Strength of Materials
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Lecture # 08
Analysis of Strain - ll

Welcome to the second lesson of the second module on the course of Strength of Materials.

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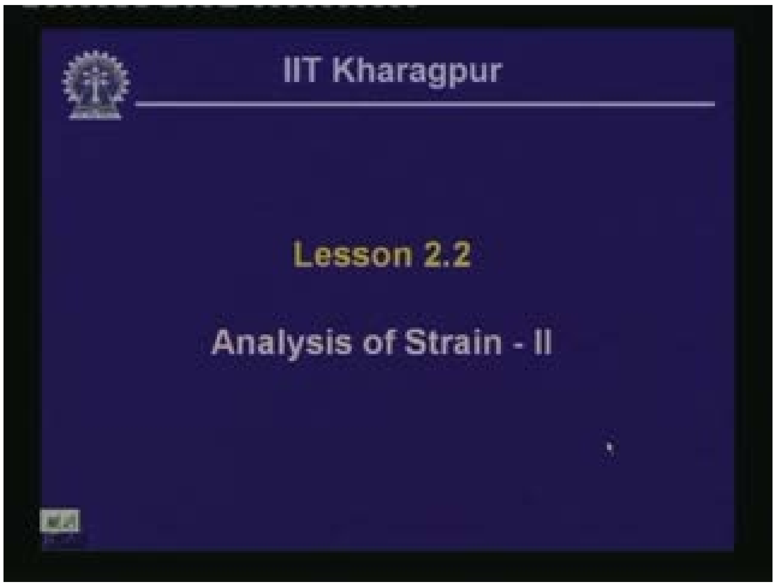

We will be discussing the Analysis of Strain II and in the last lesson we introduced the strain after discussing the different aspects of stresses in module 1.

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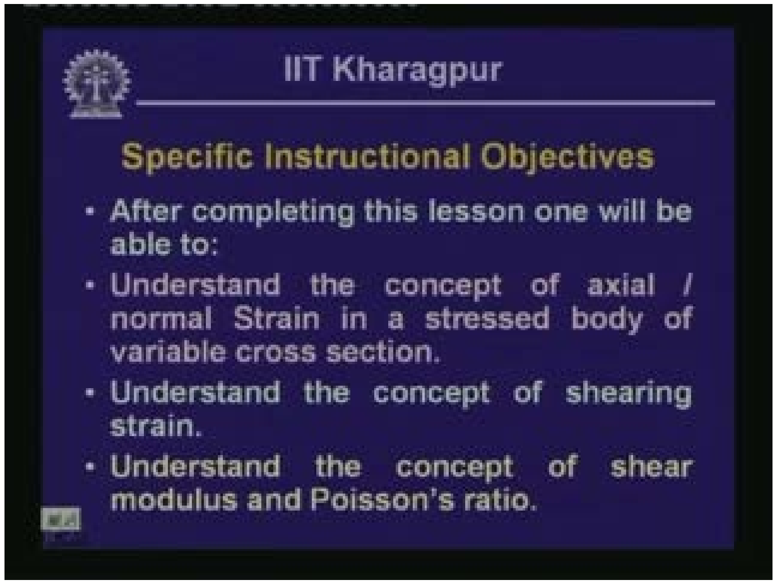

In this particular lesson it is expected that once it is completed, one should be able to understand the concept of axial and the normal strain in a stressed body of variable cross section. In the last lesson, we discussed about the strain, the axial strain, the normal strain in a bar which is of uniform cross section. In this particular lesson we are going to discuss that if a bar is of variable cross section then what will be the strain in that particular body?

To understand the concept of shearing strain also one should be able to understand the concept of shear modulus and Poisson's ratio.

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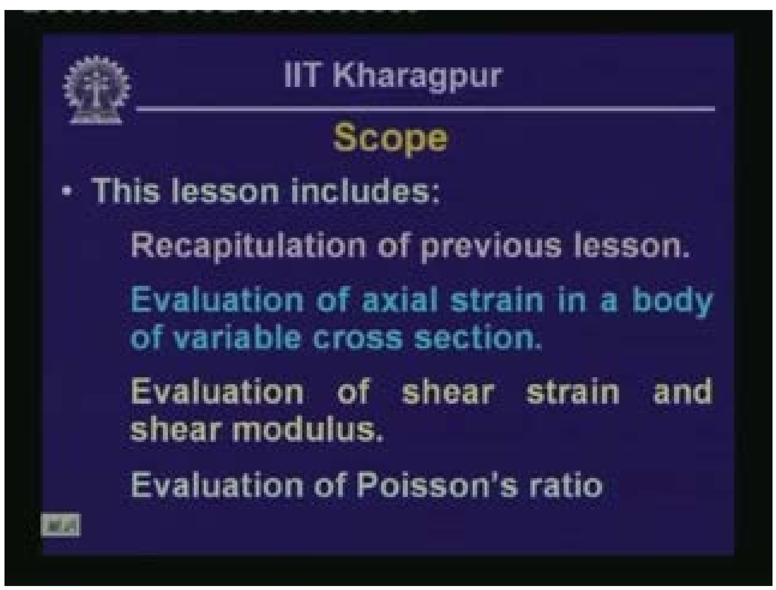

