Engineering Mechanics
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Dynamics of rigid bodies

Having understood the time derivative of the vector lets say r that we considered earlier, let's use that fact in order to derive the velocity of a particular rigid body. Where do we start from? We start from as usual, a fixed frame. We will look at the fixed frame. Let's say there is a body at a particular instant of time. The question we have basically is supposing I know the position of a particular point on the body. I wish to find out what would be the position of another point, any other arbitrary point that lies on the body. I am going to just look at the planar kinematics because that's simpler to understand, I can use the board to draw. That's pretty simple. This should be r_B and what should I call this as? If I have to find out B in terms of A, A becomes a reference.

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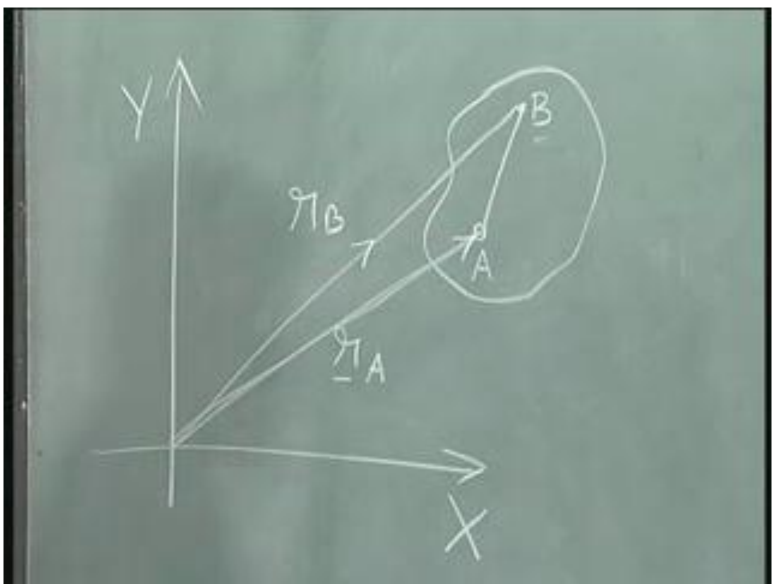

It emanates from A and goes to B. Are you with me? Therefore if I call this as r, it is r_B equal to r_A plus r. All these are vector coordinates. r_A plus r is equal to r_B. Remember, we have defined B with respect to A. As you can see, it emanates from A, goes to B and therefore this is the position of B with respect to A, an important point to note. I am going to write that over here, r of B with respect to A. We have written relative position of any other point with respect to one of the reference points on the solid. Having done this, if I have to find out the velocity relationship between these that should be simple. All I have to do is to take the time derivative of these and that is r dot B equals r dot A plus r dot B with respect to A.

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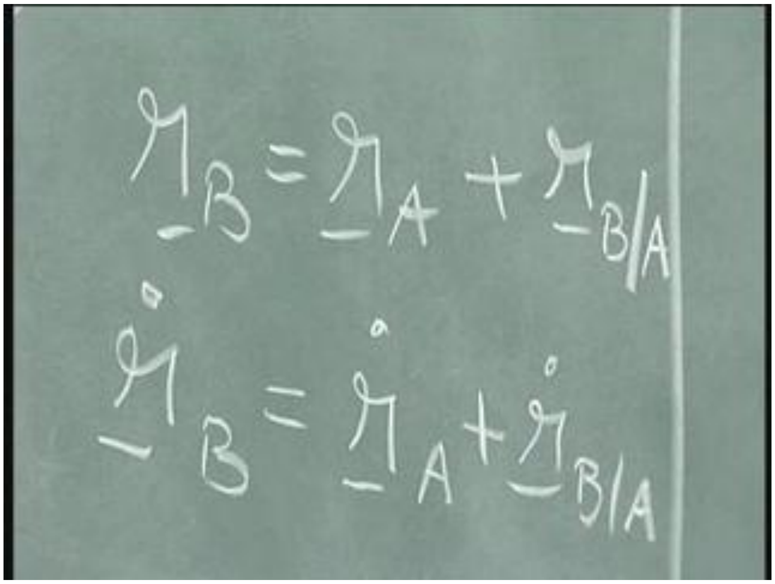

r_B is defined with respect to the origin of frame. r_A is defined with respect to this whereas, r_B with respect to A is a relative position and therefore these two, there is no problem. With respect to fixed frame this will be the velocity of B, this will be velocity of A and this will be the velocity of B with respect to A. Here is where we have to understand one important thing related to what we have written here and that is this. Let me use the same, this is a screw driver.

