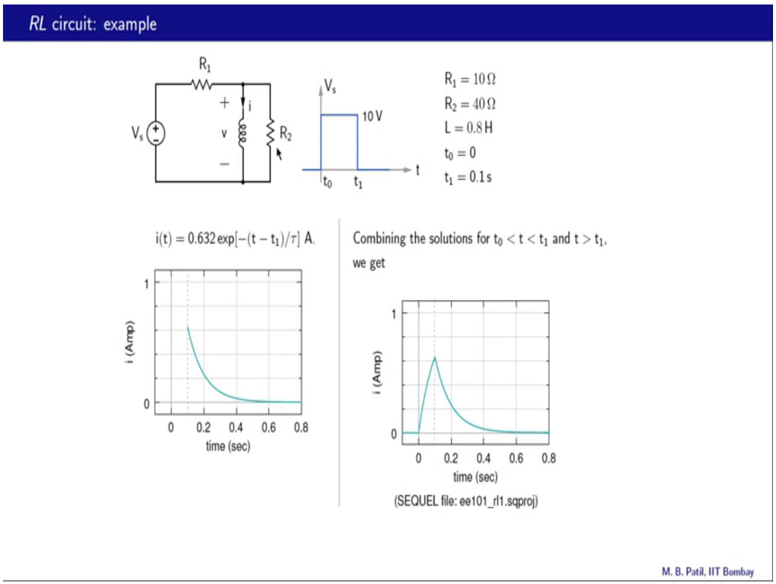

And this is what the solution looks like in the final interval $t > t_1$, at t_1, $i = 0.632$ A, and at $t = \infty$, it is = 0 A.

So, now we have different pieces of the solution available to us, one piece in this interval and one piece and this interval, all we need to do now is to put these together like that. So, this is our complete solution for i (t); the sequel file for this particular simulation is available to you given here and you can play with it for example, you can change R_2 from 40 Ω to 20 Ω, and predict first of all what should happen and then run the simulation and check that your prediction is correct.

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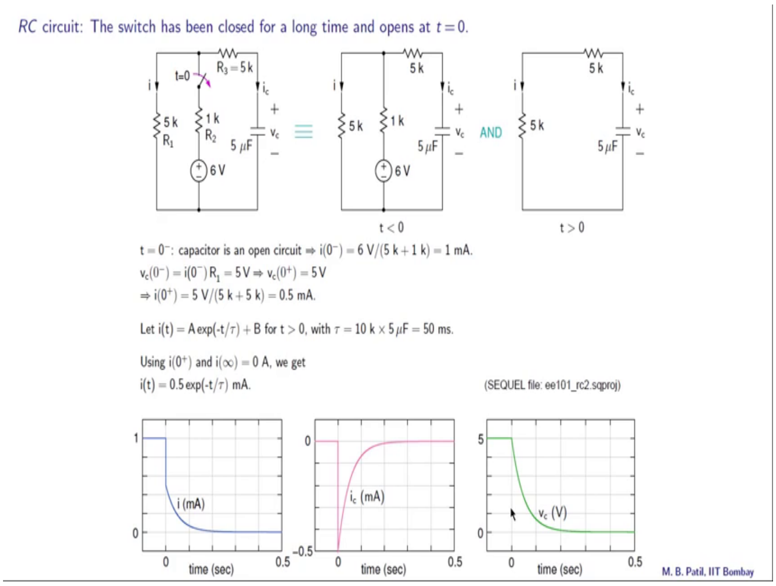

Here is our next example it is an RC circuit, with a switch that opens at t = 0. So, the switch has been closed for a long time and opens at t equal to 0, and we are interested in finding this current i as a function of time. Now this situation is very different than what we have been looking at so far, because when the switch opens our circuit changes. Before t = 0 , we have 1 circuit and after t = 0 we have another circuit. So, let us see how to handle that.

Here is the situation for t < 0 that is when the switch is closed; when the switch is closed we have a short circuit here assuming the switch is ideal and therefore, this circuit data applies. After the switch opens that is when t > 0, this switch is an open circuit and therefore, this entire branch is not in the circuit anymore and the circuit reduces to this circuit shown over here.

