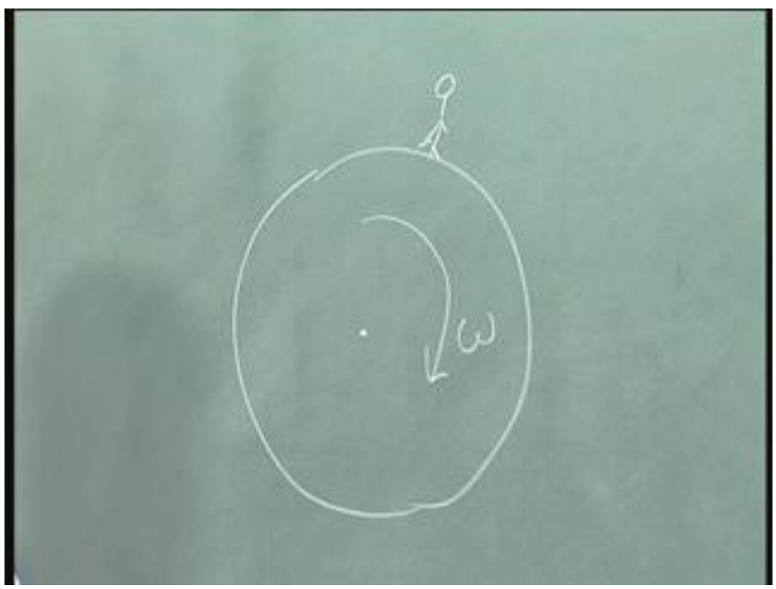
Engineering Mechanics
Prof. Siva Kumar
Department of Civil Engineering
Indian Institute of Technology, Madras
Dynamics of rigid bodies

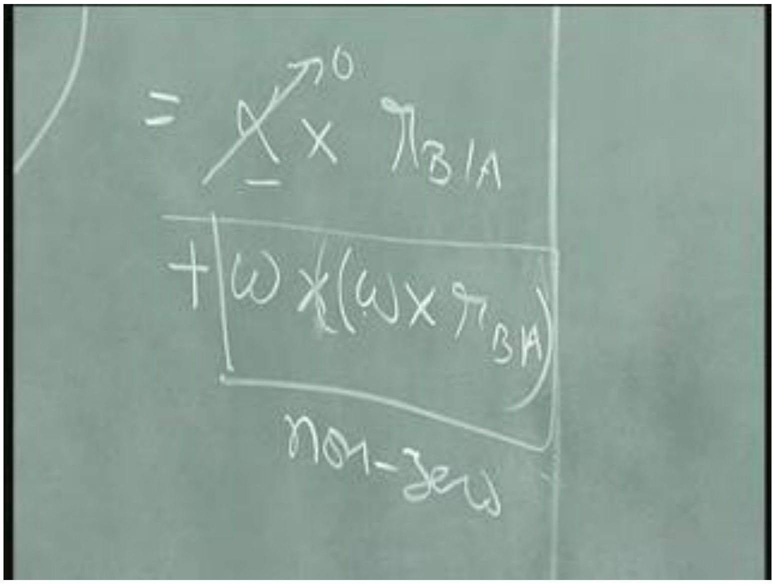
Now let's just look at a simple example. We are almost on the equator of the earth. The earth is rotating. Let's assume that this is the polar direction, its rotating. The earth is rotating, I am standing on this. Will I have an acceleration? Let's look at it. This is the center and it is rotating with an angular velocity. Is it angular velocity or angular acceleration? Almost angular velocity because it's a constant angular rotation that it is having and it's not very difficult for me to find out. It rotates about 360 degrees in a matter of only 24 hours. I can always find out omega. Now the question I have here is, is this man who is standing, I am standing on the equator, am I accelerating? Let's say the fixed frame of reference is with respect to the center. The answer is yes.

(Refer Slide Time 01:31)



Because as the earth is rotating, I am also going all round it. What would be the acceleration here? If this is given by, this is B, this is A. I am going to use the same notation so that it's easy. So that I have r_B with respect to A. A is also the origin, so it makes it easy r_A equals 0. Therefore if I have to find out r dot of B, it is nothing but r dot of B with respect to A. r double dot of B with respect to A is what I am interested in. That will be given by, as we have already derived alpha cross r_B with respect to A, which is the tangential component of acceleration plus the inward radial component of acceleration is given by this. Omega is known, omega is known, r of B with respect to A is also known here. I can find out and therefore this is not zero whereas this is zero. A man standing on the equator will see an angular acceleration inward. When he moves from here to here and that's given by this. Rest of it is not very difficult for you to find out what will be... the direction is very simple.

