Algorithms for Big Data
Prof. John Ebenezer Augustine
Department of Computer Science and Engineering
Indian Institute of Technology, Madras

Lecture – 35
The Cat and Mouse Game

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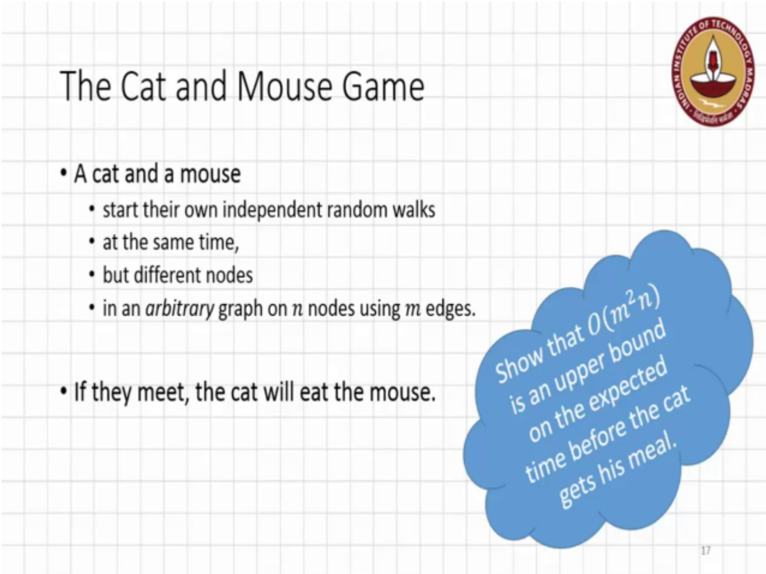

So, before we explore this cat and mouse game, let me actually motivate the reasoning for this problem, at least one reasoning. So, if you are, if we have two random walks exploring a large graph, you kind of interested, you know when will they meet next? So, this is, this problem is, may be these two random walks are walking in this graph and they are exploring in. When they meet, they have to perform some action and we want to understand how often they will meet. So, that is a kind of the motivation for this problem. But of course, you know, being good theoreticians, you know instead of facing, reposing the problem in, in that manner, we will try to post it in a little bit more fun way.

So, in this we treat this as sort of a game between a cat and a mouse. They start their own independent locations and they do their own independent random walk. They start at the same time and then we ask, I mean, that this is an arbitrary graph that they are both exploring. And you know, when the cat and the mouse meet as a, as there is a, when they both, while they are doing a random walk, while they, if they both come to the same vertex, then the cat will eat the mouse.

So, the question we are interested in, want to post in the following way. We want to show that O of m squared n is an upper bound on the expected time before the cat gets its meal, basically, the time the cat will reach a vertex where the mouse will also be present.

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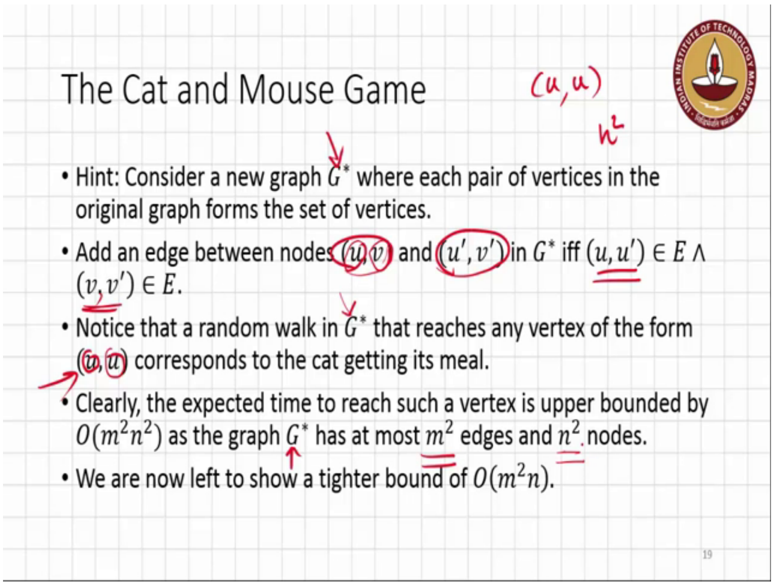