Thus the scope of this particular lesson includes recapitulation of the previous lesson. In the lesson what we have discussed, the first one, we look into it through the question and answers. Then the evaluation of axial strain in a body of variable cross section evaluation of shear strain and shear modulus and subsequently the evaluation of Poisson's ratio. We will see the meaning of Poisson's ratio and the evaluation of Poisson's ratio in the body which is subjected to through loading and undergoing stresses.

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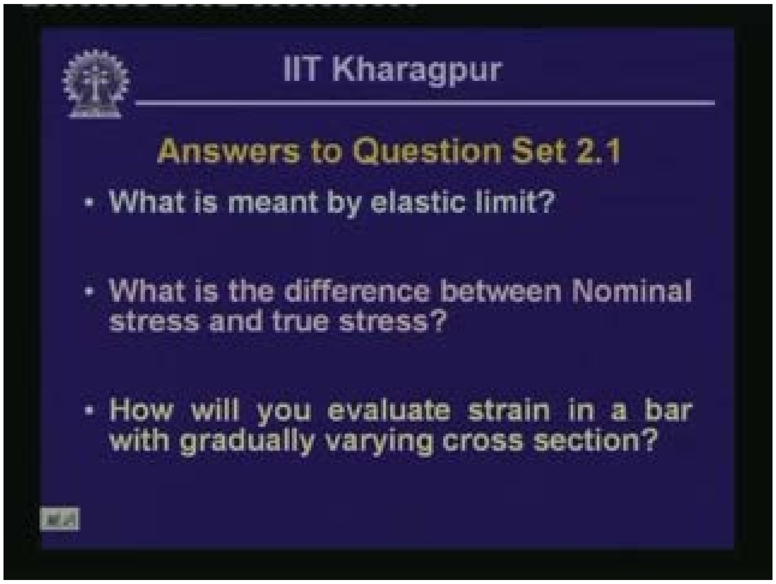

Before we proceed let us look in to the questions which I had posed last time the first question which was posed was what is meant by elastic limit? Now let me explain this with reference to the figure.

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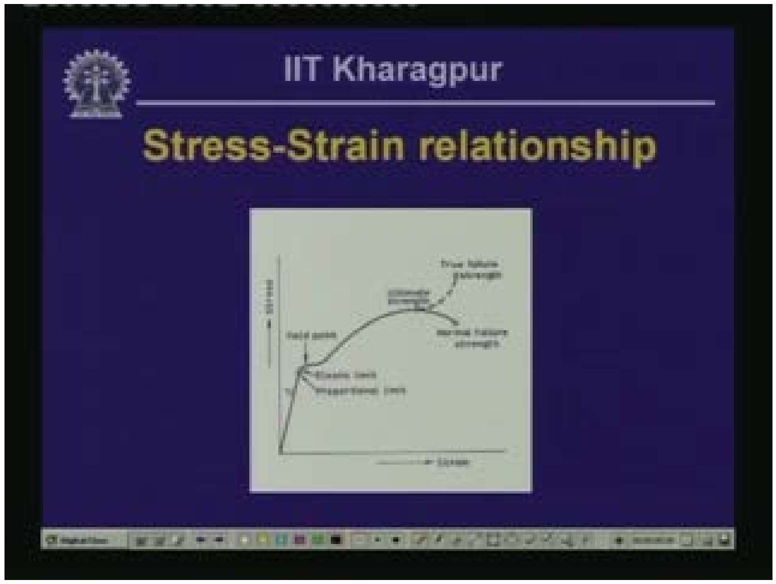

If you can remember last time I had demonstrated that the stress strain relation ship is of this particular form where this axis represents strain and the y axis represents stress. Now, say that if we keep on applying load on a bar in an axial manner if we apply a tensile pull to a bar gradually, the bar will undergo deformism and thereby you exceed that for a bar which is subjected to axial pull how to compute the strain corresponding to that port each increment of the load.

