**Engineering Mechanics**
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**Dynamics of rigid bodies**

What we'll do to start with is let's consider a vector. Let's say a vector like this so let's say this is a vector r. I am just going to represent this as that vector. This is point A, this is point B. This is point A, this is point B. Now if I have to find out the change in this vector lets say this vector has a magnitude and a direction. If I have to find the change in this vector, what all changes are possible. The changes that are possible are in the magnitude and in the direction, so this can change and this can change. When I take the time derivative of the change in a particular vector, it will involve product of the time derivative of the magnitude change and the time derivative of the directional change. Most often than not, since we most of the times think in terms of scalars, this concept of derivative of the vector becomes difficult.

Let me just introduce that particular concept first so that it's easier when we derive motion of rigid bodies. This is a three dimensional vector, let me just put it like this. Now if a change occurs in the direction only, let's look at what happens so that we can proceed forward to take the change with respect to the magnitude. Let's say it's something like this and it has shifted its direction arbitrarily in a particular direction. One thing that we find is there is an initial direction and there is a final direction and there is an angle between them.

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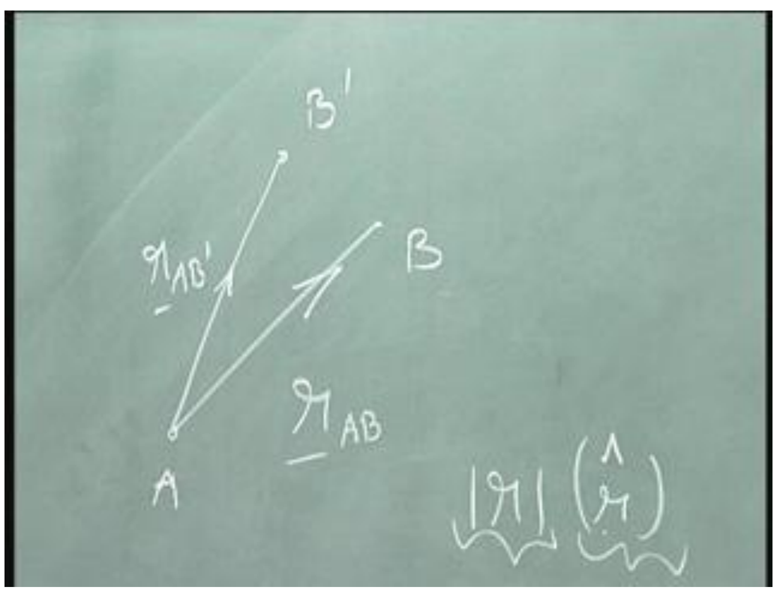


There is one like this, let's say one like this which is not exactly out of plane but a plane that is. You know that there is a plane that can pass through the changed vector and the original vector. Let us say on that particular plane, if I take that particular plane, rotate it and make it this blackboard then what I would see is it would be like this and it would have changed to something like this. Now let's just retain A to B at the same place and look at the change in B. It makes it easy for us to understand. Once we have a visual picture then it is easier to comprehend what happens to the time derivatives. Let's say after a gap of delta t, I notice that this vector has now shifted direction to something like this. Let me just draw that. Notice that I have assumed that there is no change in the magnitude, just to start with as an assumption. It would have gone from B to B prime. Do you agree with me?

Initially let's call this as $r_{AB}$, it has become $r_{AB}$ prime. I have put a line at the bottom it means a vector. So far so good. Now I need to find out how much changes occurred in this vector. The magnitude has not changed but the direction was changed. Naturally if I have to find out the change, it is the changed vector minus the original vector will give me the changed vector over delta t and lets call that as delta r. What would be delta r here? Naturally $r_{AB}$ plus a vector like this will give me $r_{AB}$ prime and therefore this should be delta r. So far it's okay?

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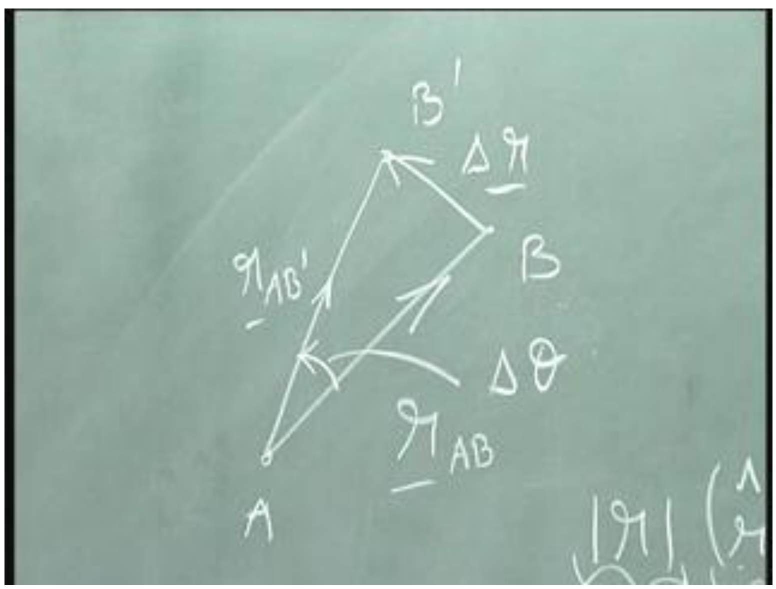


