Engineering Mechanics

Prof. Siva Kumar
Department of Civil Engineering
Indian Institute Of Technology, Madras

Lecture No: 1.2

Statics

System of Planar Rigid Bodies

Degrees of Freedom

Now let's look at little more complexity in it. We looked at one single rigid body earlier. Now if I have a look at a system of planar rigid bodies, how do I find out the total number of degrees of freedom. That's very simple. Suppose I have a body here, another body here and I call these two lets say this is A and this is B. Let's say this entire thing is called as system of rigid bodies. I know that the rigid body A will have 3 degrees of freedom and rigid body B will have 3 degrees of freedom and as we can see here, these two independent of each other and therefore 3 degrees of freedom here. I am just going to call it as dof and 3 degrees of freedom over here which means I have 6 degrees of freedom for this system of equations.

Now supposing I had one more body over here, let's say C. It's easy for me to find out the total number of degrees of freedom this system constitutes and that is nothing but, I had 3 degrees of freedom more and so that this entire system will have 3 times 3 degrees of freedom which means it is equal to 9 degrees of freedom. Supposing I have n rigid bodies in a system. I am only talking about planar rigid bodies here. I know that each one of them will have 3 degrees of freedom. I have n rigid bodies which are independent of each other, these are very important statement. They have to be independent of each other means that I will have 3 n degrees of freedom that this system of rigid bodies have. This system of rigid bodies which has n rigid bodies in the system has 3 n degrees of freedom in it. So far it is clear there is no problem.

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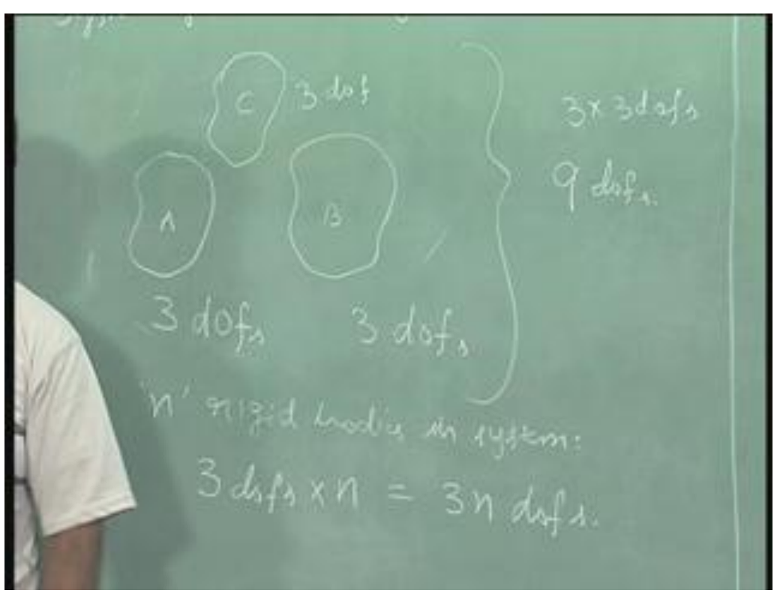

Please remember if I have to define degrees of freedom here, I have to introduce what is called a fixed frame. In order to illustrate that lets say I have these two rigid bodies. Now let’s say these two rigid bodies are moving in such a way that one rigid body does not see the other body moving. But yet they are moving with respect to some other frame. If you look at it more carefully, we need to fix a particular frame with which we talk about all these things. That particular frame is the Galilean frame of reference and for now we will just take it as a fixed frame that I define. If I have two rigid bodies, I have to introduce the notion of a particular frame with respect to which I will be talking about the motion of the rigid body. This is x, this is y, I can have rigid body over here which is A. This is rigid body B, rigid body C and so on. So that the system of rigid bodies will be there and there motions will be defined with respect to this fixed frame of reference.

In this particular case I will have 3 times 3 which means 9 degrees of freedom. What does this mean? If I need to define the motion of this system of rigid bodies, I need to have 9 different values. Again repeating it, 9 different values independent values that I have to use in order to define completely the motion of this system of bodies. Let’s just add a little bit of complexity to this. Here we assumed that each of these rigid bodies that we have or completely independent of each other. Supposing I introduce connectedness, I am just going to call it as connectedness. Let’s say these two rigid bodies don’t move independent of each other but there is some connection that I have established or it could also be with respect to fixed frame of reference. In other words unless I say that these are completely independent bodies, if I have some connectedness that I define, the number of values that I have to use to define the motion will come down. In a sense that way we have maximum of 9 quantities that we have to use in order to define the motion of this system. Now let’s just look at some examples here.