If we compute the strain for a particular load and compute the stress corresponding to that load, we can get a plot of stress versus strain. Now up to a particular limit if we keep on applying the load, on removal of the load the bar is expected to come back to its original position. But beyond a point if we are applying the load, the state of stress and the strain in the bar will be such that it will not come back to its original position after the removal of the load. Now the limiting point up to which on removal of the load the material comes back to its original position, we call that as elastic limit and this is what is indicated over here and up to that limit we expect that the stress is proportional to the strain which is popularly known as the Hooke's Law.

Second question was what is the difference between nominal stress and True stress?
When you compute the stress that any point in the stress strain curve, the stress at any point is given by the applied load divided by the cross sectional area. When we take the original cross sectional area, the stress thereby which we get P divided by cross sectional area, we call that stress as nominal stress. Whereas we are applying the load in the specimen and beyond elastic limit the power is expected to have a reduced cross sectional area.

If we calculate the stress based on this reduced area will get P divided by the actual cross sectional area which is little higher than the stress, this is higher than the nominal stress and that stress p divided by actual cross sectional area is called as True stress. So P divided by the original cross sectional area is an nominal stress, and P divided by the actual cross sectional area will give us the True stress.

Now the third question posed was, how you will evaluate strain in a bar with gradually varying cross section. First let us look into how we perform test on a tensile specimen in tensile pulling equipment. This experiment is primarily the tensile testing of a HYSD bar. HYSD stands for the High Yield Strength Deformed bar, commonly termed as tar steel.

We are interested to primarily evaluate three aspects of it, the hill strength of the bar, the ultimate stress and the percentage elongation the member undergoes. The bar will be tested under constant axial pull and the load will be applied till the member fails. The whole length of the bar has been divided into a number of segments and each segment measures 40 mm. After the specimen is tested, after failure, we will re-measure the length and will see the how much extension this individual segment has under gone which will give us the measure of elongation that the material has undergone.

The objective of this particular test is to evaluate again the hill stress of the material, the ultimate stress at which it fails and the elongation it undergoes which gives us the measure of the ductility of the specimen. This is the control panel from which the load will be applied to this particular specimen at a constant rate till the member fails and the type of load that is being applied on to this particular specimen is a tensile pull.

Now we look into how the load is being applied on the specimen till its failure. After the bar has been tested if we look into the failure of the bar, we will see that one side has been found shape of a cone and the other side is that of a cup and this kind of failure is generally called as cup and cone failure. If we look into the whole length of the bar you will find that the centre part it has under gone extension and because of elastic deformacy the length has been extended.

In the beginning we had divided the whole length of the bar and the segment of 40 mm now if we measure the extended part of that particular segment now, after it has elongated we find that this particular length which was originally 40 mm is now 54 mm. This indicates that this particular bar will be extended by 14 mm. This gives us the measure of percentage elongation and thereby the ductility of the material. This is the experiment which has been performed on a bar applying a tensile pull and then snapped at a point and the type of failure we observed we call as the cup and cone failure.

If you remember in the last lesson we were discussing the equi apply at tensile pull in a bar, the body is expected to undergo deformation gradually and at certain points of time it undergoes failure and that is what this is represented in this stress strain curve.

The stress strain curve says that if we apply load gradually up to elastic limit, if we release the load the load the member comes back to its original position. But we keep on applying the load it undergoes through the maximum stress and then fails at a lower value. This part we call it as a failure stress and this part the maximum stress we call as ultimate stress. Also, in the last lesson we said that in a particular area in the bar we generally reduce the cross sectional area from the entire original area. The purpose of that is to concentrate the failure in that particular zone so that we can observe how the failure is occurring in that particular member.

In this particular experiment there we have not reduced the cross sectional area as such but we are applied a pull to the bar but the whole bar was marked with segments of 40 mm. After the application of the tensile pull the segmental lengths which was originally 40 mm was measured again and the segment in which the failure had occurred we measured the extension which we found to be around 14 mm. Now this gives us a measure of how much elongation the member can undergo before it really fails.