Now, this situation for t < 0 has been there for a long time. So therefore, this circuit is in steady state, you can use that fact to find V c over here, and that capacitor voltage value serves as the bridge between these 2 situations, because as we know the capacitor voltage must be continuous. So, Vc at 0 + must be = V c at 0 -.

To begin with let us look at the situation at t = 0 -; that means, we are looking at this circuit, and at 0 -, the switch has been closed for a long time, so this circuit is in steady state; what is the meaning of steady state? That means, all currents and voltages have become constant, so therefore, this voltage Vc has also become constant and therefore, i c which is C d V c dt = 0. So, this is an open circuit and the current path is given by this path here and that tells us what this current i should be. It is simply = 6 V/ total resistance in the circuit that = 1 k + 5 k, so 6 V/ 6 k = 1 mA.

So, we have i at 0 - = 1 mA, what about V c at 0 -? We have seen that this current = 0 at 0 -, and therefore there is no voltage drop across this resistance and then this voltage V c is the same as this voltage here, that is i *5 k, and i = 1 mA so therefore, we have 1 mA * 5 k or 5 v. So, V c at 0 - = 5 v and because the capacitor voltage must be continuous, V c at 0 + in this circuit must also be = 5 V.

So, therefore, we have $V_{c\,0^-}$ = 5 V and $V_{c\,0^+}$ = 5 V; and that tells us what i at 0 + should be let us look at this circuit because now we are talking about t > 0, this voltage = 5 V and i therefore, = 5 V/ 5 k + 5 k that = 0.5 mA.

Now, let us find the current i as a function of time, for t > 0, and let us begin by assuming that i (t) has this form $A\,e^{-t/\tau}$ + B. So, we need to determine 3 things: τ , the time constant and then these constants A and B, what is the time constant for the circuit? It is= R_{th} C, where R_{th} is the thevenin resistance as seen by the capacitor and in this case the thevenin resistance is simply these 2 resistors in series so therefore, 5 k + 5 k or 10 k. So the time constant; then = 10 k * 5 µf, that = 50 msec.

What about A and B? To find A and B we need 2 conditions on i, we already have 1 condition here i at 0 + = 0.5 mA. Let us now find, i at ∞ and that give us the second condition. To find i at ∞, let us look at this circuit what is i here? i is = V c / 10 k; now as this current flows the capacitor gets discharged so that will keep happening until the entire charge on the capacitor there is exhausted. So, finally, we have no charge on the capacitor, V c becomes = 0 and therefore, i becomes = 0.

Another way to find i at ∞ is to use the fact that the circuit would be in steady state as t → ∞, and all quantities currents and voltages would have become constant at that time, so therefore, this V c would have become constant, and the current i c which is c d V dt = 0 if, i_c = 0 then of course, i is also = 0.

So, now we have 2 conditions i at 0 + = 0.5 mA, and i at ∞ = 0; and using these 2 conditions we can find A and B and we get this expression, finally i (t) = $0.5\,e^{-t/\tau}$ mA.

Here is a plot of i as a function of time, and the current i is in milliamps here this is = 0 and that is = 1 mA. At 0 -, i =1 mA and in fact, it has been 1 mA for a long time, because the switch has been closed for a long time, so therefore the current i has been constant. When the switch opens the current changes and at 0 +; that means, just after t = 0, where i =0.5 mA, 0.5 mA is right here.

So, therefore, at t = 0 , there is a discontinuity, before 0 ,we have 1 mA and after 0 ,we have 0.5 mA; and subsequently i (t) = 0.5 $e^{-t/\tau}$ mA and that part of the curve is shown over here. Eventually of course, as $t \to \infty$, this term becomes 0 and $i \to 0$ as we see here.

Now, how long does the transient last we expected to last for 5 time constants, what is the time constant? The time constant = 50 msec.. 5 times that is =250 msec that = 0.25 sec. Here is 0 , here is 0.1 sec, 0.2 sec and so on. So, 0.25 sec is here and we do observe that after 0.25 sec the current does not change it becomes constant.

