

**INDIAN INSTITUTE
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**NPTEL
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Technology Enhanced Learning**

Probability Methods in Civil Engineering

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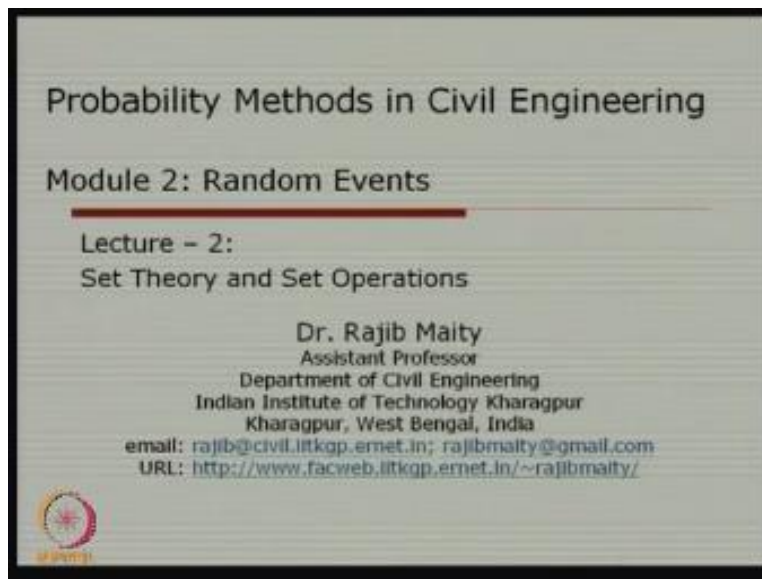
Lecture – 03

Topic

Set Theory and Set Operations

Hello and welcome to the second lecture of our second module random events.

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Module 2: Random Events

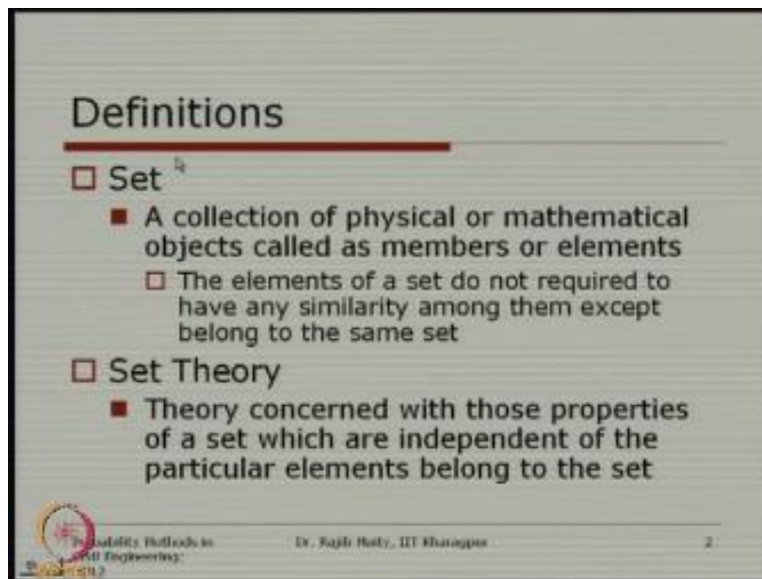
Lecture – 2:
Set Theory and Set Operations

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In today's lecture, we will cover set theory and set operations, which is very important in the understanding of the concept of probability theory. So in this class, we will know the concept of set and different set operation which will be useful to describe the different probability to different set and subset of a outcome of particular experiment.

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Definitions

- **Set**
 - A collection of physical or mathematical objects called as members or elements
 - The elements of a set do not required to have any similarity among them except belong to the same set
- **Set Theory**
 - Theory concerned with those properties of a set which are independent of the particular elements belong to the set

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First we will know the definition of the set and set theory, basically a set is a collection of physical or mathematical objects called as members or elements. So the elements of a set do not require to have any similarity among them except that they belongs to the same set. So, basically a set if we want to want to define that it is a collection of any, a collection of a particular group of elements and those elements can be physical or mathematical in terms of different outcome of a particular experiment, if we think in the context of our probability theory.

Now the set theory says the theory concerned with those properties of a set, which are independent of the particular element belong to the set. So when you say that set theory those theories generally applicable to the whole set, it is not related to a particular element of a set. So, the overall the set what are the different operations that we can do, what are the different theories that is applicable to the full set, and that consist that is, we call as set theory.

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Example

- If a coin tossed twice, the resulting four outcomes or events hh, ht, th, tt form the set
 - $S = \{hh, ht, th, tt\}$
- If an experiment is carried out to determine the strength of concrete, the all the experimental values form the set.
- If the stage height is recorded at a river gauging site, all possible values of stage height form the set.

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So with this if we see that different examples, so if we start with the that our classical example one for example, if we take a coin and if it is tossed twice, then the outcome of this experiment can be of a either the first and second both are head or head tail or the first one is tail and second one is head or both are tail, so the outcome of this events that is the full all the collections of this experiment is one set.

In an experiment is if a if an experiment is carried out to determine the strength of concrete in particularly in the civil engineering, then all the experimental values, they form A set. In the other example of the stage height generally, we measure the stage height at a river gauging site there if we take that the river gauging site, if we take the stage height at a particular gauging site, then the all possible values of the stage height form the set.

So here you can see that these things can have any value depending on the physical range of the strength of the concrete depending on various factors its water cement ratio or the time and type of curing and all these things it can take any values. For this type of set, this we can say whenever we are saying that the stage height, this is generally called one side it is bounded by 0,

so physical or in the mathematically, it can take any value starting from 0 to the class infinity. So all this outcome if we take a total this is generally this forms generally a set.