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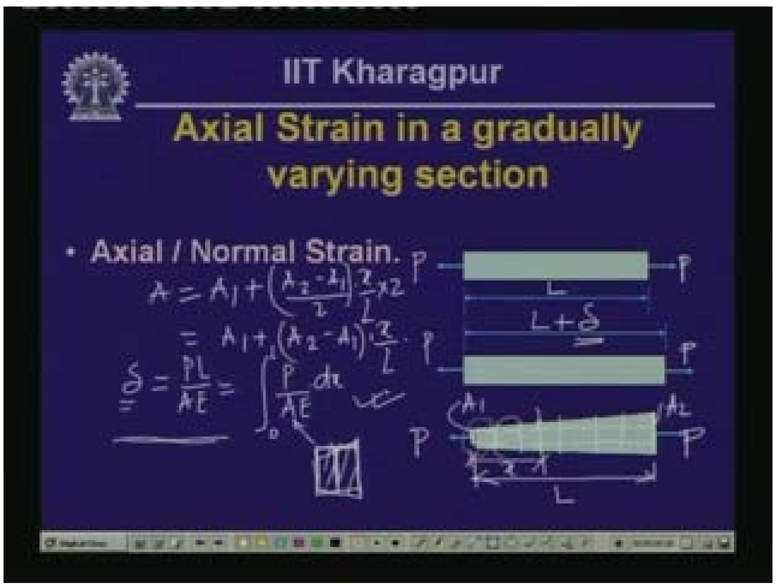

Here is the third question:

What happens if we apply axial pull in a bar for which the cross section is no longer uniform but varies gradually as that of this particular one as it is indicated in this particular figure.

In the previous lesson we said that if we apply a load axial pull p in a bar it undergoes deformation and from its original length L it becomes L plus delta and this extension delta we had seen how to compute. Now if instead of having such uniform cross section if the cross section of the bar is varying, say we have an area here which is A_1 and the cross sectional area here is A_2 and this bar is having subjected to an axial pole p we assume that this area is varying linearly between A_1 and A_2 over the length L. Now this cross sectional area of this bar can be of cylindrical type thereby we can compute diameter, diameter could be varying, or we can have a rectangular cross section in which the thickness is uniform and thereby because of change in the width, the cross sectional area is varying.

Now at any cross section which is at the distance of x from the left end, the cross sectional area of this bar is going to be equal to, this is A_1 plus if we take a line down this is also A_1 so this particular length is a A_2 minus A_1 by 2 so this is (A_2 minus A_1 by 2) x by L and since we have this unit plus this unit which will give me the total area over here so star 2 so this is equal to A_1 plus (A_2 minus A_1)x by L.

Hence the equation is:
A is equal to A_1 plus (A_2 minus A_1 by 2)x by L into 2 is equal to A_1 plus (A_2 minus A_1)x by L. So this is cross sectional area over this particular segment. Now, if we like to find out the elongation of this particular bar which is deltaAs we have seen in the last lesson delta is equal to P l by AE since here the cross section is varying at every segment we may take a small segment which is dx is equal to ∫ P by AE dx. AE is function of x now d x which is varying from 0 to l now we substitute the value of a here which is the function of x and integrate it over the length which will give me the deformation delta. Also, sometimes we can compute the extension by considering the whole of the bar into number of segments. If we divide the whole bar into a number of segments and if we consider one of the segments on an average, this is area A_1 this is area A_1‘. On an average here we can say the area is average of this area and this area by two which is constant over this particular segment.

Here, we are adding something and here we are subtracting something. Therefore on an average this satisfies for the whole segment. For these if we compute delta for this particular segment likewise we can compute delta for each of these segments and if we sum them up we get the total delta for the whole of the bar. But since we are dividing the whole bar in to a number of segments, it is expected that and we are approximating that the area is average of the two ends, so it is expected that we may get a little error in to it .

However, we can compute instead of going for a rigorous calculation, we can sometimes compute the elongation in the bar in this manner. However, for e precise analysis the deformation delta should be computed in this expression.

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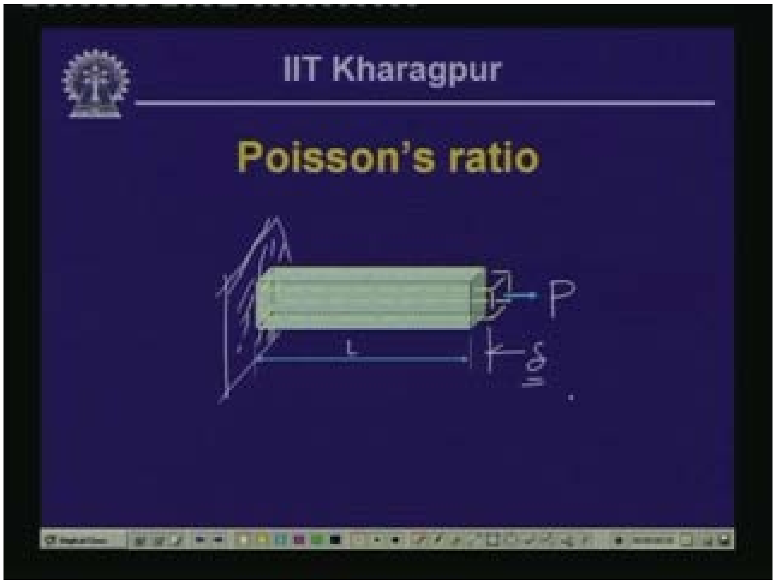

If a bar is fixed at one end and it is been pulled by an axial pull P this is of length L. When the bar is been pulled it is undergoing extension this is the extension delta and we have seen how to compute this extension as a function of this axial pull P in terms of the cross sectional parameter A and the length L. Now when this is being pulled, the bar undergoes deformation in other directions as well. In fact when it is being pulled or elongated in axial direction it undergoes contraction in other directions.

