Algorithms for Big Data
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Lecture – 33
Two Different Random Walks on Lines

Let us now look at two different Random Walks.

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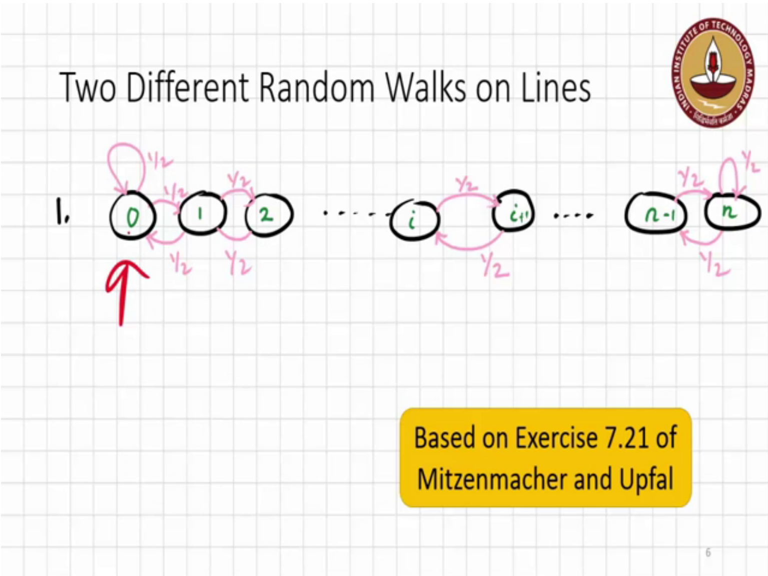

Basically, on a graph that is very linear in structure. But we will see that the two random walks have very different characteristics in particular, very different stationary distributions. So here is one, we have vertices in, you know arranged in a linear order starting from 0 to n. Here the transitions probability have few more forward and probability have few more backwards except for the two very last vertices the 0th vertex and nth vertex. For example, in the 0th vertex is no way to go backwards. We have instead a self-loop with transition probability half. Now this is one random walk.

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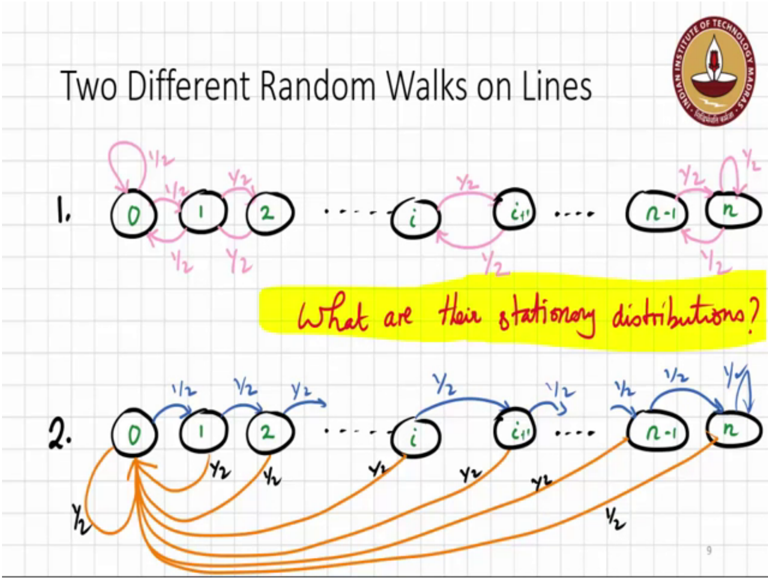

Let us also look at the second random walk that has some structure. Again you have vertices from 0 to n, and similar to the previous one you have forward transition probabilities half. In this case instead of just stepping back one step we transition all the way back to 0 with probability half. The previous or first random walk can be taught of as a sort of undirected random walk, this one is the direction is exploited, so you actually returned back all the way to 0.

These are two stationary distributions. Let us try to understand I mean these are two Markov chain or Random walks rather and we try to understand what their stationary distributions are.