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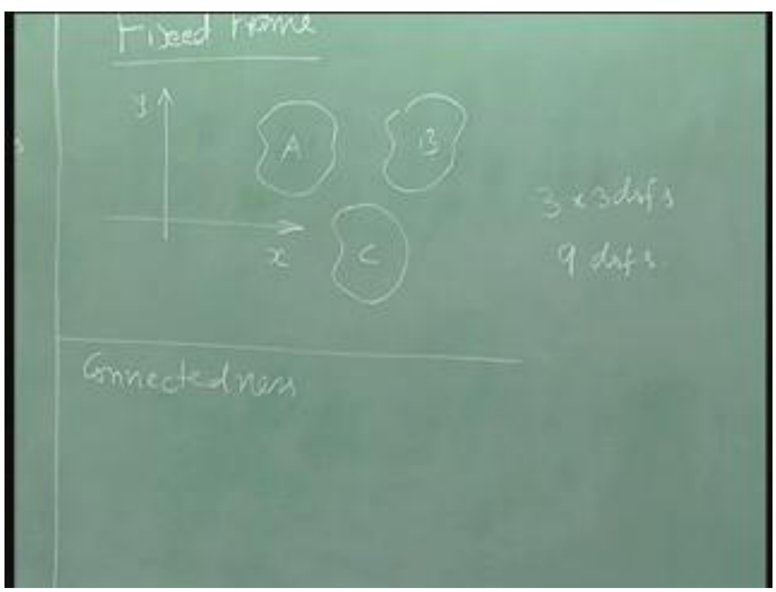

In order to show connectedness, let me call them as constraints. Some of the general constraints could be supposing I have a rigid body like this.

I could probably have some kind of connection like this. Whenever I hash like this, it means that I am fixing it to the fixed frame of reference. This particular point alone I am fixing it with respect to fixed frame of reference. This is what it means or in other words, if I have this rigid body and I say this is fixed frame of reference and I am just pinning it. It's easy to understand that it has only one degree of freedom here. Instead of doing this, supposing I have something like this. I am just going to start with certain examples and then go on with understanding it properly.

Supposing I have some other rigid body, two rigid bodies A and B and they are connected to each other by a single pin over here. In a while I will just explain that to you. Its pined like this, one of the ways to find out the degrees of freedom pertaining to this system of rigid bodies A and B is first to think of no constraint at all. This as a possibility of 3 degrees of freedom, this is for A and for B again I have 3 degrees of freedom possible, if there were no constraints at all. That's very simple to understand. But if I put a constraint how many degrees of freedom will be lost or in other words what is the minimum set of quantities that I will need in order to describe the motion of this particular body.

There are two ways of understanding this. Let me just explain the first way of understanding it. Supposing I take this particular set of bodies and make sure that this particular point is fixed to the fixed frame of reference. Let me just think of it like this then I know about this particular point, this B can rotate and about this particular point again let me call this as o. About this particular point again this A can rotate independent of B. So I have one degree of freedom possible for the B rigid body to move or rotate and for A rigid body to rotate either this way or this way independently. I have one independent rotation of B, one independent rotation of A possible. Mind you I have fixed this particular point o of both the bodies to fixed frame of reference or in other words if I release this particular point with respect to fixed frame of reference, I can move with this point around.

This point has two degrees of freedom possible, it can move in x and y direction in a particular way and therefore there are two degrees of freedom possible that I can move this o with, if I remove the constraint over here and therefore in total I have… I am going to call this as 3 and 4 and I have 4 degrees of freedom in total that this system of rigid bodies A and B will have. One other way of understanding is this. Body A can go through 3 degrees of freedom independently, B can go through 3 degrees of freedom independently if this constraint was not there. What does this constraint do?

Supposing I fixed this body A to fixed frame of reference then this particular point arrest the motion of body from moving, translating at this point o in x or y direction. Or in other words I have two constraints of point o, x and y directions and therefore I have 6 which can be independent and 2 constraints that we have added equal to 4 degrees of freedom. This will have a few problems associated with that. I will ask those questions in a separate clipping.