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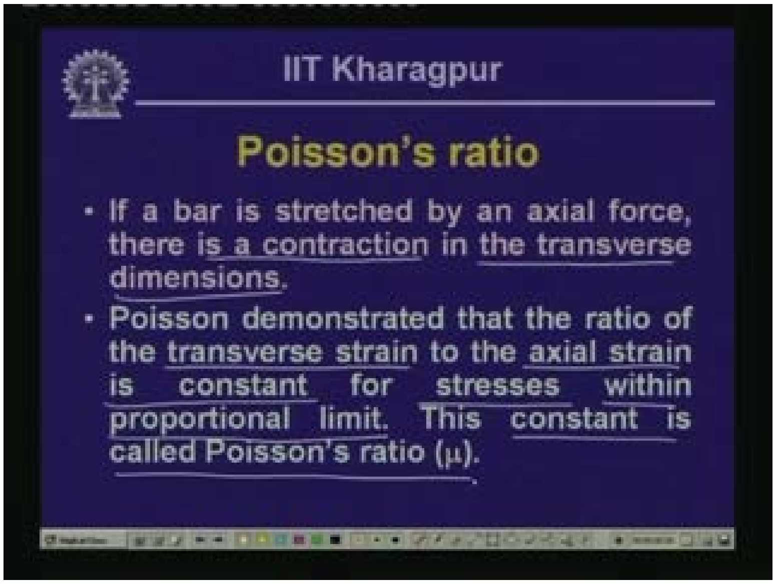

If it is stretched by an axial force then there is a contraction in the transverse dimensions as well. Poisson demonstrated that the ratio of this transverse strain to the axial strain is constant for stresses within proportional limit. So within elastic limit, within proportional limit where stress is proportional to the strain the transverse strain which we get because of these axial pull, the ratio of this transverse strain to the axial strain or longitudinal strain is constant and these particular constant is called as Poisson's ratio.

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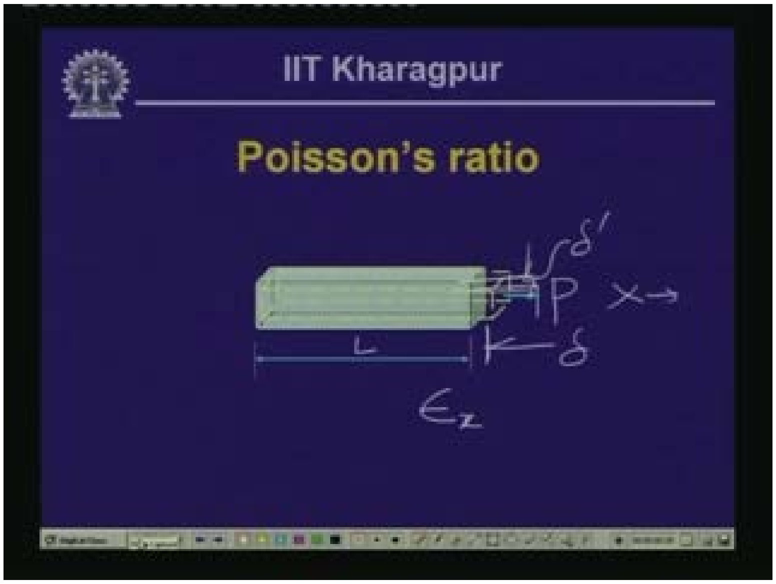

Therefore when the bar is being pulled by axial force p it undergoes deformation in the axial direction. If we call this as x direction then it undergoes deformation in the x direction and thereby the strain which it will designate by epsilon we call the x direction strain as epsilon$_x$ which we can compute in terms of Pl cross sectional area and modulus of elasticity of that member. Also, it undergoes deformation from its original, if these dot line is that the deformed configuration so from its original depth the depth also deduces thereby it undergoes some deformation over here delta prime. So the deformation as Poisson had observed is that these deformations in the transverse strain the transverse strain to the axial strain remains constant for stresses up to the proportional limit and this is what we designate as Poisson's ratio commonly designated as mu sometimes it is designated as the Greek symbol mu as well. So mu is called as Poisson's ratio