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Properties

- Two different sets S and T are *equal* if all elements of the set S belong to set T and vice-versa

The diagram shows two overlapping circles, S (black) and T (red). The intersection contains both red and black dots, indicating that elements of S are in T and vice-versa.

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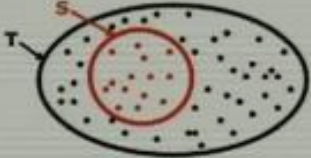
There are different properties that will see now one after another, the first thing is that when we say the both the sets are equal. So two sets are we generally say these are equal if all the elements of one set belong to the other set and vice-versa. Here in this pictorial representation, there are two sets are there, one is one is drawn by the red circle and another one is drawn by the black circle. If you see that these are exactly sums of whatever the entries that belongs to the red set, there are some red dots and the black dots and also there are some.

So all theses dots belongs to both the set S as well as set T , now so we can say that this set S and set T are equal. In the example of the throwing a dice it has having the six different output, now if I say the one set is even and the outcome is even number and if in other words, if I say the outcome consists of 2, 4 and 6, then we can say that these two sets are equal.

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Properties...contd.

- A set S is proper *subset* of set T when all elements of set S belong to set T , not vice-versa
- denoted as $S \subset T$



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This is very important which is known as the subset. A set S is a proper subset of another set if we can designate that T when we say that all the elements of this set T belongs to the former one that is S , but the opposite is not true that is which is the vice versa is not correct. So if you see this pictorial representation the bigger ellipse is the T set and inside this there is another set, which is shown in a red circle.

So this S belongs to this T , so this S is known as the subset of subset of this the larger set T . So in the example of the river gauging site if I say that the extreme values of the stage height, then I can say that if I say that all the stage height that is above say 10 meter or I can say that another set, which takes the all possible values. So the all possible values means that can take any value from 0 to whatever the physical maximum in available height in the historical record or it can be even higher than that.

And another set is that the just extreme value which I put a threshold of the 10 meter, then I can say that the threshold, which is more than 10 meter is that is a subset is a subset of the full set, which can take any value from 0 to mathematically plus infinity.

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Properties...contd.

- An empty or null set is defined as the set contains no elements.
- A null set is denoted by $\{\emptyset\}$

S

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The second thing that is known as a null set, when we say that there is a particular set, which does not have any element in it, then we say that this is a null set. And it is defined as that when the set contains null element and this is denoted by this kind of symbol you can see here that there is set where there is no element then we can say that this is a null set. In the probability concept, we can say that this null set is generally is a set that is the impossible set that is impossible outcome.

For example, if I say that the strength of concrete, then if I say some value which is negative, then we can say this is an impossible event and this is this set is a is a null set which does not have any entry, which is which belongs to then negative value less than 0.

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Properties...contd.

- Sets are *classified by size*, according to number of elements they have
- A set may be *finite* or *infinite*
- A *finite* set contains a *whole number* of elements, called Cardinal Number
- Any set with Cardinal Number is called Countable Set

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Generally the sets are classified by its size according to the number of elements that they are having a set a may be finite or infinite, a finite set consists of a whole number of elements which is known as the cardinal number. Now if any set that is having the all the number with the cardinal numbers is called the countable set and infinite set generally infinite set, when we cannot count or the number of elements that belongs to is infinite or sometimes the another term is used that is called the countable infinite. It can be countable, but it can be infinite in number those set those size of set is known as countable infinite set.

So, these are all the basic concept that is will be useful to understand the other properties and the operations of a set.

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Properties...contd.

- A set is said to be *ordered* if a relation ('>' or '<') can be defined for any two elements such that:
 - either $a > b$ or $a < b$ is possible
 - $a < b$ and $b < c$, then $a < c$

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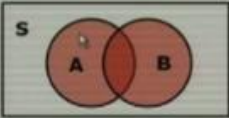
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The second and one, another important thing is ordered if a set is generally said to be ordered, if a relation like the $>$ or $<$ than can be defined for any two elements of the set. For example, if I say if I take two elements a and b , I can say that $a > b$ or $a < b$ is possible, then this kind of set is known as the ordered set. We can even conclude that if we say that $a < b$, and $b < c$, then I can say that the element $a < c$, so this kind of set is known as the ordered set.

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Union and Intersection

- Union
 - A union of two events A and B is
 - denoted as $A \cup B$
 - read as "A union B"
 - the event which consists of all of the outcomes either 'A alone' or 'B alone' or 'both A and B'



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These two are two operations, which is very important and frequently used these are known as one is union and another one is the intersection, this concepts are very important to understand the later part of this of this class. First, we take this union a union of two events that is A and B, here you can see that this is your, full set this is your full sample space and in this space, there are two events one is this the left hand side circle and another one is the right hand side circle, which is denoted as B and the earlier one denoted as A.

So, this two are events in the in the sample space S now the union of A and B is generally denoted as $A \cup B$, this is this is generally read as the A union B this A union B consist of all the outcomes that is either in A alone or B alone or both in A and B. So you can see this three regions, where I am pointing now this area is belongs to A only, this area belongs to the B only, and this in between belongs to both A and B. So if I say that A union B, then this area which is highlighted in red, this area in total is known as the A union B.

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Union and Intersection ...contd.