Now, to understand this problem and find a way to get a handle on this is what we do is, consider a new graph and we call that graph G star and this graph, there are more vertices, there are actually n squared vertices. Basically, every pair of vertices in the original graph forms the set of vertices. In fact, every pair where each of the pair is drawn with replacement, so you, you, so even pairs, you, even, even have pairs of performed u, u, where it does not, we do not need both the vertices to be distinct.

So, that means, we really have n squared vertices in this new graph G star, but how do we place the adjacent in this new graph. So, you, so you consider two vertices, u and v and u prime, v prime in this new graph G star. Now, there will be an edge fitting these two vertices if and only if u, u prime was, was a, was an edge in E and also v, v prime was an edge in E.

So, in other words, intuition is following. Now, first if you look at any such vertex, the first one gives you the current location of the cat in the original graph G and v kind of gives you the current location of the mouse in the original graph G. So, so that is the intuition and you can immediately notice that a random walk in G star that reaches any vertex of the form (u, u) corresponds to the cat getting its meal, why? Because well, both the cat and the mouse are in the same vertex, so then the cat will eat the mouse.

Now, we just saw a little while ago and rather, we reminded ourselves little while ago, that the expected time to cover the entire graph and therefore, also reach a vertex of the form (u, u) is going to be at most m squared n squared because this graph G star has at most m squared edges and we saw that it has n squared nodes, but we need to show something a little bit title.

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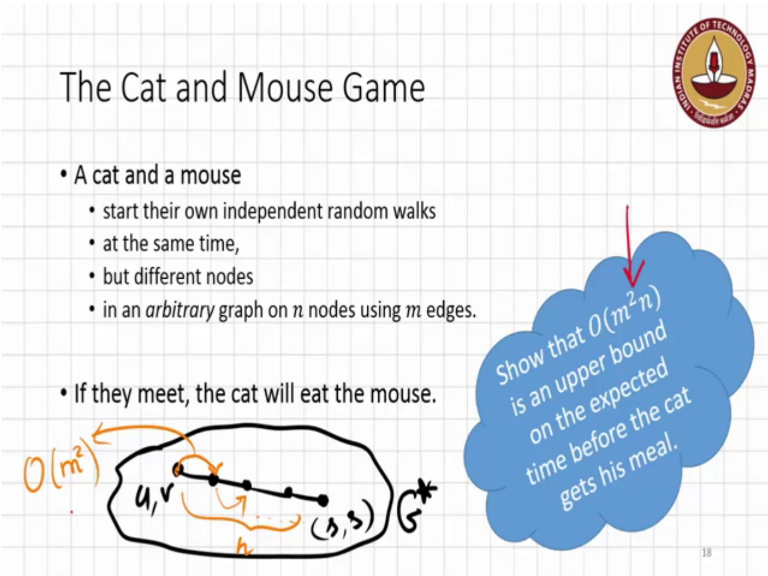

If you recall what we need to show is, that the, the cover time is m squared n. So, we need to shave off a factor of n. So, how do we do that is the question. So, let us, let us think about why that might be the case. I leave you with one intuition.

So, now let us look at this graph G star here. Now, wherever be your starting point (u, v), you first argue, that there is a path of length at most n that will then lead you to a vertex of the form (s, s) and now we know what is the, the time, the expected time to go from here to here when you are doing a random walk and similarly, you know the expected time to go from here to here and so on.

You can add them all up, this whole thing will be of length n, each one of them, each one of these components we need to argue is going to be at most O of m squared and therefore, we should be able to get the final result. So, that is the sort of the technique that we will need to follow in order to be able to prove this claim.