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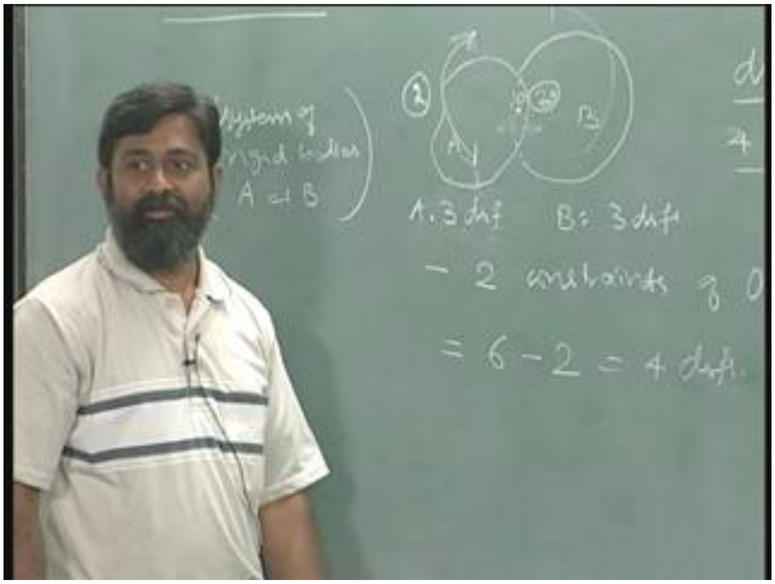

Let us look at some examples so that we will have some clarity. Look at this, there are 3 rigid bodies, rigid body A B C; A and B are connected at point o_1 pinned at o_1, B and C are pinned at O_2. I am just defining a fixed frame of reference. Now the question is how many degrees of freedom does this system have? The simplest way of looking at it is A has 3 degree of freedom, B has 3 degrees of freedom, C has 3 degrees of freedom and total of 9 degrees of freedom if the constraints $o_1$and o_2 are not present or in other words there is no pinning occurred here. This pinning is going to restrain the motion between A and B in both x and y direction which means I would have lost 2 degrees of freedom pertaining to A and B.

In a similar way with o_2, we would have lost 2 degrees of freedom or in other words you don't need two more quantities in order to define. The minimum quantity you will need, will be 5 degrees of freedom, if you find the total. This is the very simple method of doing it but if we have to look at in a different fashion there is one more way of understanding this and that is this.

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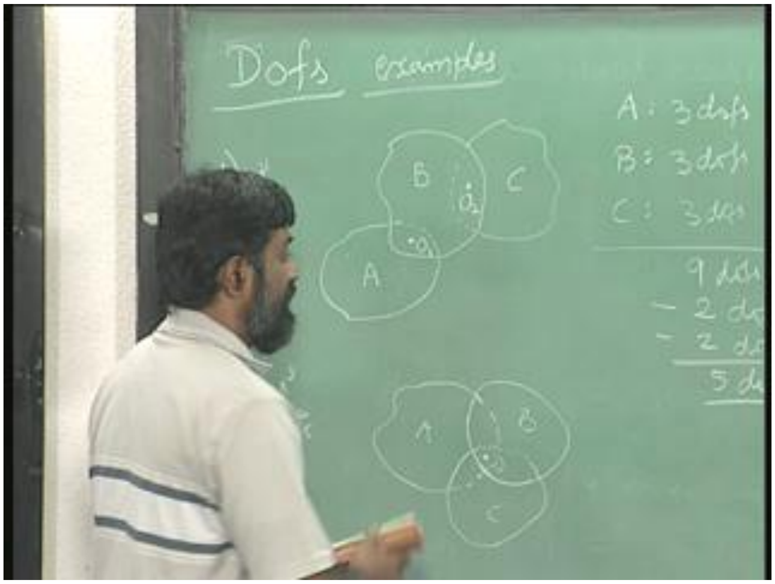

Let’s look at this particular body A. This has 3 degrees of freedom. Now what we are going to do is with respect to the fixed frame of reference, we will just fix this body A and look at what motion body B can have with this constraint. It’s very simple to understand. This body can move, forget about the body c, now this body can move in only a rotational form and therefore this body can have one degree of freedom. Let’s fix A as well as B with respect to fixed frame of reference and ask the question how many ways will this body C move? Very simple to answer, it can only rotate about this point o_2 which means it has one degree of freedom available to it. If you take the total, we have 5 degrees of freedom that we need, minimum of 5 quantities we need in order to define this. The second method is very simple method and it has a physical understanding to how we comprehend. The first one is mathematical. In the next example make it clear to you that there will be a problem if we use the first method.