- Intersection
 - An intersection of two events A and B
 - denoted by $A \cap B$
 - read as "A Intersection B"
 - The event which consists of all outcomes those are in 'both A and B'



The diagram shows a rectangular box labeled 'S' representing the universal set. Inside the box are two overlapping circles labeled 'A' and 'B'. The area where the two circles overlap is shaded in red, representing the intersection of events A and B.

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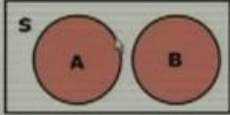
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On the other hand, if I say the intersection again this two events are defined here one is A and another one is B, and this is it is intersection is denoted as A this symbol B, and it is read as the A intersection B. So, we call that intersection is defined as all the outcomes, those are in both A and B, so it must be both in A and B. So obviously the highlighted area, the red highlighted area is known as the intersection between A and B. So once again the union is the total area that consist of either B or A or both A and B that is the union and intersection means, it is both on A and B.

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Mutually Exclusive Sets

- Two events A and B are mutually exclusive when none of the outcomes in A are in B
 - also as termed as A and B are disjoint
 - denoted as

$$A \cap B = \{\phi\}$$


The diagram shows a rectangular universal set labeled 'S'. Inside 'S', there are two separate, non-overlapping circles labeled 'A' and 'B'. This visualizes that the intersection of A and B is an empty set.

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There are other few concepts that is also useful here one is the mutually exclusive set, this mutually exclusive set between two events A and B, we can call that this A and B are mutually exclusive, when no outcome, no element in A are in B and of course vice versa. So there is no overlap between this two these two sets. So if we say that there is no overlap between these two sets that means. If I take the intersection between A and B, now you know this symbol is an intersection that means in this intersection there is intersection A is a null event, where there is no element exist in this part.

So you can say that if this two are completely separated, there is no overlap area, these two events known as the mutually exclusive event in the example of a throwing a dice. If I say that one set consist of all the even numbers and another set consist of all the odd numbers, then you can easily say that there is no intersection between these two events, and so all the set consist of the even number and the set consist of the odd number are mutually exclusive sets.

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Collectively Exhaustive Sets

- Union of all sets comprise the whole set
 - denoted as $A_1 \cup A_2 \cup \dots \cup A_n = S$



- Note that intersection of any two sets need not be null set

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There is another very important and very frequently we use the concept, which is known as the collectively exhaustive sets. Now the collectively exhaustive sets means, when I take some subsets or some events or some sets and if I take the union of all this events then I can say, if the union of this sets comprise the whole set, whole set means that whole sample space if it consist of now you can see that there are few subsets has defined here say for example, this area is your A_1 , this area is your A_2 , in this way it is going on, and there are some subsets like this. Ultimately, if I take the union then obviously, the full union consists of this whole set it red sided area.


So this whole red sided area is nothing but you have the full sample space. Here one thing is important to note that the intersection of any two sets need not be null sets, so you can see that this a A_1 and A_2 the intersection is null but this A_1 and if I say that this is another set which is A_3 , so there are some intersection point. So we are not considering the intersection point here we are just considering the union. So, if I can somehow say that all this unions consist of the full set then I can say that this is a this is collectively exhaustive set.

In the example of a throwing a dice if I say that one event is less than 4 and another event is greater than two and if I take the union of this two obviously the union will consist of all possible outcomes starting from 1 to 6. So the two sets which is that less than 4 and greater than two there maybe some intersection. But if I take the union of this 2 it consist of 4 possible sets. So these two events are mutually exclusive sets.


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Collectively Exhaustive Sets

- Union of all sets comprise the whole set
 - denoted as $A_1 \cup A_2 \cup \dots \cup A_n = S$



- Note that intersection of any two sets need not be null set

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Partition of a Set

□ A partition U of a set S is a collection mutually exclusive and collectively exhaustive subsets A_i of the whole set, such that,

$$A_1 \cup A_2 \cup \dots \cup A_n = S$$

and

$$A_i \cap A_j = \{\emptyset\} \quad \text{for } \forall i \neq j$$


The diagram shows a large irregular shape labeled 'S' representing the whole set. It is divided into six smaller, non-overlapping regions labeled A1, A2, A3, A4, A5, and A6. A dashed line indicates the boundary between A3 and A4.

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Then the partition of a set, now this partition of a set suppose that we have one set in hand and I want to make it partition here I just make this is a another subset A1 this is A2 and in this way it is going on an up to n, so n number subsets are there in such a way that there are two conditions must be satisfied, the first condition is that if I take the union of all the sets it should consist of the full set.

Which is here shown as S the full set and if I take the intersection of any two sets as long as $i \neq j$ then this intersection $A_i \cap A_j$ should be a null set you see here no events are intersecting each other, so the intersection between in any two event any two subset is the null set then we can say that this S is partition by A1, A2, A3.... An. So a partition U of the set S is the collection of collection of that will be one word of the collection of mutually exclusive.

And collectively exhaustive, so this is a new word again we will just explain it in a moment so the mutually exclusive this mutually exclusive means all this unions are the are consist of this full set again mutual exclusive and collectively exhaustive subset are A_i , A_i is of the whole set such that as I told there are two conditions one is that A1 to A2.

If I A_1 to A_n if I take the union of all the set it consist of the full one and any two set in the section is 0. So in this way if satisfying this two condition if we can make the if we if we sub divide the sets then this is the this is known as the known as the partition of a set.

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Set Operations

- Complements
 - A complement of an set A are the outcomes in S those are not contained in A
 - denoted by A^c or A' or \bar{A}

S

A \bar{A}

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Now, the set operations there are different set operations the first thing is the complement. A complement of an set A are the outcome in S those are not contained in A so suppose that this is your the full sample space A and there is one event A so this is your event the complement of A is the area is a set which is not belongs to A. So, if I say that the even numbers of throwing a dice experiment.