(Refer Slide Time 03:09)



The direction is, it is directed along, acceleration is directed along the centre of the earth or towards the axis. Is this clear? Let's see if we can answer this question. What is the man doing? The man is just standing still, so no problem. What if the man is moving on the equator?

Now you have two things, the earth itself is moving and there is this man who is also moving. Probably he may be accelerating along lets say the equatorial line. Can I find out what is the actual acceleration, velocity of that particular person? Let's do that exercise and see whether we can do that. What is the difference right now compared to what we did earlier? In the earlier exercise this point B and point A had one particular relationship. What was that? What was the particular relationship that we knew? Let's make it a little more complicated so that you can think a little differently.

Let's say he fitted himself, like our shastra, he fitted himself with a particular rocket booster and he is going like this. He just shot up like a bird and he is going like this with an acceleration. Can I find out the actual acceleration of this particular person? Let's try to answer the question. One another way of looking at it is supposing I have a body and let's say there is a small marble on this body which is moving independent of or let's say there is an insect. There is an insect that is moving around with a velocity or acceleration.

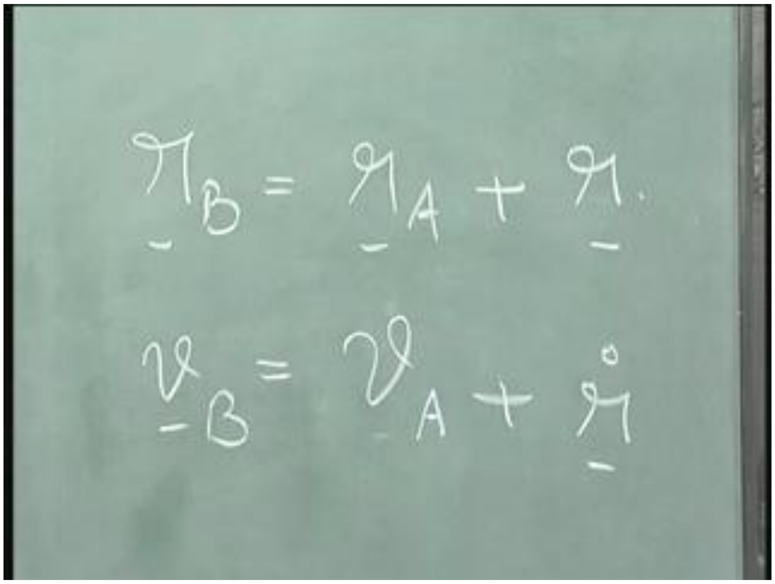
Now remember this body also is moving and accelerating and can I find out with respect to fixed frame of reference, what would be the acceleration of this particular insect? Why am I interested in acceleration of that particular insect or velocity of that insect with respect to the fixed frame? What's the reason, why can't I just live with what ever acceleration that I am seeing, when I am standing on this? Because the acceleration that I am seeing, when I am standing on this rigid body is a relative acceleration whereas the Newton's law can be applied only if I have been able to fix a particular frame, a non-accelerating frame.

In this particular case those are these capital X, capital Y frame. If I can define with respect to this frame then whatever acceleration that I find out, can be used for f equals m a equation. The entire exercise is to make sure that I am able to use the equations of motion, that's the idea. One thing that I already know is if I have a point and relative to lets say this is A, relative to this point A we already found out what would be the relationship of any other particle that is fixed to this rigid body with respect to A. Now what we want to do is we want to find out, what would be the acceleration of this guy, actual acceleration or acceleration with respect to fixed frame rather than a frame attached to this, which means first thing I have to do is I have to attach a frame with this body.