Now let’s look at second problem. This is clear to us. If I use the method of saying that each one of these bodies will have 3 degrees of freedom and if we look at the constraint then we can answer the question. We will have 9 degrees of freedom if there is no constraint and we have one constraint to over here which restricts the motion in x and y direction. Therefore I should have 9 degrees of freedom but in a moment I will show you that this is a wrong comprehension. Now simple to understand if I look at the second method which is a physical method of understanding.

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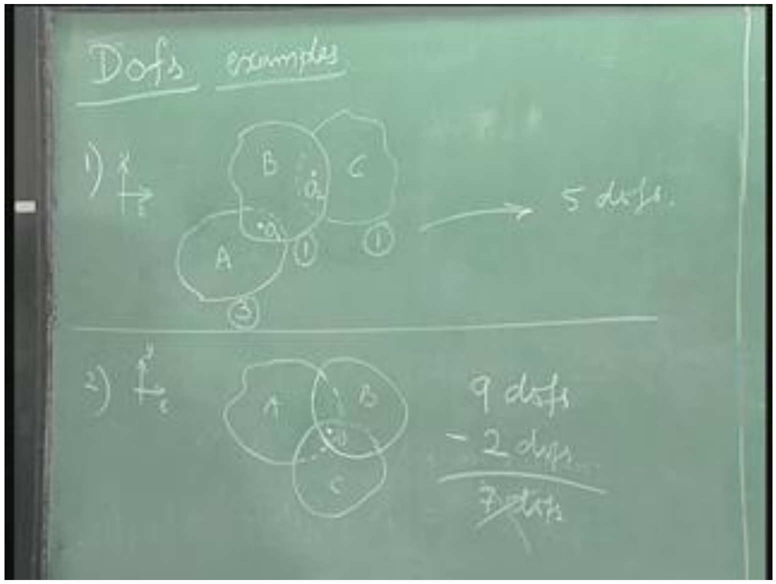

So as we started earlier, if we look at this body A, this will have 3 degrees of freedom, no problem. If I fix this body to the fixed frame of reference and ask the question how many ways can B move? B can move independent of any other body in one way or in other words it is just a rotation possible. From B you can have one degree of freedom. Now I will fix A as well as B and look at C and ask the question whether there will be moment at all. The is answer is yes because this is a pin and the body C can move independent of A and B or in other words this have an additional degree of freedom to move. If I add the total, I will have 5 degrees of freedom possible. The earlier method, you have to take caution in using. The second method is what I would suggest is the best method to find out what are the degrees of freedom that you can find. Let’s look at some more examples.

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Let’s look at some more examples. Let this is a single rigid body, naturally if I don’t have any constraint, unconstraint motion, I will have 3 degrees possible. If you look at it, I am arresting motion of this particular body A with respect to this direction let me call this as o_1 and this is o_2. I am arresting a direction motion here which means there is one degree of freedom that is lost. What A would have had? Two degrees of freedom. Since I am doing this also if you look at this, this is a pin joined or a hinge and this arrest the motion of o_2 in x as well as y direction. So in all if I look at this particular rigid body and asked the question, how many degrees of freedom it has? Or in other words whatever ways can it move, the answer is if I do this, it cannot move at all. Now in this particular situation it is so, that it cannot move at all and therefore this has zero degrees of freedom.

The same thing I can answer again by the first method. The rigid body has three degrees of freedom because single rigid body and I have one constraint here in terms of motion along this which is minus 1 and there are two translational motions arrested which means minus 2 equals 0. In this particular case, it becomes simple. Please remember there are some cases where you have to take precaution. I will ask you the questions at the end. Usually we will solve simple problems and ask complex problems when comes to exams.

Let’s look at a few more examples so that you are familiar with this. How about this? Very simple, there are two translations arrested. One translation arrested here, total of three translations arrested and this body cannot move in x or y direction. As well as it cannot rotate because about this particular point connecting these two lines, if I take this point and try to rotate, this will arrest the motion around this direction which means it cannot rotate. Very simple and therefore this has zero degrees of freedom. In this case again it is easy to show that it will have zero degrees of freedom by this kind of calculation.