The if the even number consist of one event then the complement of this set is the odd number of the outcome. So, one thing is very important to note here is that if I take the union of A set and its complement then we get the full set full feasible set which is S. This complement is generally denoted by either A^c or A' or $A - \text{bar}$, so all this notations are just a notation are different but these are all the meaning are all for same the complements of A particular set.

In this class we will use this concept which is A bar to denote that which is the complement of a particular set.

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Set Operations...contd.

- De Morgan's Law
 - When in a set identity, all the subsets are replaced by their complements, all unions by intersections, and all intersections by unions, the identity is preserved
 - If A and B are subsets of a set S, then
$$\overline{A \cup B} = \overline{A} \cap \overline{B} \quad \text{and} \quad \overline{A \cap B} = \overline{A} \cup \overline{B}$$

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Another set operation which falls on this De Morgan's law, okay before I go to this I should tell that this unions and intersections are also the also the different set operation which is also very important that just it is covered and the complements also you covered and using this two things we will now explain a very important law which is known as the De Morgan's law and that is very helpful for different kinds of operations on different sets.

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Set Operations...contd.

- De Morgan's Law
 - When in a set identity, all the subsets are replaced by their complements, all unions by intersections, and all intersections by unions, the identity is preserved
 - If A and B are subsets of a set S, then
$$\overline{A \cup B} = \overline{A} \cap \overline{B} \text{ and } \overline{A \cap B} = \overline{A} \cup \overline{B}$$

So, this is De Morgan's law, when in a set identity all the subsets are replaced by their complements all unions by intersection and all intersections by unions the identity is preserved. May be when I am just telling this one it may not be very communicable but we will just explain it what all these things means. Before that just look at this diagram, where there are two sets one is A and another one is B.

And if we just tell once again the recapitulate what just now we have seen. So this is your A this area is your B, so we know that this is the this in between the intersection between this area which is little bit gray looking this is the A intersection B. Here when we write that A B side by side it indicates that this is the intersection and when we write this symbol this is generally this is generally the union.

So, this area is your $A \cap B$ now so if I say this pink area around this one then it is $A' \cap B'$. So what is A' here? So, whatever the set that is that we can see here outside of this area is your A' and here it is B and outside of B is your B' . Now all this outside area then outside B that is the complement B and complement A if we take the intersection between these two then only these area where my cursor is moving now.

Only this area is your $A \cap B$ as $A' \cap B'$. Now if I say that $A \cap B$ that complement, then it consist of this set this area as well as this area which is not in this intersection but the rest of this A and rest of this B which is not in this one. So this understanding of these two that is $A' \cap B'$ and $A \cap B$ that complement is important. So, in this $A \cap B$ consist of this pink area as well as this A area apart from this intersection.

And this B area not belonging to this intersection so these three area consist of $A \cap B'$ and only this outside area which is not in A not in B and not in A and B, that is that $A' \cap B'$. Now if we know these three area, then we can just appreciate these two relationship you can see that if A and B are the sub sets of A set S, then $A \cup B$ complement so $A \cup B$ complement means this A first of all we will see where is the $A \cup B$, this $A \cup B$ is nothing but this area and it is complement means this area what just now I explain which is nothing but here that A complement intersection B complement so this relationship.

We can show it pictorially similarly, if I say that A intersection B then the A intersection B is nothing but only this area now if I take the complement of A intersection B, which is shown by this bar here which is nothing but the summation of three areas one is that this area + this area + this area and this three area is nothing but your A, so this three area is shown here now again if I see that this one the A complement U B complement that is that A outside B outside, if you take the union of this two area.

Then this three area will be added so this area + this area + this area so this two relationships also holds good now again I am just reading this De Morgan's law, when in A set identity all the sub sets are replaced by their complements, all the unions by their intersections, all the intersections by unions, the identity preserved so, I am just making all the symbols just opposite in the sense that if I say that intersection is opposite to the U or the subsets are opposite to their complements, then if I just make this conversion. Then this identity preserved this will be more clear in the next slide.

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Set Operations...contd.

- De Morgan's Law
 - Demonstration
 - $A(B \cup C) = AB \cup AC$
using the identity $\overline{AB} = \overline{A} \cup \overline{B}$
 $\overline{A(B \cup C)} = \overline{A} \cup \overline{B \cup C}$
using the identity $\overline{A \cup B} = \overline{A} \overline{B}$
 $\overline{A \cup B \cup C} = \overline{A} \overline{B} \overline{C}$
similarly, $\overline{AB \cup AC} = (\overline{AB})(\overline{AC}) = (\overline{A} \cup \overline{B})(\overline{A} \cup \overline{C})$
 $A(B \cup C) = AB \cup AC$, their complements are also equal, so $\overline{A \cup B \cup C} = (\overline{A} \cup \overline{B})(\overline{A} \cup \overline{C})$
Thus, finally $\overline{A \cup B \cup C} = (\overline{A} \cup \overline{B})(\overline{A} \cup \overline{C})$

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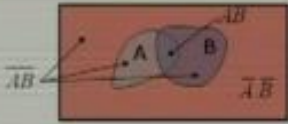
When I just see this relationship this is a demonstration of that what just now we told is that this is you are $A(B \cup C)$ and $A \cup AB \cup AC$ you can even draw another figure just to see that whether these two quantity are same or not you can see by a simple Venn diagram that these two quantity are same now you see this relationship, and you just see the last line first before you go through all these steps just see the last line here.