(Refer Slide Time 10:08)

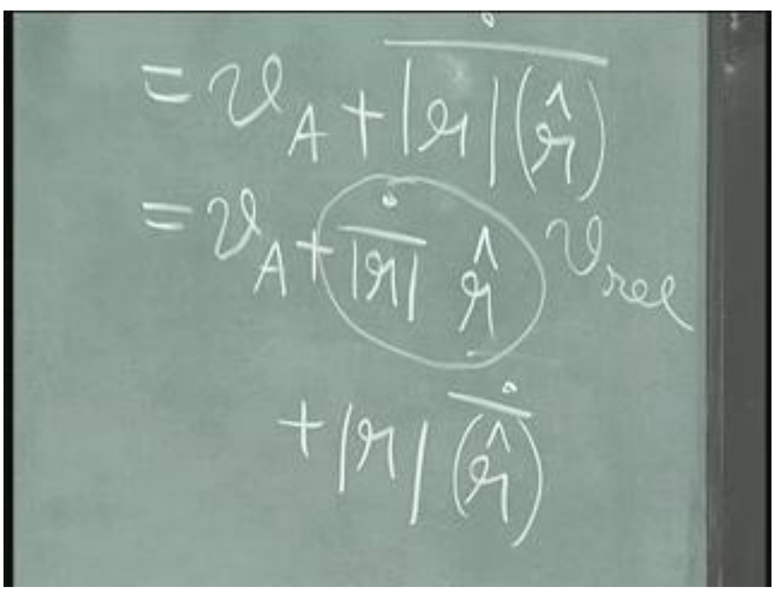
Let's attach a frame. So we attach a frame like this. What about this frame? If this rigid body is rotating, this frame will also be rotating, that understanding should be clear. Let me call this as small x, small y and this can be taken to be r, the position of the insect with respect to this frame that is attached to the rigid body. If this insect were not moving, it's easy for me to find out what it is. Because this distance is maintained. The only difference that I see is that this distance is not maintained as far as the insect is concerned. I have to consider that in my equations. Let's proceed to do that exercise. Let's take this as r_A, let's say this is, shall we call this as B. Let's just call for now this particular insect to be positioned at B and this B may be moving with respect to A. Let's say this is r and this is r_A and this is r_B, like what we have done. How do I find out r_B? r_B is given by r_A which is a location of the point at which we have fixed the axis plus r. What is r? r tells me the location of this particular insect. Now I have to find out the velocity and velocity is not very difficult. If I have to find out velocity of this insect with respect to fixed frame, what would that be? It would be r dot A plus r dot. This is very simple, this is nothing but V_A because I have already defined r_A with respect to fixed axis.

(Refer Slide Time 11:04)



There is no problem in defining V_A. Now we have an r dot, remember this has a magnitude and a direction. I have V_A plus, I am going to now write this as r dot magnitude or rather r magnitude times the direction, the entire thing dot. It makes it easy for us now to understand, which is nothing but V_A and if I differentiate by parts, it is the derivative of this. Let me just call that as r dot times r cap plus magnitude of r times r cap dot. Is it confusing? It’s fine. Let me just write it a little differently, plus r magnitude times r cap dot.

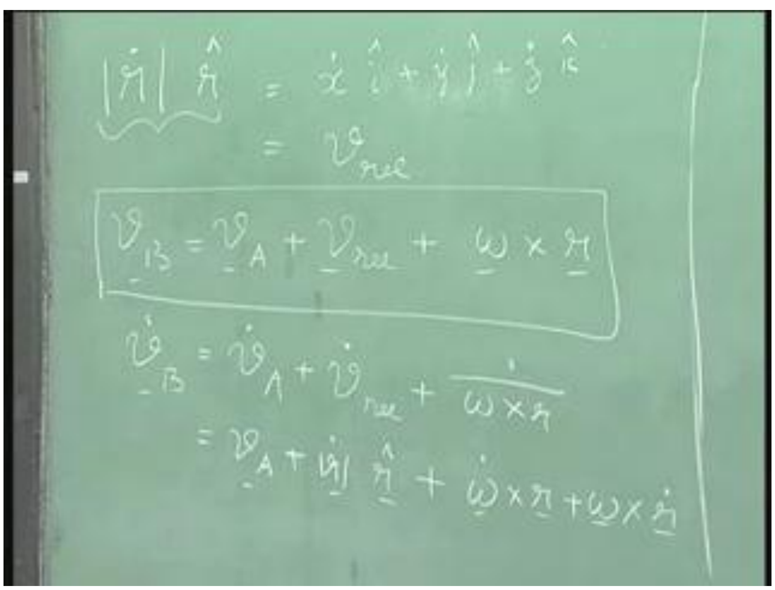
(Refer Slide Time 13:46)