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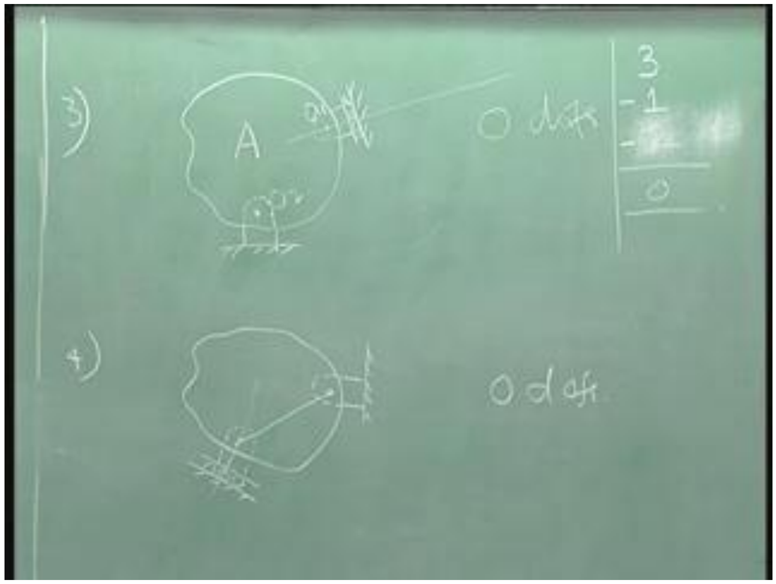

Let’s move on to one more problem. This is again very simple one. You have the rigid body that is rigidly fixed to the fixed frame. Mind you when I hash it like this, it means I am referring to the fixed frame. If this were not there, this body would have moved in three different ways, two translation on rotation but if I fix it like this, it cannot move in the x or y direction. It can also not rotate about this because it is rigidly fixed like this and therefore it has zero degrees of freedom. Simple to understand.

Let’s move on to the next. In this particular case, if you notice carefully the motion in this direction is arrested. This is a straight line and this is the fixed frame of reference. In this direction there is a translation that is arrested. About this particular point if I try to rotate this body, it cannot rotate and therefore let me say x translational is arrested. One rotation which means the entire body cannot rotate at all. Only possible movement is in the vertical direction and therefore it has one degree of freedom.

One last example here this is a system of bodies A and B. Mind you there is a slot on this body A and the two are connected through a small pin that can slide along the slot. Then I asked the question how many degrees of freedom can it have? First way can I start with is I can say, I have three degrees of freedom possible for this rigid body, if I don’t consider anything else. By fixing A I am going to look at B. This point here can slide and this body B can also rotate. There are two ways which we can understand. Let’s fix this particular body A.

Let’s take this particular pin and another body and that pin can have one translational along this direction. So one more degree of freedom and I am going to call that as one more degree of freedom. If I fixed this pin as well as this A, the body B can rotate about the pin axis. Therefore it can have one more degree of freedom and therefore this will have 5 degrees of freedom. The only motion here that is arrested between A and B is the translation which is vertical with respect to the slotted line that is shown here. I hope it is clear as far as number of degrees of freedom is concerned. This will help us understand how to write the equations of equilibrium.

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Now I have a few challenges for you. So far what we have done should be enough for you to answer these questions. This is the rigid body, let me call this is A, a single rigid body which is supported on a straight ground three rollers supports. Question B, this time with two rollers supports but at some angle to each other as you can see here. Third one is three roller supports, two rollers support like this in the horizontal and one roller support for vertical direction. Here there are 3 roller supports on something that is curved. Fifth problem is I have a slider over here but it is not a slider which is straight. I need to find out degrees of freedom for this system of bodies A and B. I am sure you will be able to get the answers.

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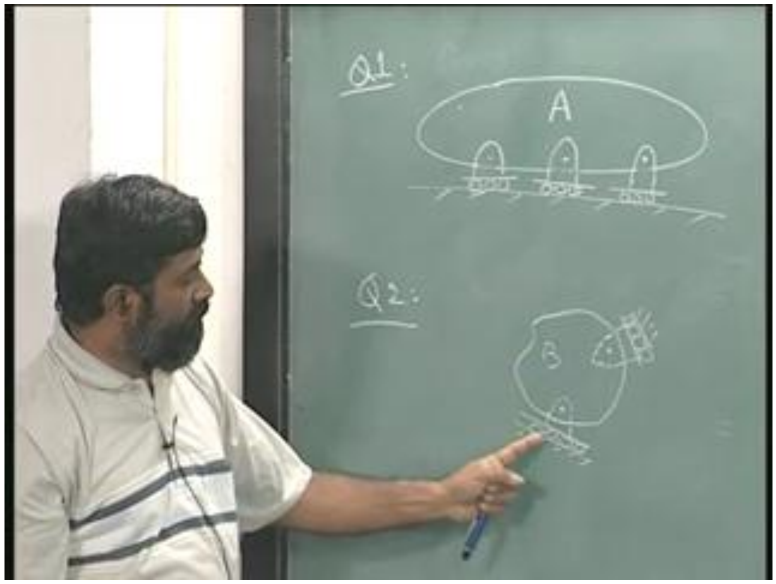