Now this A here, this A is replaced by it is complement this intersection is replaced by the union this B is replaced by its complement, this union is replaced by intersection this C is replaced by this C complement similarly on the right hand side this A is replaced by it is A complement intersection is replaced by union B replaced by B complement similarly this union replaced by this intersection A by A complement intersection by U and C by C C complement so according to the De Morgan's law if this two are equal, then again this two this two also will be will also be equal now, let us see that in between how we can show this thing just in the previous slide we showed.

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Set Operations...contd.

- De Morgan's Law
 - When in a set identity, all the subsets are replaced by their complements, all unions by intersections, and all intersections by unions, the identity is preserved
 - If A and B are subsets of a set S, then
$$\overline{A \cup B} = \overline{A} \cap \overline{B} \quad \text{and} \quad \overline{A \cap B} = \overline{A} \cup \overline{B}$$




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That this $\overline{A \cap B} = \overline{A} \cup \overline{B}$ intersection B complement is A complement U B complement. So using this one and we described this one with this simple Venn diagram, using this relationship.

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Set Operations...contd.

- De Morgan's Law
 - Demonstration
 - $A(B \cup C) = AB \cup AC$
using the identity $\overline{AB} = \overline{A} \cup \overline{B}$
 $\overline{A(B \cup C)} = \overline{A} \cup \overline{B \cup C}$
using the identity $\overline{A \cup B} = \overline{A} \overline{B}$
 $\overline{A \cup B \cup C} = \overline{A} \overline{B} \overline{C}$
similarly, $\overline{AB \cup AC} = (\overline{AB}) (\overline{AC}) = (\overline{A} \cup \overline{B})(\overline{A} \cup \overline{C})$
 $A(B \cup C) = AB \cup AC$, their complements are
also equal, so $\overline{A \cup B \cup C} = (\overline{A} \cup \overline{B})(\overline{A} \cup \overline{C})$
Thus, finally $\overline{A \cup B \cup C} = (\overline{A} \cup \overline{B})(\overline{A} \cup \overline{C})$



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Here that A intersection B complement = A complement U B complement if we use this identity and we want to put this one is that A intersection B U C full complement so this is one set A and this is B union C is another set so this is a corresponds to A here and B U C corresponds to another set B here then using this relationship we can write that this A complement U B U C complement again this in the previous slide another.

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Set Operations...contd.

- De Morgan's Law
 - When in a set identity, all the subsets are replaced by their complements, all unions by intersections, and all intersections by unions, the identity is preserved
 - If A and B are subsets of a set S, then
$$\overline{A \cup B} = \overline{A} \cap \overline{B} \quad \text{and} \quad \overline{A \cap B} = \overline{A} \cup \overline{B}$$

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Identity that $A \cup B$ complement = A complement intersection B complement using this identity.

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Set Operations...contd.

- De Morgan's Law
 - Demonstration
 - $A(B \cup C) = AB \cup AC$
using the identity $\overline{AB} = \overline{A} \cup \overline{B}$
 - $\overline{A(B \cup C)} = \overline{AB \cup AC}$
using the identity $\overline{A \cup B} = \overline{A} \cap \overline{B}$
 - $\overline{A(B \cup C)} = \overline{AB} \cap \overline{AC} = (\overline{A} \cup \overline{B}) \cap (\overline{A} \cup \overline{C})$
 - $\overline{AB \cup AC} = \overline{AB} \cap \overline{AC} = (\overline{A} \cup \overline{B}) \cap (\overline{A} \cup \overline{C})$
 - $\overline{A(B \cup C)} = \overline{AB \cup AC}$, their complements are also equal, so $\overline{A} \cup \overline{B} \cap \overline{A} \cup \overline{C} = (\overline{A} \cup \overline{B}) \cap (\overline{A} \cup \overline{C})$
 - Thus, finally $\overline{A} \cup \overline{B} \cap \overline{A} \cup \overline{C} = (\overline{A} \cup \overline{B}) \cap (\overline{A} \cup \overline{C})$



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We are we can even this $B \cup C$ from this right hand side, we can just write that A complement $B \cup C$ using this one now, this B corresponds to A and C corresponds to B we can write that A complement B complement intersection C complement similarly so from this one I just prove that this is equals to this quantity similarly if I use that A intersection $B \cup A$ intersection C and its complement this will be equal to using this identity again they are the A intersection B corresponds to A and A intersection C corresponds to B then using this identity i can explain that this is A intersection B .

It is complement intersection A intersection C its complement which is again equal to by using this back sorry using this identity using this identity $A \cap B = A$ complement B complement we just put this one here so that we will get A complement union B complement and A complement $B \cup C$ complement now you see this relationship that is this one is equals to this and this entity this set is equals to this entity now, I have started with the relation that this set.

This full set is equals to this full set now so if these two are equal, then their complements are also equal now this is nothing but the complement of the left hand side and this one is nothing but the complement of this one so if this two are equal then this two are also equal so I can write

that this entity is equals to this entity so that way what I can write is that I am just writing this one here and this entity here so I am finally getting this statement which if I compare with the first statement.

Then all the sets are replaced by their complements all the intersections replaced by their union, all the unions replaced by their intersection and still we see that if we do all this replacement then still the identity is result this is what is the De Morgan's law there is base using this De Morgan's law we can utilize this De Morgan's law for another concept further to further take this concept which is known as the duality concept and that duality is written here.

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Set Operations...contd.