What is this? You are taking for example if I am doing this, it is nothing but, what is this? This is i, j, k taken here and this is nothing but the magnitude of r dot. It is x dot i plus y dot j plus z dot k. Do you understand this? This is what it means. If I write this as this, let me call this as $V_{relative}$. Why? Because if I am standing on this and having this fixed frame of reference on the rigid body and I am seeing the insect moving, that will be the derivative of x, y and z and therefore I am going to call that as $V_{relative}$. What I know now is this is $V_{relative}$. Correct? There is one more term here, which is the magnitude of r times the derivative of the direction r cap. How do I find out the derivative of the direction? If there is a rotation occurring, so I know that this is rotating. If it is rotating, how do I find out this? It is nothing but omega cross r cap. So using that I will get V_B equals V_A plus $V_{relative}$ plus omega cross r, that's it. As you can see if I don't have this, it is nothing but V_B equals V_A plus omega cross r which is something that we already knew for a rigid body. What is added to that is $V_{relative}$.

Now I seek to find out accelerations, acceleration of B and I want to relate through acceleration of A. Acceleration of B I want to know, given acceleration of A and the rigid body movement. How do I get it? Let me just translate this over there because it's easier. What I have here is V dot B equals V dot A plus V dot relative plus omega cross r dot, simple. Now instead of writing V dot relative, I am now going to use what we have already written here, which is the r dot time's r cap. It is V_A plus r dot times r cap plus omega dot cross r plus omega cross r dot. Is this okay?

(Refer Slide Time 16:41)

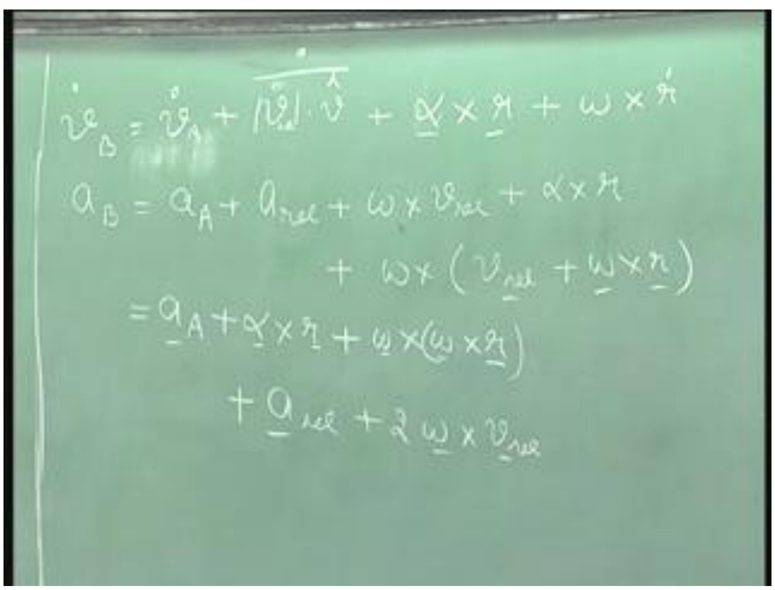


Remember V dot relative is a vector again and if I take a derivative of that, I will have to take the derivative of the magnitude as well as the direction and that's an important thing we have to do, this whole thing dot. V dot relative, V dot B equals V dot A. Here V dot A (Refer Slide Time: 17:18). Let's just rewrite V dot B equals V dot A plus, this v dot relative again I have to now look at the magnitude and the direction.

Let me write it as either this way or this way, I have V dot magnitude, let me write it like this V dot magnitude times, lets write it as $V_{relative}$ magnitude times V direction. This whole thing taken derivative of plus omega dot is nothing but alpha, alpha cross r that does not change. Again I have omega cross r dot. Now this is acceleration of B, this is acceleration of A plus this I have to now resolve, how do I do that? This will give me two terms, one term is related to derivative of $V_{relative}$, so I will get an $a_{relative}$ which is nothing but a derivative of $V_{relative}$ magnitude times the direction which is $a_{relative}$. I am going to write it as $a_{relative}$ plus $V_{relative}$ magnitude times which will give me what? Directly I can write this to be omega cross. Is that okay? Plus alpha cross r plus, what is this?