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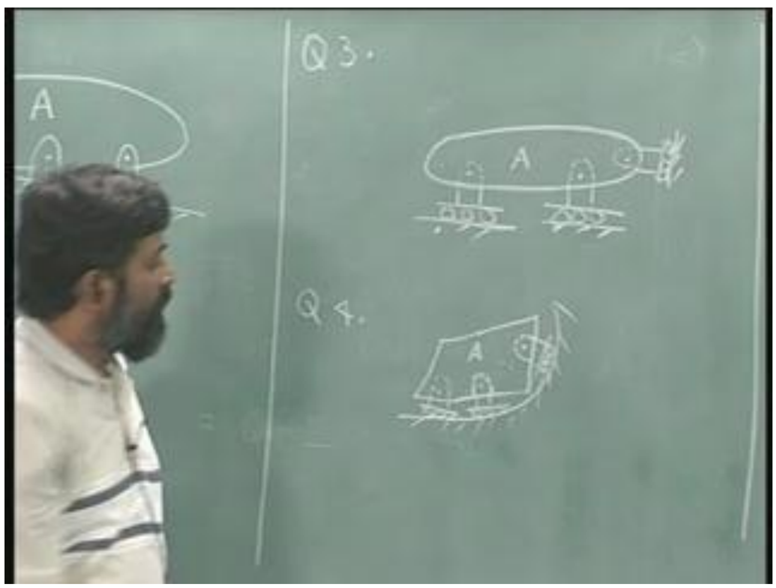

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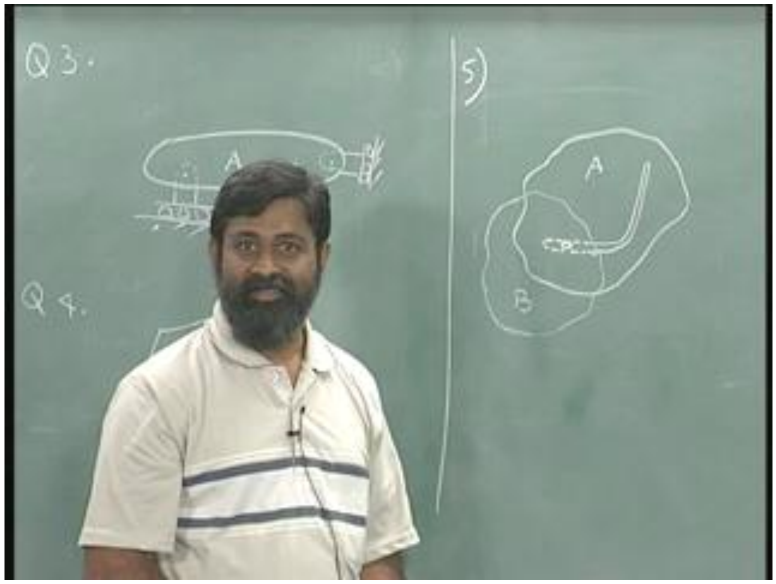

Let’s just look at whether we are correct in this particular problem. We have this body A and this body can move around or rotate which means it has 3 degrees of freedom. What we have there is one pin over here like this, there is one more pin that connects A and C. So you have A, B and C. Let me just connect them. This is what we have. As you can see A, B and C.