□ **Duality Principle**

- This is an extension of De Morgan's Law
- Since $\overline{S} = \{\emptyset\}$ and $\overline{\{\emptyset\}} = S$

If in a set identity, all unions are replaced by intersections, all intersections by unions and the set S by $\{\emptyset\}$ and the set $\{\emptyset\}$ by S , the identity is preserved.

Thus $A(B \cup C) = AB \cup AC$ leads to $A \cap (BC) = (A \cap B)(A \cap C)$
and $S \cup A = S$ leads to $\{\emptyset\}A = \{\emptyset\}$

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It is a, this is an simple extension of the DE Morgan's law; here, this is the S is the full set, so if I say that S is the fullest, then if I take its complement, which is nothing but a null set. So S consists of everything, so if I take that its complement, obviously its complement will be a null set and vice versa if I take the complement of a null set, then I will get back the fullest, which known as S. Now if in a set identity, all the unions are replaced by intersection, all the intersections by their unions, the set S by the null set and the set and the null set by its set, then the full set S, the identity is preserved.

So, using this $A \cap B$, which is equal to $A \cap B$ just in the previous slide we have just shown the this one, then this will lead to I will just simply changing this intersection as union and union as intersection on both ends both side left hand side and right hand side, then we get the another side another identity which is also preserved.

Then $S \cup A$ is equal to S obviously the full set if I take the union of any sub set then S , then that is also leads to this S , which leads to if I just replace this whatever I told here the S by the null set union by the intersection and A by this one, then it will written as this S is null set

So you see that the null set intersection A is the null set. So this is known as the duality principle, which is an extension of the De Morgan's law.

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Set Operations...contd.

- Symmetric Difference
 - For any two sets A and B, the set of elements which exist only one of the sets (not in both) is called Symmetric Difference Set

The diagram shows two overlapping rectangles, A and B. Rectangle A contains a black square, a red square, and a black circle. Rectangle B contains a red circle, a black square, and a red square. The symmetric difference set is highlighted by a blue oval and includes the black square in A, the red circle in B, and the red square in B.

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There are few other concepts also which are also useful in the set operation say for example, that first one is the symmetric difference, the symmetric difference for any two sets A and B, the set of elements which exist only one of the set not in both called the symmetric set.

Here you can see there are two sets; one is one this one is the black rectangle which is denoted as A and another set is denoted by B which is a rectangle; here there are few elements, which are common both in A and B in this area that you can see and which are not in not common, which are located in this area or in this area. So now if I prepare another set, which consists of all the elements, which falls either in A either in only A and or only in B, then the set that is the symmetric difference between this A and B.

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The slide is titled "Set Operations...contd." and contains the following text:

- Cartesian Product
 - For any two set of events A and B, the set comprise all the possible *ordered pairs* (a,b) ; where a is a member of A and b is a member of B, is called Cartesian Product of A and B
 - Denoted as $A \times B$
 - an *ordered pair* is a collection of objects having two coordinates, such that one can always uniquely determine the object. Thus, the first coordinate is a and the second coordinate is b then the ordered pair is (a,b) , which is different from (b,a) .

At the bottom of the slide, there is a logo on the left, the text "Probability Methods in Civil Engineering" in the center, "Dr. Rajib Nath, IIT Kharagpur" on the right, and the number "19" in the bottom right corner.

The second one is the Cartesian product; for any two sets of events A and B, the set comprise of all possible ordered pair this is a new term will just explain it in a moment; the all possible ordered pair A B, where A is a member of A and the small B is a member of B, B set, then this all the possible ordered pair is called the Cartesian product of A and B, and which is denoted as a cross B.

Now, what is this ordered pair? An ordered pair is a collection of objects having two coordinates, such that one can always uniquely determine the object; thus the first coordinate is A and the second coordinate is B, then the ordered pair is A B. So always the first entity comes from the

first set and the second entity comes from the second set, which is obviously different from the B A.

So if we maintain the first entity of all the sets, all the elements, the first entity always comes from the first set and second entity always comes from the second set, then this is known as an ordered pair. So once again the Cartesian product if we see it states that for any two sets of events A and B, the set comprise all possible ordered pair A , B, where A is a member of A and B is A member of B is called the Cartesian product of A and B, which is denoted as $A \times B$.

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Set Operations...contd.

- Power Set
 - The Power Set of any set A, is the set whose elements are all possible subsets of A
 - The Power Set of {5, 6} is
 - q $\{\{\emptyset\}, \{5\}, \{6\}, \{5,6\}\}$

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The third one is the power set the power set sometime this kind of terminologies are used frequently to have the concept is the power set of any particular set A is the set whose elements are all possible subsets of A. I repeat the power set of any particular set A is a set whose elements are all possible subsets of A. So if I just take one set that is 5 6, then the power set of this particular set can consist either the null set or the one element, the first element or the second element or another subset can be consists of the full set, the 5 6, so there are 4 possible subset that can happen.

So this is known as the power set of this set 5 6. Generally, we say that this any particular set is one subset of its own power set, which is obviously true.

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Open Set and Closed Set

- **Open Set**
 - A set S is called *Open Set*, if any point a in S always belongs to the set, even though it moves to some particular direction
- **Closed Set**
 - A set is called *Closed Set*, if its complement is a *Open Set*

The points (a, b) satisfying $a^2 + b^2 = r^2$ are colored blue. The points (a, b) satisfying $a^2 + b^2 < r^2$ are colored red. The red points form an open set.

The Union of red and black points form closed set.

(source: http://en.wikipedia.org/wiki/Open_set)

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There are the there are two more thing that we say that open set and the closed set, the open set a set S is called open set, if any point A in S always belongs to the set even though it moves to some particular direction; and the closed set, a set is called the closed set if its complement is an open set.