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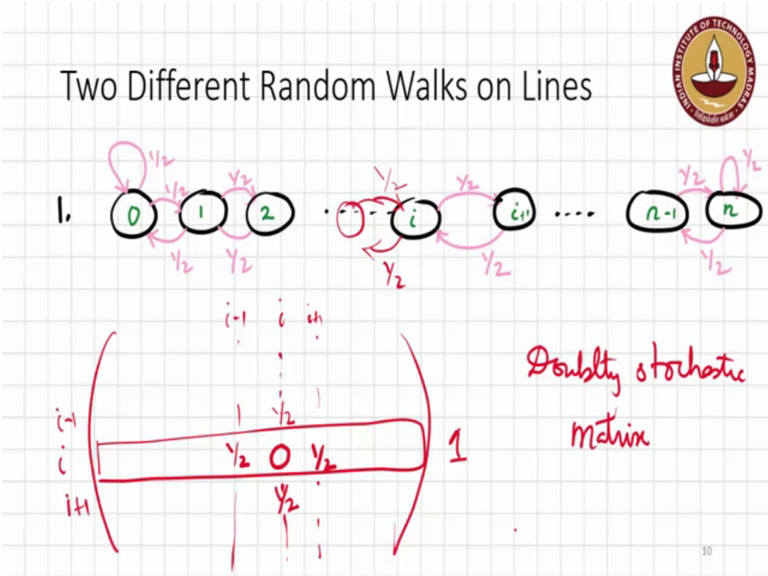

In this case here, what do we notice about this graph or about this random walk? Well, if you look at, let us try to work out the transition matrix. Now the transition matrix let us look at we have node i, the node i can move with probability half, so this will be let say i minus 1 i and i plus 1. If you notice the probability of staying in i itself is 0, but if you look at the probability of transition into i plus 1 that is a half and moving backwards to i minus 1 will again be a half. So, the probabilities in any row will add up to 1.

So, clearly this is a Stochastic Matrix. What about the columns? Well, if you look at the, to fill in the columns we will have to know how the probabilities are coming into vertex i. From i plus 1 you can move to i with probability half, so that will be actually half over here, so it is i plus 1 here. And from i minus 1 you can also move in with the probability half.

Again if you look at the columns the probabilities add up to half. So what we have here is a Doubly Stochastic matrix, which immediately you should tell you that the stationary distribution is the uniform distribution.

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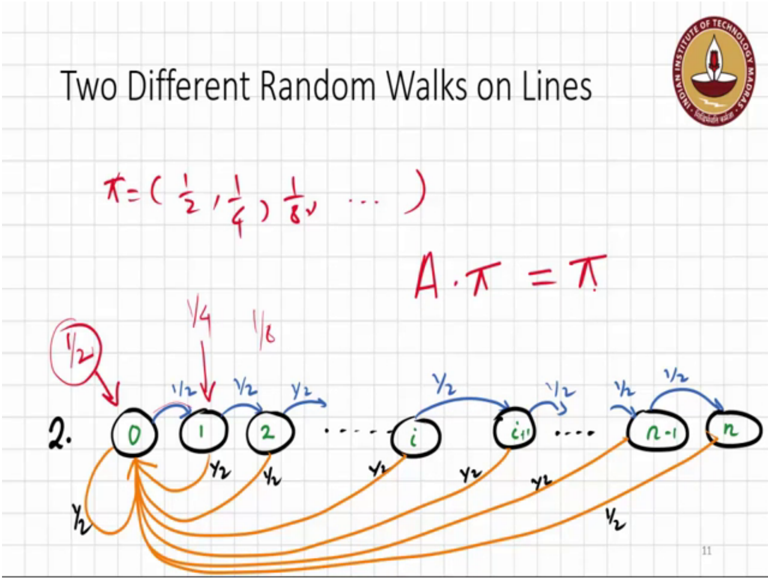

How about this random walk? Now here, if you think about it, there is a much higher probability of a being in this vertex. In fact, wherever you are with probability half you will be coming to that vertex. So that immediately tells you. Now, we are speaking more intuitively and you should verify this more carefully, immediately tells you that the stationary distribution entry corresponding to this particular the 0th vertex is going to be the half. So, what is the probability with which a random walk will be in this vertex? think about it. The only way you can get to that vertex is by being at vertex 0 and then making a transition into this vertex.

So, you will be in vertex 0 with probability half and we kind of made an intruder statement towards that effect already, which means that you will be in this vertex 1 with probability half times a half. So that is going to be one-forth, and then here with probability similarly one-eighth and so on. And you should now verify that this is in fact a stationary distribution for this Markov Chain.

So now, for that you have to actually write out the transition matrix multiplied with pi and ensure that it actually gives you pi again. So, that is something that is left for you to verify.