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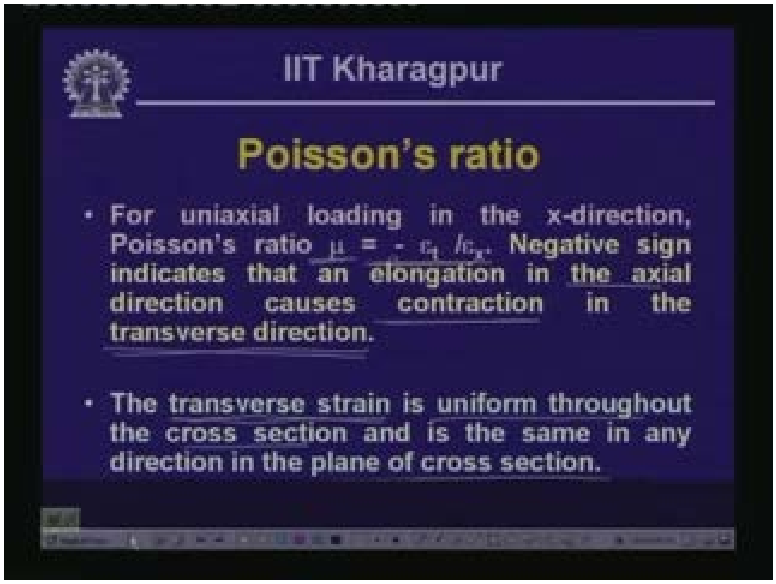

For uniaxial loading when the bar is subjected to uniaxial pull in the x direction then as defined by the Poisson's ratio it is equal to the lateral strain to the longitudinal strain. Here we have introduced a negative sign in this particular expression and this indicates that when the bar is being pulled in the axial x direction in the other direction it is undergoing contraction and that is why this negative symbol has come over here. The negative sign indicates that an elongation in the axial direction causes contraction in the transverse directions and the transverse strain is uniform throughout the cross section and is the same in any direction in the plane of cross section.

An important point to be noted is that the transverse strain which could be the y and z direction is uniform throughout the cross section and is the same in any direction in the plane of cross section. So this leads to the fact that for uniaxial loading that means a bar when it is loaded in the x direction with an axial pull p the contraction or the deformation in the y and z direction if we call this as x direction this as y direction and this as z direction so the strain in the y direction which is deformation to its original length in the y direction epsilon$_y$ is the same as that of epsilon$_z$ as we said that in all directions it is the same is equal to minus mu star epsilon$_x$ where mu is the Poisson's ratio. So lateral strain to the longitudinal strain is equal to the Poisson's ratio so there by the lateral strain the y or in the z direction are equal to the Poisson's ratio times the axial strain.

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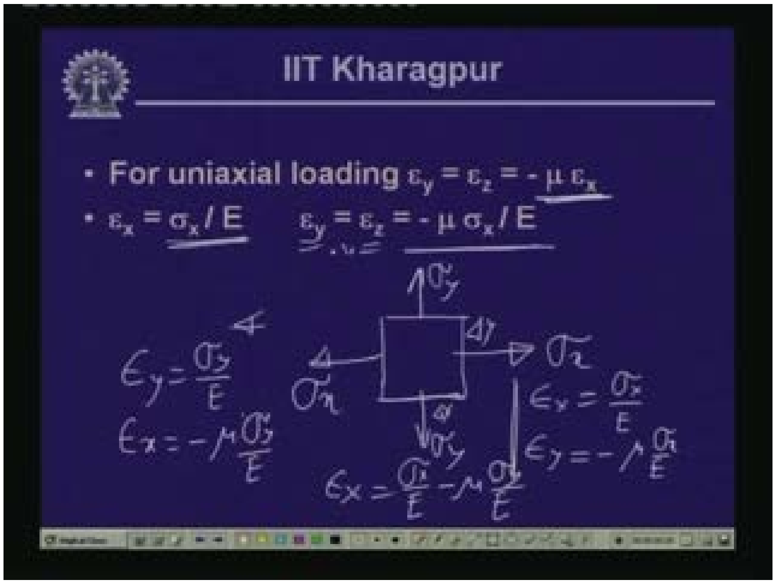

In the previous lesson we have seen that when we relate the stress to the strain after the proportional limit or after the elastic limit the stress sigma is proportional to the strain or sigma$_x$ is equal to E epsilon$_x$ thereby the strain epsilon is equal to sigma$_x$ by E and the strain in the transverse direction epsilon $_y$ or epsilon$_z$ in trans of the axial strain is equal to minus mu sigma$_x$ by y in place of epsilon $_x$ we substitute sigma$_x$ by E which gives as mu sigma$_x$ by E. Now this particular concept can be extended to a planar body.