So the concept of open set is more important more important and the closed set we generally say that its complement, if its complement, if the complement of a set is open set, then that particular set is closed set. Before I go to this example if simply we say that a particular set that consists of all the integer numbers between 5 to 10, then this is a closet, and if I say its complement then it consists of the count ably infinite number a infinite numbers and that is obviously an open set.

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Open Set and Closed Set

- Open Set
 - A set S is called *Open Set*, if any point a in S always belongs to the set, even though it moves to some particular direction
- Closed Set
 - A set is called *Closed Set*, if its complement is a Open Set



The points (a, b) satisfying $a^2 + b^2 = r^2$ are colored blue. The points (a, b) satisfying $a^2 + b^2 < r^2$ are colored red. The red points form an open set.

The Union of red and black points form closed set

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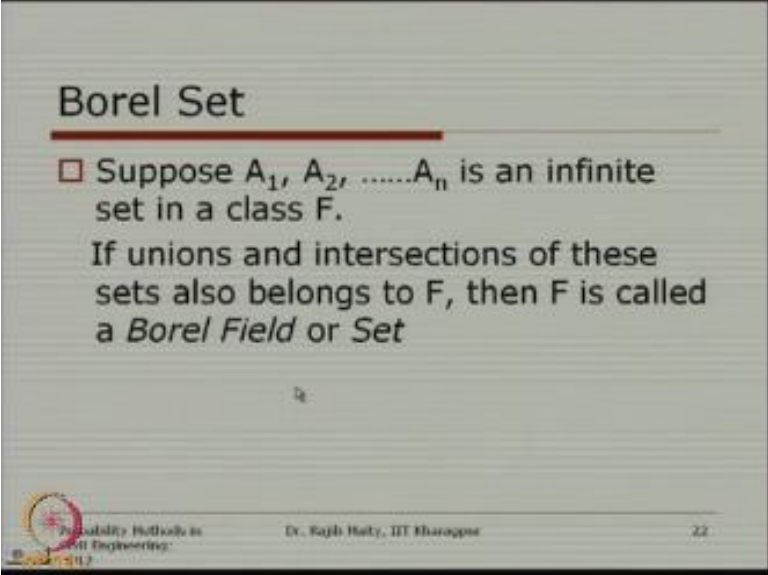
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Coming back to this particular example, which is taken from the Wikipedia the point (a, b) if they satisfy this relationship that is $A^2+B^2=R^2$ which is nothing but an equation of one circle and which is shown by this blue line here this circle. Then A^2+B^2 is exactly equal to R^2 is color blue here though the points a, b satisfying the A^2+B^2 less than equal to R^2 these are colored red.

So there are this red area consists of many points, which satisfies the relationship A^2+B^2 less than less than R^2 , and this circle, this blue circle also consists of many points where it which satisfies this relationship $A^2+B^2 = R^2$. This red points forms an open set this red points, what we can see that in between the inside this blue circle that forms an open set. The union of this red and the sorry this will be blue, the red and the blue points forms a closed set.

The union of red and blue points forms the closed set, because if I take the complement of this union so this union is nothing but this full area red are including this blue area if I take the complement of this set, then it is completely outside. So that is so that is an open set so that is why this union is a closed set.

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Borel Set

□ Suppose A_1, A_2, \dots, A_n is an infinite set in a class F .
If unions and intersections of these sets also belongs to F , then F is called a *Borel Field or Set*

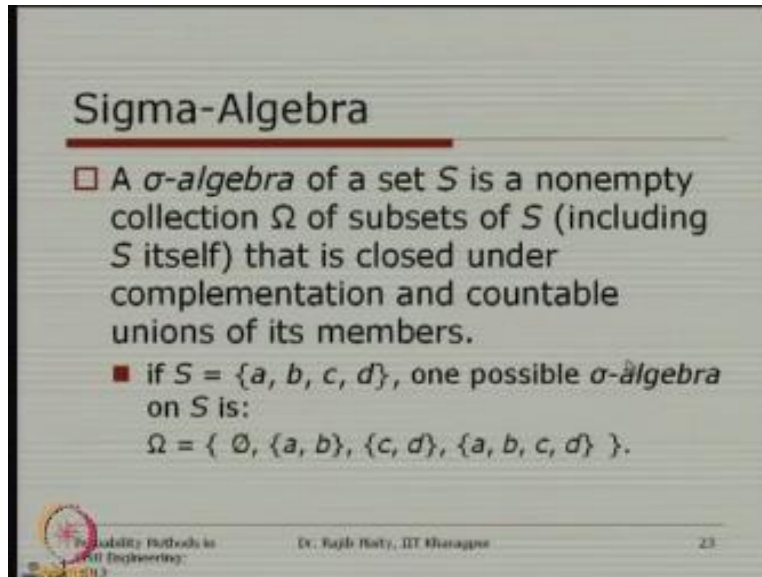
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There is another important concept that is generally say that this is borel set, the concept like this, suppose there are some events n number of sets A_1, A_2 is an infinite sets. In some class F . Now, if you say that the union of any number of sets here from this A_1 to A_n or intersection between any two or more than two sets, all this possible cases, all the unions all the intersection of this sets also belongs to same class F , then the F is known as the bore l field or borel set.

So this is important, when we say that we in particularly in the probability theory, when we say that there are some events, and which belongs to A class F whether that probability can be applied to their unions to their intersection, so to apply that one this condition must be satisfied that is that particular that particular set should be a borel set or a borel field.