Here is the capacitor current as a function of time, and that is also plotted in milliamps this = 0, and this = - 0.5 mA, what is the situation at 0 -? At 0 -, we have this circuit and it is in steady state V c is constant and therefore, the capacitor current = 0 and that is what we see over here, and it has been 0 for a long time because the switch has been closed for a long time. So, that explains this part of the plot. What about 0 +? After t = 0 , this circuit comes into picture, and then we have i c = - i.

And since i starts off at 0.5 mA and goes to 0 eventually, i c starts off at - 0.5 mA and then goes to 0 finally; and notice that there is a discontinuity at t = 0. Let us now look at the capacitor voltage, what is V c at 0 - we already looked at that V c at 0 - = 5 V, and it has been 5 V for a long time and that explains this part of the plot. What about t > 0, for t > 0, we have this circuit and V c now is =i * 5 k + 5 k, that = 10 k * i. Now i = 0.5 mA at 0 + and then eventually it goes to 0. So therefore, V c would go from 0.5 mA * 10 k that = 5 to 0 like that.

And as we would expect the capacitor voltage is of course continuous, there is no discontinuity at t = 0. The sequel file for this particular simulation is given over here and you can run this simulation and look at all of these variables, and also some other variables that you may be interested here.

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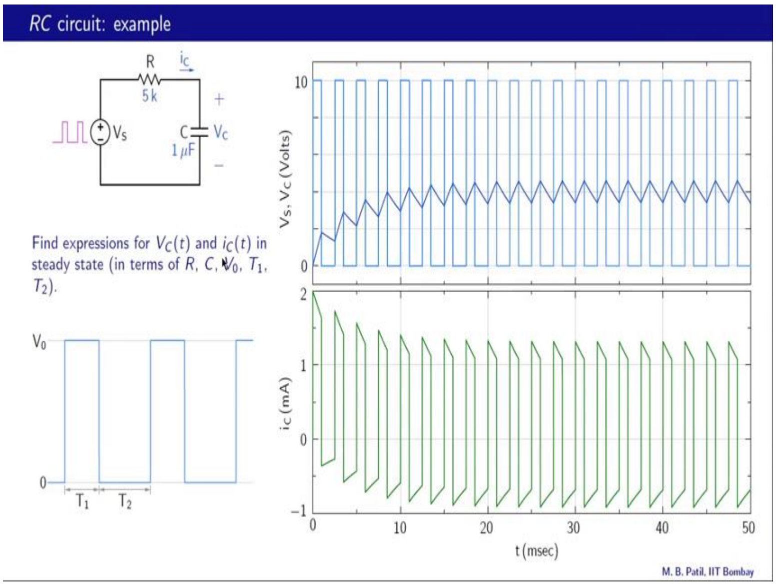

Here is our next example it is a series RC circuit with R = 5 k, and c = 1 µf and the input voltage is a square wave, in this interval the input voltage is high that is 10 V, and in this interval it is low that is 0 V and then it repeats it is periodic.

Let us say that our capacitor is initially uncharged; that means, V c = 0 V, at t = 0 as shown over here; and at t = 0 , the input voltage goes high to 10 V as a result of that the capacitor is going to start charging like that toward 10 V, but that does not happen of course, because at this point the input voltage goes back to 0 V and now the capacitor starts discharging towards 0. And that discharging process is also not completed because the input voltage once again becomes high. So, therefore, the capacitor starts charging again, and this process repeats until finally we have a steady state situation.

This kind of steady state is called periodic steady state as suppose to the DC steady state, that we have looked at earlier what happens in the periodic steady state? Let us take this V c as an example, V c starts off at this point it does change within one period, but then after one period is over it comes back to the same value where it started. So, that is the meaning of periodic steady state.