No problem. What is this angle? Let's take this angle to be some delta theta. Mind you, we will use this in order to find out the time derivative. Now, limit as delta r by delta t as delta t tends to 0 will be nothing but limit as this tends to 0. Correct. Is this okay? So limit as delta t tends to 0, $r_{AB}$ prime minus $r_{AB}$ vector by delta t. Perfectly alright.

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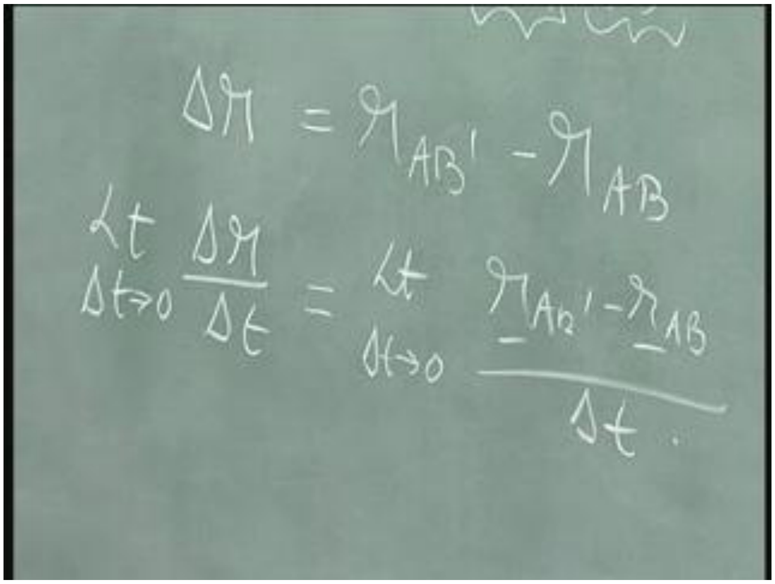

Now how do I find out the magnitude of delta r? In this particular case, if I look at this delta r, delta r again has a direction and a magnitude. Let's look at the magnitude. What would be the magnitude of this delta r vector? Can I find out? That's the question I will ask. I know that this is a delta theta sweep that has occurred about A and I know delta theta is small because I am taking a limit. So for small delta theta that has occurred over delta t, can I find out this. Since the lengths of $r_{AB}$ and $r_{AB}$ prime are the same, it is equal to this one of the lengths, times delta theta.

Let me just call that as r times delta theta. And what is this r? It is the magnitude of the vector for which we are finding the time derivative. So it's easy to find out the magnitude. How about the direction? Let's look at this particular plane. Any vector on this particular plane, can I say is perpendicular to a vector that is outward normal to this plane. The answer is yes. It is perpendicular to this r, it is perpendicular to delta r. When we found this particular r times delta theta, we took this to be tangential to $r_{AB}$ or in other words this is perpendicular to $r_{AB}$.

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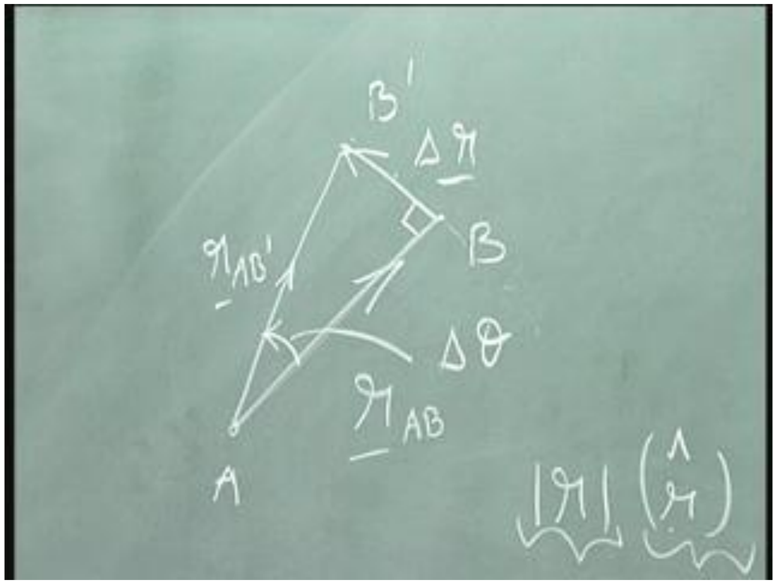