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Sigma-Algebra

- A σ -algebra of a set S is a nonempty collection Ω of subsets of S (including S itself) that is closed under complementation and countable unions of its members.
- if $S = \{a, b, c, d\}$, one possible σ -algebra on S is:
 $\Omega = \{ \emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\} \}$.

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Another important thing is the sigma algebra, a sigma algebra is a set S is a non empty collection of non empty collection capital sigma here of sub sets of S , including S itself obviously S is also included that is closed under complementation and countable unions of its members. So there importantly there are three conditions; one is that that set must be non empty this is the first condition, second condition it should be closed set under its complementation.

If I take its complements, then the complements should also be closed; and the countable union of the set also is it is closed under this countable unions also. So there are three condition for a set if this three condition are satisfied, then that set is known as the sigma algebra. So suppose here that this S consist of a, b, c, d is one possible sigma algebra on S is that this particular set the null set first two second two and these full set.

So this is a possible sigma algebra on the A^2 this your that collection of the particular set, this is a sigma algebra.

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Probability Space

- The sample space, S or Ω is called certain event, its elements experimental outcomes, and its subsets events.
- The empty set $\{\emptyset\}$ is called the impossible event
- The event $\{\zeta\}$ consists of a single element ζ is an elementary event

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The probability space, this probability space here is important, because that this is the way how we link between the set theory and the probability concept. So in this probability concept the sample space S or sigma is called the certain event, it is elements experimental outcome and its subsets are events. The empty set null set is called the impossible event, and the event joy concept of a single element is an elementary event.

So the full have sample space or its sigma algebra is certain event. In from the set theory the full sample space is consist of the certain event, its elements of the experimental outcomes, so in the, whatever the experimental outcome that we have taught in the in the previous classes. So its elements the elements of the set is the experimental outcome and its subsets are correspond to some events.

The empty set again here if I say some set is empty that means that is an impossible event. So for that experimental outcome that particular outcome is not possible, and obviously if it consist of a single elements single outcome that is an elementary event.

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Probability Space...contd.

- ❑ In the *applications of probability theory to physical problems*, the *identification of experimental outcomes* is *not always unique*.
- ❑ A single performance of an experiment is called *trial*.

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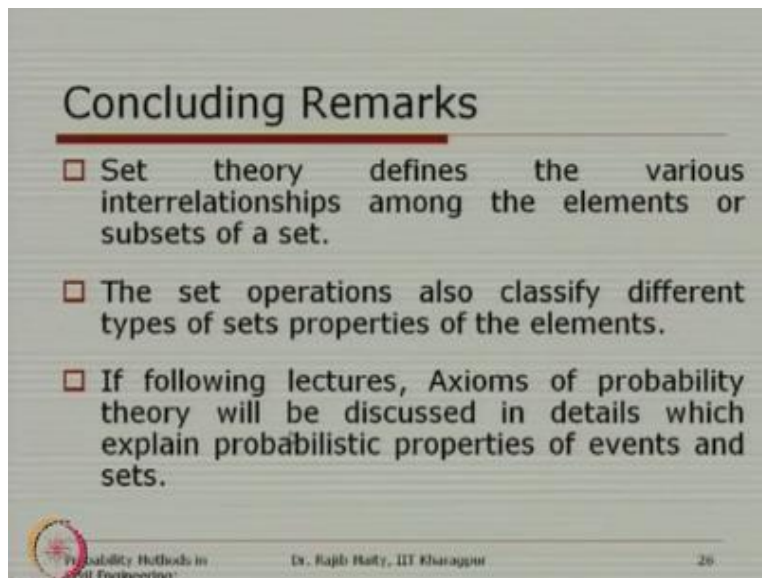
In the application of the probability theory to the physical problems its identification of the experimental outcome is not always unique. This is important in the sense that is how you are interpreting the particular outcome of an event that generally consist of, how you are representing that particular outcome.

For example, if I say that I am just interested to know whether the outcome of a throwing a dies is even number or odd number, then my possible sets or possible outcomes are two either it is even number or it is odd number. But if I say that whether the outcome is 2, 4 or 6 we I am also looking for the even number but my possible outcome here that 2, 4 and 6, so there are three possible outcome in this representation.

Again if I say that if I through a die and its coordinate if I just fix up some coordinate system and say that the outcome is two and along with this location, where the two is comes. So, that location of die itself it is coordinate, it is coordinate that is x coordinate and y coordinate on a plane obviously and the outcome is two. Then the number of possible outcome is infinite.

So this is the way how a particular representation of a particular the application of this probability theory to a physical problems problem generally that this identifications of the outcome is not unique, the way we want to represent it is it becomes in that particular way. A single performance of an experiment which we generally call is as a trail. So, a particular trail whether a particular outcome will come that depends on this how the experiments is conducted, and that is the way we generally assign that particular probability. So a single performance of an experiment is known as a particular trail.

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Concluding Remarks

- Set theory defines the various interrelationships among the elements or subsets of a set.
- The set operations also classify different types of sets properties of the elements.
- If following lectures, Axioms of probability theory will be discussed in details which explain probabilistic properties of events and sets.

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In the concluding remarks, in this class we have seen that different set theory the definition of set and the set theory that defines the various inter relationship among the elements or the subset of subsets of a set. The set operations also classify the different types of sets, property of set the elements. So with this, in following lecture in the next lecture, we will explain about the axioms of probability theory which will be discussed in the next class in details that will very important for the probabilistic properties are event sand set, thank you.

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End of Lecture 03

Next: “Axioms of Probability”

In Lec 04

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