Let us now look at the current waveform; the current is denoted by i c here the capacitor current, in this first interval when the input voltage is high, the current is positive and in fact, that is responsible for this increase in the capacitor voltage. In the second interval, the input voltage has gone back to 0, and the current becomes negative as shown over here, and this negative current is responsible for this decrease in V c. So, this keeps happening the current is positive here, negative here, positive again, negative again and so on and finally, the capacitor current waveform also reaches the periodic steady state; that means, within one period there is a change in i c, but after one period is over, the i c value comes back to where it started all right and our problem has to do with finding V c (t) , and i c (t) in the periodic steady state.

Let us look at the problem statement now, this is our input voltage waveform, it is a square wave, wave from 0 to V_0 and then back to 0.

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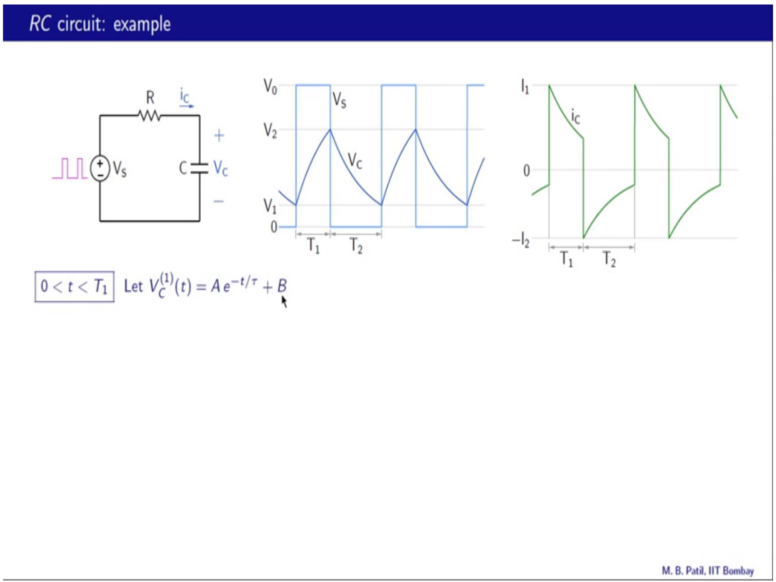

T_1 is the interval in which the input voltage is high that is V_0 and T_2 is the interval in which the input voltage is low, that = 0 V. So, the problem is to find expressions for V c (t) that is the capacitor voltage, and i c (t) the capacitor current, in the periodic steady state and these expressions will be in terms of R, C, V_0 ,this value T_1 and T_2.

Here is a schematic diagram showing the capacitor voltage and the capacitor current as a function of time in the periodic steady state. The light blue graph is the input voltage V s during this interval marked by t_1, the input voltage is high = V_0 and during this interval marked by T_2 here the input voltage is low that = 0. During the t_1 phase the capacitor voltage rises from V_1 to V_2 and during the T_2 phase the capacitor voltage decreases from V_2 to V_1. So, within one period, the capacitor voltage comes back to where it started off namely V_1.

In the T_1 phase when the capacitor voltage is increasing we have a positive capacitor current, and in the T_2 phase when the capacitor voltage is decreasing; that means, when the capacitor is discharging, we have a negative capacitor current and the capacitor current is also periodic.

Now, the maximum value of i_c is denoted by i_1 over here, and the minimum value is denoted by $-i_2$. Now this minus sign is introduced to make our algebra easier, so note that this i_2 is a positive number, so $-i_2$ is negative. Our objective is to find expressions for $V_c(t)$ and $i_c(t)$; let us begin with V_c and we should note first that V_c will be represented by 2 expressions, one expression in this t_1 phase and another expression in this T_2 phase. So, let us begin with the t_1 phase and in that phase let $V_c = A e^{-t/\tau} + B$ and we will call that $= V_c^1$ where the superscript 1 indicates this t_1 phase. So, we now need to find A and B the time constant of course, in this case is = R C.

To summarize we learned how to treat an RC circuit with a switch, we also looked at the salient features of the current and voltage plots for that circuit, we then started with an RC circuit with a square wave input, we looked at the meaning of the term periodic steady state and started analyzing the circuit in that condition. In the next class we will complete the analysis, and also compare the results with plots obtained by circuit simulation until then goodbye.