Supposing I rotate it with a point as the pivotal point or if I take this point and rotate it, let's see what difference it makes as far as rotation itself is concerned. Let's say I am going to take a 90 degree rotation with respect to a central point, it goes like this. With respect to an end point if I take like this, it goes like this. Now supposing I know in this concurrent position, one of those points with respect to a fixed frame and I know there is a rotation that is occurring on this rigid body. Every point on this rigid body will be seeing the same rotation. If it is seeing the same rotation, it will also see the same rate of rotation and a double rate of rotation.

We are going to use that fact over here. We are looking at this body rotating and let's say the axis is perpendicular to this. We already derived a relationship with respect to doing that particular thing. Let's make use of that after having written this. Now the question is what is V of B with respect to A? Remember in the earlier exercise we had A B pivoted about A, there was a rotation occurring and we wanted to find out what would be the rate of this rotation. Here the exercise is similar because given that this point is fixed and there is a rotation occurring, I would want to find out the rate of this.

One important observation here is how much is it changing? I have a body that is rotating, a rigid body that is rotating and I have one point here, another point here about A it is rotating. What would be the change in the magnitude of r_B with respect to A? Only the magnitude, 0. Correct? What is the other aspect that I should look at? Change in direction. When we did that exercise earlier, we looked at change in direction alone, the rate of change in magnitude was considered to be 0. We separated magnitude and direction. Right now we have a situation where we have to look at only the directional change of this vector. It becomes easy for me or if I know that this is rotating with an angular velocity omega then, it is nothing but omega cross, so let me write it in the style that we have already written.

If I have to find out derivative r dot of B with respect to A, it is omega cross r_B with respect to A. Therefore if I substitute in this, I will get r dot B equals r dot A plus omega cross r of B with respect to A. I have written a relation between two points of the body, V_B equals V_A plus omega cross r of B with respect to A. One step and that's it over.

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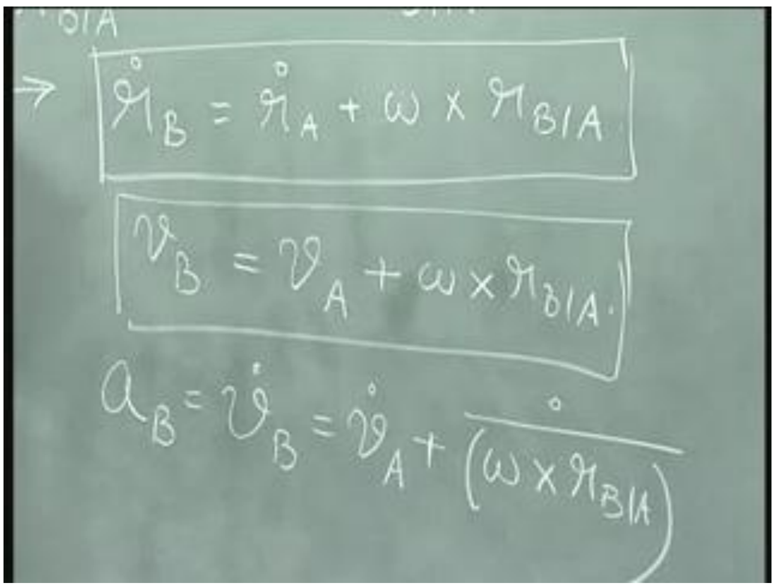

Let's complicate it a little bit. I want to find out the acceleration. What should I do? Naturally if I want to find the acceleration and I am looking at, with respect to the fixed frame then, all I have to do is to take derivative of this particular thing. As simple as that. Let's do that. If I take the derivative, so let's say a_B, acceleration of B is nothing but V dot B and that is equal to, based on this, V dot A plus omega cross r B with respect to A, the derivative of the whole thing.

I am just putting a sort of line over here on top to represent that I am taking the entire derivative. Now this implies, so V dot B equals V dot A plus what should happen to this? I will have omega dot cross r B with respect to A plus omega cross r dot B with respect to A. Looks okay? What is omega dot? The rate of angular velocity is angular acceleration. Therefore this will be, acceleration of B can be written as acceleration of A plus, let me just denote that as alpha, angular acceleration, cross r_B with respect to A plus, now you notice that this r dot B with respect to A is revisiting and I know that is equal to omega cross r_B. I can just substitute over here. I will get omega cross omega cross r_B with respect to A.