The fact of the matter is number one, delta r vector is perpendicular to outward normal. Let me call it as n of the board, it is also perpendicular to $r_{AB}$. Do you agree with me? What am I interested in now? I am interested in finding out only the direction because I know the magnitude of delta r. If I need to find out the direction, let me just put a cap over here, just to indicate that I am looking at a unit vector along this direction. Let me call the one that is perpendicular as n, lets call this as lets say m vector without loss of generality. What I am basically saying is this is perpendicular to m cap. When I put a cap it means it's a unit vector. If this is the case, what do you know about direction of delta r?

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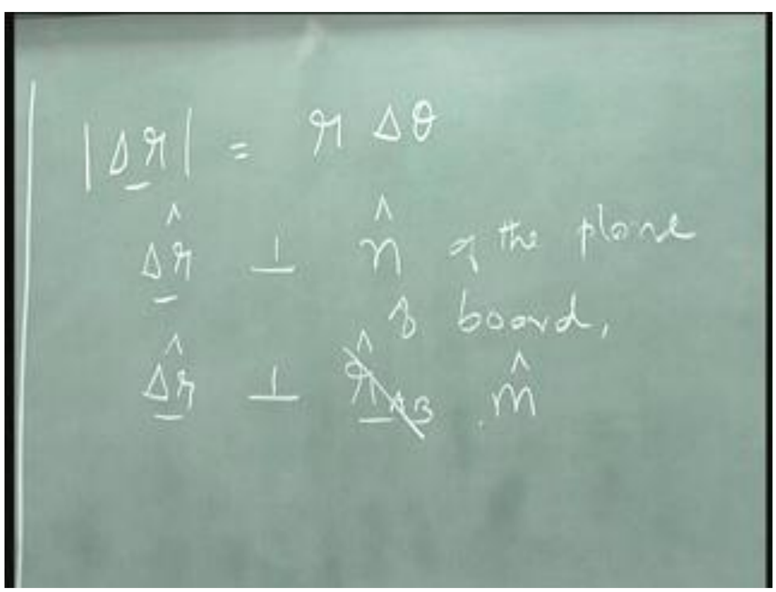

What would be the direction? Very simple. If I know a little bit of vector algebra, it should be related to the cross product of these two, n cross m will give me delta r. With the consistency in direction, I should be able to find out. If I take n and then cross it with m, I should get delta r and therefore I will say delta r cap is equal to n, unit vector along the perpendicular direction cross with m. So far so good, no problem. Let's put them together, I know the magnitude. I know the direction. Now what is delta r equal to?

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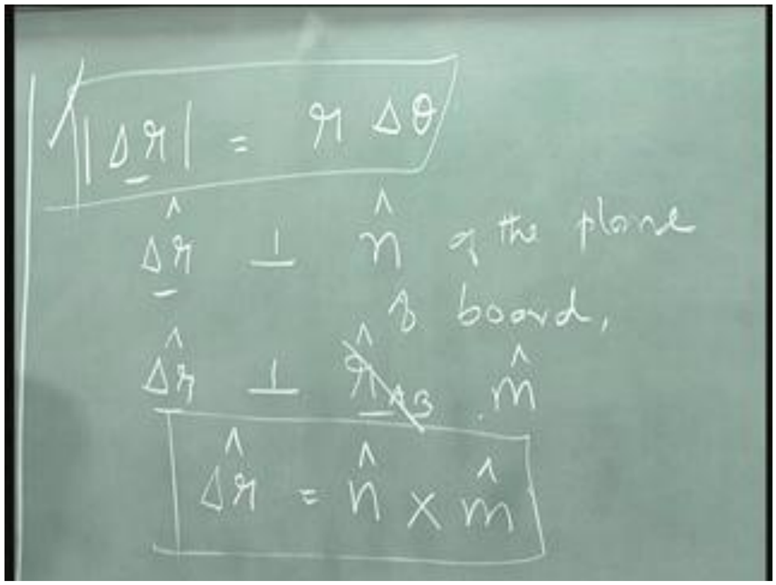


It is equal to delta r magnitude times delta r direction. Putting them together, it is r delta theta times n vector cross m vector. Any question from this? Now we will proceed to the time derivative. Let me write this as limit, as delta t tends to 0. What is this? I am going to take that, which is r times n cross m times delta theta by delta t, just substituting this expression over here. Limit as delta theta by delta t will give me theta dot and therefore this will be r theta dot n cross m. Is this clear? Let me call that as r dot of AB.