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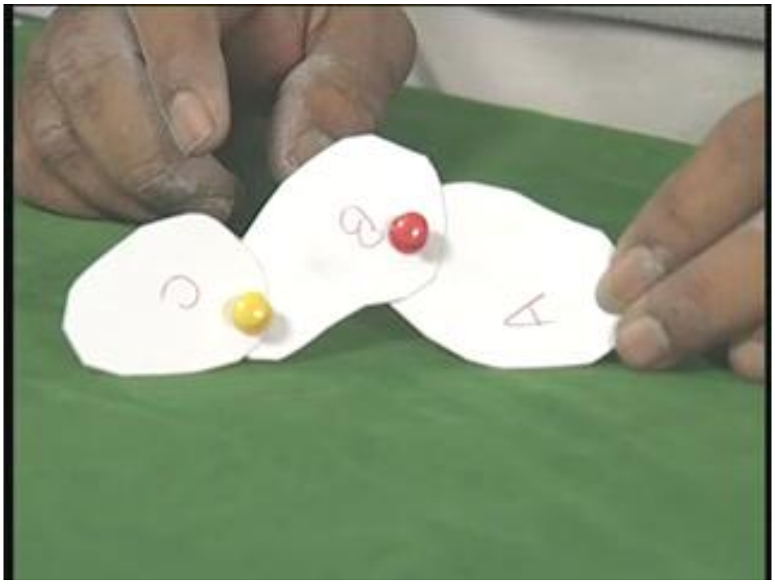

We need to find out the degrees of freedom of the system. We can see that you can move around like this. Now how do we find it out? We take this body A, we know that it can rotate, it can translate and it can move which means this body as such we can say as three degrees of freedom then I fixed this to the fixed frame of reference. Just assume that my hand is a fixed frame of reference, left hand is of reference. I am just holding is A completely and don’t let it move in any way possible. If I look at only B, only possibility for B is to rotate about this particular joint which means if I focus only on B, B has only one degree of freedom or in other words only one quantity needed to describe the motion of that with respect to A. If I fix this particular body B also with respect to the fixed frame of reference, body C has only one way in which it can move or in other words just one single of degree of freedom which is rotation, can describe what is the motion of this with respect to A and B.

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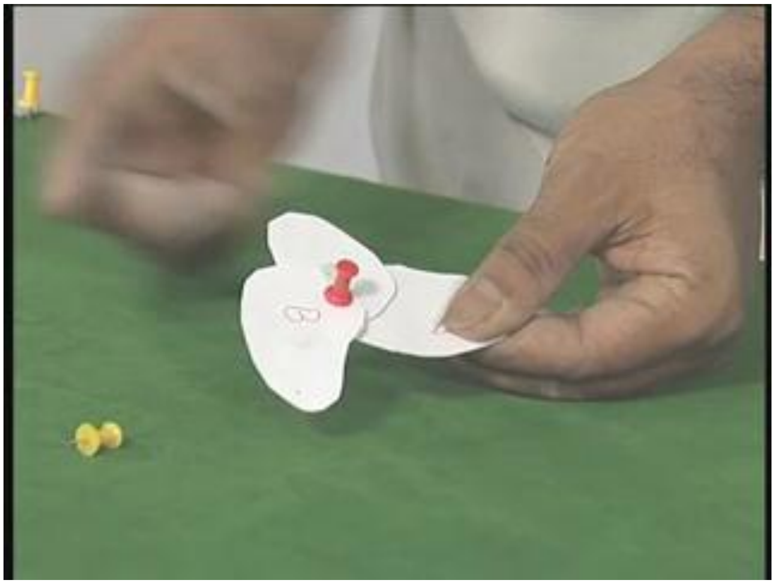

In other words 3 plus 1 plus 1, we have 5 degrees of freedom whereas the other problem is like this. I have body A, body B, body C but all the three are connected through a single pin. Let me just put it properly so that it is easy for you to understand. Let's examine how many degrees of freedom it will have, like before we will have 3 degrees of freedom for body A. Let me just fix it. Now let's look at B, B has one degree of freedom and body C has another degree of freedom, if I fixed A and B which means it has 5 degrees of freedom.

I have put a small slot over here through which a pin can move. I can just show you like this or like this. It cannot move this way, it can move this way along this particular slot.
Now what I am going to do is I am going to attach through this particular pin, the rigid body B and ask the question what is the total degree of freedom that this particular system of bodies A and B will have. Very simple, again I will do the same thing supposing I take only this body A. This has three degrees of freedom, I will take this pin has another body, this has one degree of freedom.

Now I am going to attach this B and like before if I fix A and look at only this pin, the pin will have one degree of freedom. So 3 degrees of freedom plus 1 degree of freedom and if I fix the pin as well as the rigid body A, B has only one rotation possible.
Therefore A has 3, the pin has 1 and B has 1, total of 5 degrees of freedom.