If we take a planer body in which we have the stresses normal stresses sigma$_x$ and sigma$_y$ and if we say that it is original dimension is deltax and deltay, now when there have been pulled they will undergo extension. When it is being pulled in the axial direction that is the x direction it will undergo elongation in the x direction and it will undergo contraction in the y direction. When it is being pulled in y direction it will undergo elongation in the y direction and there will be contraction in the x direction.

Now, if we take independently that when it is undergoing elongation in the x direction which is epsilon$_x$ is equal to sigma$_x$ by E and correspondingly the strain in the y direction epsilon$_y$ is equal to minus mu sigma$_x$ by E as for the definition of the Poisson’s ratio. This is the case when sigma$_y$ is not there the planar element is being pulled by the axial pull in the x direction. When the plate element is been pulled in the y direction and having no load in the x direction it will undergo elongation in the y direction and thereby there will be contraction in the x direction as per the Poisson’s definition. So when it is being pulled by sigma$_y$ we have the strain epsilon$_y$ is equal to sigma$_y$ by E and correspondingly epsilon$_x$ is equal to minus mu star sigma$_y$ by E.

Now if we consider the case when these plate element is been subjected to sigma$_x$ and sigma$_y$ simultaneously when it is under the action of whole in the x as well as y direction then we can superpose the effect of this individual results because we are within the proportional limit or within the elastic limit and as a result we an superpose the results of the two and thereby the epsilon $_x$ finally is for simultaneous action of sigma$_x$ and sigma$_y$ is equal to epsilon$_x$ from sigma$_x$ by E which is the direct effect and because of y is equal to minus mu star sigma$_y$ by E and likewise epsilon$_y$ is equal to sigma$_y$ by E minus mu star sigma$_x$ by E. These are the strain in the simultaneous action of axial pull both in the x and y.

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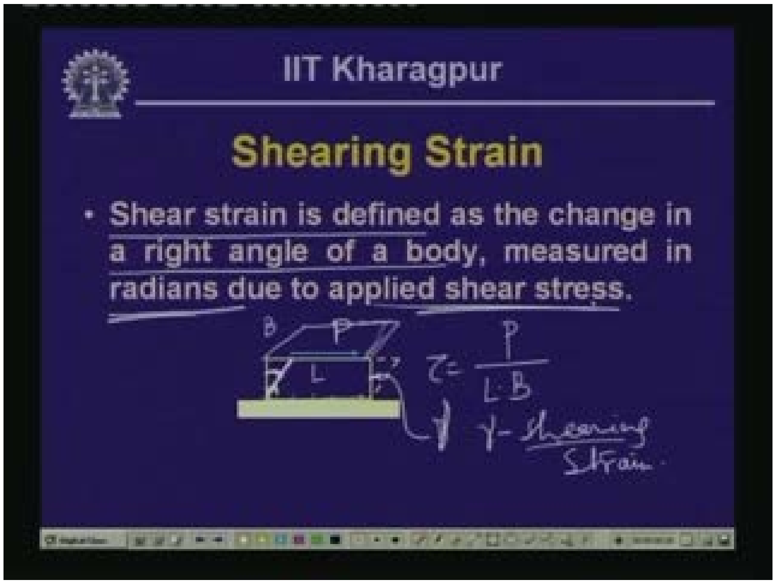

Now having looked into the strength in the axial direction when it is acting only in one direction or acting in the two directions, where in we have the strains epsilon$_x$ epsilon$_y$. We discussed the stresses we encounter, the stress which is in the x direction in the y direction and also the body undergoes at a point a stress which we call as a shearing stress which acts between the two planes.

If we look into such a case where a particular body is subjected to, for example this body is fixed at this particular end and it is subjected to a horizontal force p which is acting on this plane let us assume that this plane has deformed as something like this may be this length is L, this width is B hence the force which is acting on this plane will cause a shearing stress which is P by deltaLB. This is the shearing stress.

Now in this, this particular body is fixed at this particular end and this horizontal planar force will try to cause a deformation in this particular body and this is expected that it is deform state it will undergo deformation as shown in this dotted line. Now why this deformation the angle which we get here this we call as the shearing strength and denoted by the symbol gamma. So gamma is the shearing strength and this is what is defined here, the shear strength is defined as the change in right angle of a body. You see that when this particular body which has right angle over here after the action of this planner force which is causing deformation in this body undergoes a deformed state and thereby this right angle undergoes the deformation and this angular deformation is turned as the shearing strength and generally it is measured in radians due to the applied shear stress.