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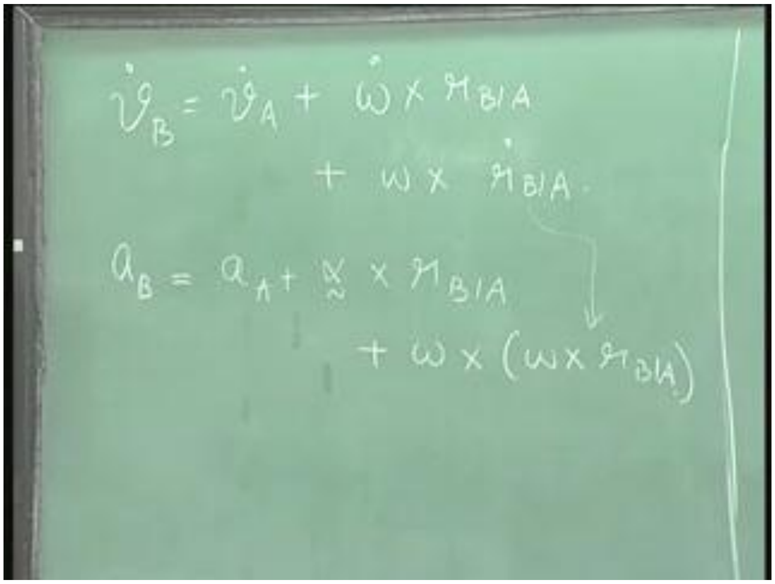

That's it. I have a_B is given by the acceleration at A plus angular acceleration cross r_B with respect to A plus angular velocity cross omega cross r_B with respect to A. What is the direction of this and what is the direction of this? If we get an idea then we are talking little more sense. Let's just go back to this. When I say omega cross r_B with respect to A, omega is perpendicular. r_B with respect to A is like this, perpendicular and this, which means if I draw a line that is perpendicular to this, an axis that is perpendicular to this, this is the direction in which omega cross r_B with respect to A should occur.

The direction of r dot B with respect to A is this. I am just showing the direction. Sorry to write this small. It is r, do you agree with me? This is perpendicular. Now, observe this expression. What's the direction of omega cross r_B with respect to A? It is direction of r dot B with respect to A, which is perpendicular to omega and r_B with respect to A. Now I am taking omega cross this particular, this is the velocity here. This is the velocity we already know. Omega cross this will give me what? Perpendicular to these two but I am looking at omega cross this and therefore it will be in this direction. This is the direction of omega cross omega cross r_B with respect to A. do you agree with me? I am just looking at one term in that particular equation.

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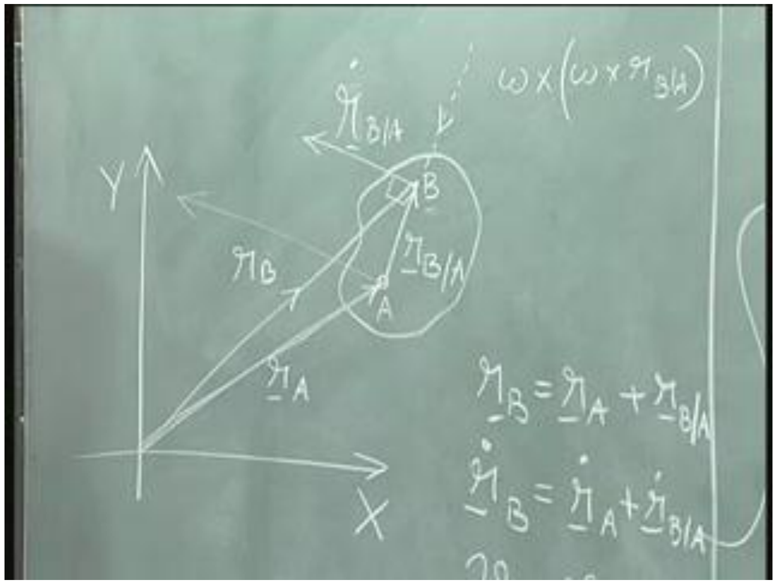

How about this? What's the derivative of omega dot? The perpendicular direction is not changing at all. If I have theta dot times n, alpha is nothing but theta double dot times n. This is the direction along the perpendicular to the board, cross r_B with respect to A. What should I get? It has to be perpendicular to those two and that's how I will get this and this is alpha cross r_B with respect to A. Are you with me? Now what do I notice about this particular point, if I see this direction it is towards A. It is going to rotate like this. If I draw an arc about this A, a circle about this A this seems to be along the radial direction and this seems to be along the tangential direction.

I am going to call this as a, what shall I call this as, t or B with respect to A. This I can call as a_n or let's say r, a_r is fine; a_r B with respect to A. So going back to this, I know this is along the tangential direction. I am going to call it as a_t and this is going to be along the negative of r direction, I can call it as a_r or a_{-r}. Therefore acceleration at B is equal to acceleration at A plus a tangential component and a radial component. Supposing I know that the body is rotating with a constant angular velocity, immediately alpha is 0. It is like having a thread, so I am taking this, I am just rotating with an angular velocity. What will happen? There will be a centrifugal force, and therefore this will be holding it in place. Now or in other words this acceleration is related to basically that. It is trying to tend inward, the acceleration is tending to go inward. Later on we will understand how this translates to a centrifugal force.