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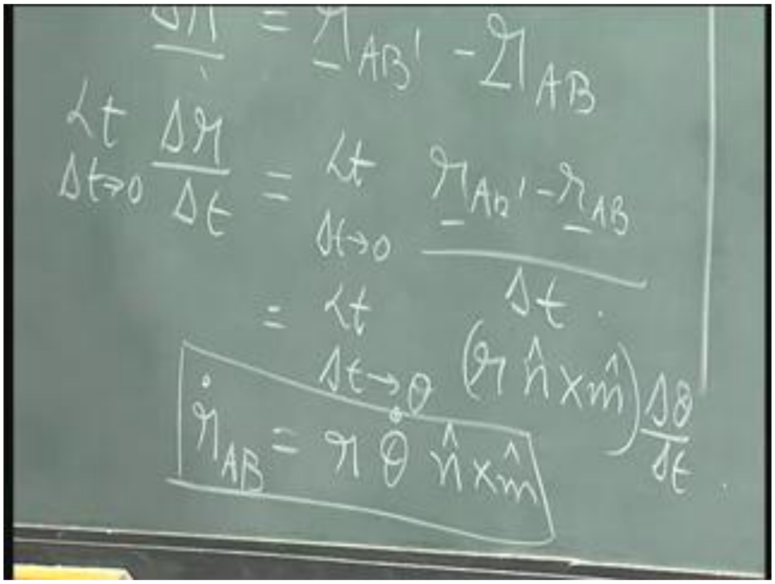

I can just drop this r dot here and just say or instead of r dot A B, I will just say r dot is equal to r theta dot n cross m. We are not done yet. We just need one more step so that we understand what is happening here. So far so good. Let's just look at it again. Therefore I have r dot. What is r dot? It is derivative of the vector r that is equal to, borrowing from this, it is r theta dot n cross m. What is this m here? This is the direction along r. What is this? Magnitude of r. If I combine these two, it is nothing but theta dot n cross r.

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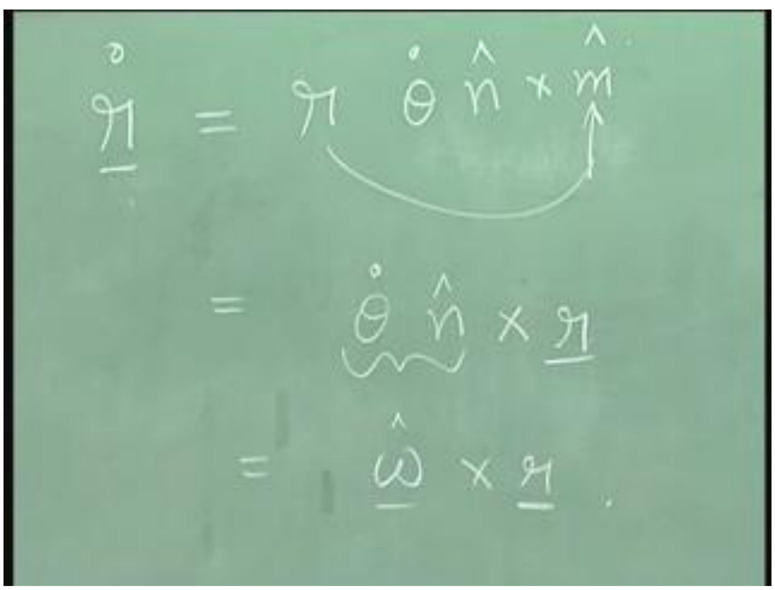

Now what is theta dot times n? As I told you earlier, when I had a three dimensional vector and it changed the position, I took the plane of that plane that is formed by the two vectors to be, this plane. And therefore the rotation is actually occurring along an axis that is perpendicular to this plane. Therefore this is nothing but angular velocity. The velocity with which it is rotating about an axis and therefore this is omega cross r. One important thing that I have to understand is I have pivoted about this point A. That's an important thing to understand. The reason is, supposing the point is somewhere else then I will have to rewrite in a different fashion, but to give you an idea that r dot involves omega cross r is an important result that we will be using over and over in kinematics of rigid bodies.