What is r dot? r dot again I can take from here, I have already written r dot here, r dot happens to be $V_{relative}$ plus this. I have omega cross, I know that this is the thing that is related to r dot. I will write it as $V_{relative}$ plus omega cross r. Is this clear? This is clear because if you look at this particular expression r dot, this r dot manifests as V relative plus omega cross r. Therefore here that's how it manifests. Now we just have to group them together. What do we get is nothing but A plus let me write this, plus notice here I have two terms omega cross, omega cross r, omega cross $V_{relative}$. Let me write omega cross omega cross r plus I have this term which is $a_{relative}$ plus what else is left out? Omega cross $V_{relative}$, omega cross $V_{relative}$ added together is two times omega cross $V_{relative}$.

(Refer Slide Time 21:23)

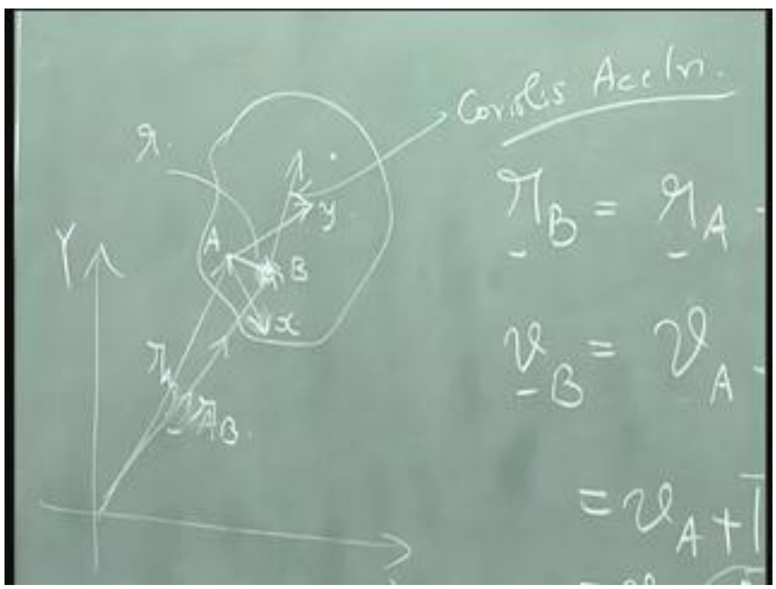


What was this? What is this that you know of? a_B equals a_A plus alpha cross r plus omega cross omega cross r, it is even that we found out for a rigid body. If there is an insect that is moving with an acceleration or whatever, it will be the relative acceleration with respect to the rigid body on which I am standing stationary and then looking at it, plus one more term. And this one more term is, what is counter intuitive.

One would think, if I have a point over here and this is a fixed point with respect to the point that I am looking at as the pivot and when it is rotating, this acceleration is given by a_A plus alpha cross r plus omega cross omega cross r. Therefore if I have an insect that is moving with an acceleration, I should expect an additional $a_{relative}$. But what is counter intuitive is, there is an additional term that comes here which is omega cross $V_{relative}$. This counter intuitive acceleration is often called as coriolis acceleration. Is this clear? Otherwise it's a very simple idea that I can understand, but upon doing this derivation, we find that there is an additional acceleration.

We already found out what would be the direction of this, what would be the direction of this. The direction of $a_{relative}$ is not very difficult to understand. It is along the direction of r and how about this? It is perpendicular to the direction of omega which is nothing but the perpendicular to the board and it is also perpendicular to $V_{relative}$. Understood? If I take a perpendicular direction with respect to those two, so this and this, this is the direction in which I will expect coriolis acceleration to occur.

(Refer Slide Time 24:14)



Now a simple example. Let's say I have on top of a tower, imagine I am the tower. On top of it, there is a rotating restaurant. This is a rotating restaurant, this is the center of it. There is a person that moves with a constant velocity. What will happen is, there will be an acceleration that he will experience which will be perpendicular to his velocity that he sees and perpendicular to the direction of the rotation. So perpendicular to this and let's say he is moving towards the center, perpendicular to this and perpendicular to this is a direction like this. He will see himself moving, being pushed like this and that's what is called Coriolis force. When you are in the relative realm, you will seemingly see that acceleration. When you are outside with the fixed frame of reference, this seems an acceleration that does not give any sense of feeling that you see when you are inside the restaurant.