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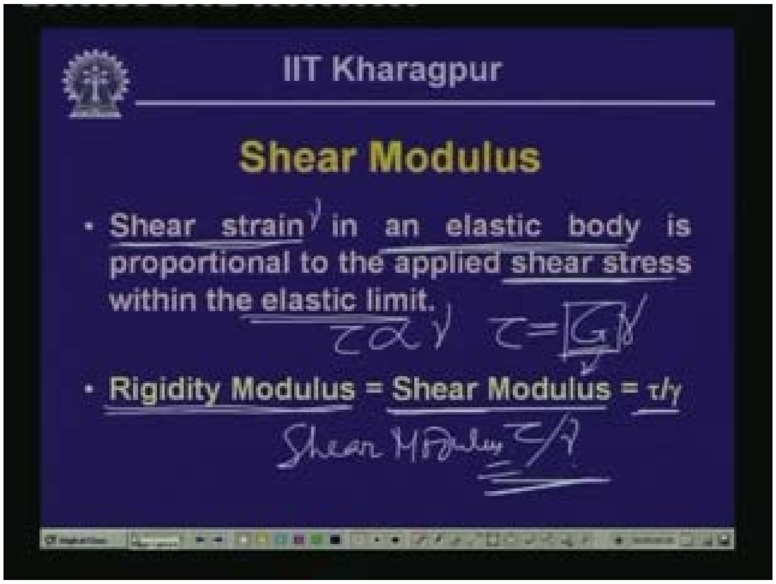

As we had observed in case of the stress and the strain, in an actual pull up to the proportional limit or up to the elastic limit the stress is proportional to the strain. Likewise here the shear strain also is proportional to the applied shear stress. So in an elastic body up to the elastic limit the shearing strain is proportional to the applied shear stress or the shear stress we designate it by tau and shear strain we designate that by gamma. So tau is proportional to gamma within elastic limit and thereby if you remove this proportionality constant we can say tau is equal to G gamma and this proportionality constant G is commonly known as shear modulus or many a times we called that as Rigidity modulus. So the G is the shear modulus is equal to tau by gamma so the shearing stress by the shearing strength gives as the shear modulus G. This is an important result where we look into how as we go along to apply this or to evaluate the stress or the strain in a body when they are subjected to different kinds of loading.

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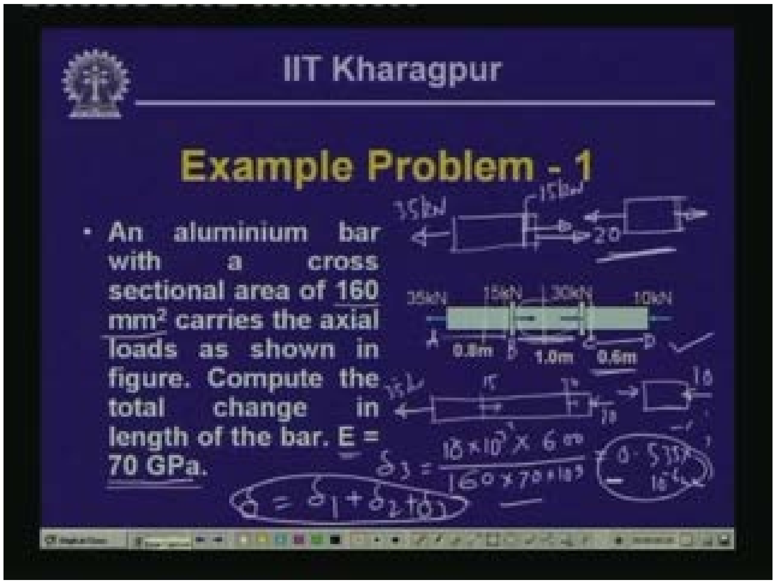

Here are some example problems based on the aspects discussed earlier. An aluminium bar which is having a cross sectional area of 160 mm square carries the axial load as shown over here. We will have to compute the total change in length of the bar and the modulus of elasticity E is given as 70 GPa the Gigapascal.

If you remember, we had solved one problem which is similar to this but we had different cross sectional areas for the different segments in which we had computed the extensions of the different segments and finally we added the extensions to get the final elongation in the bar. In this particular case also since there are variations of the loads at different points the first thing that should be checked is whether the applied loads keep the body in equilibrium or not.

If we look into that 35 Kilo Newton is acting in the negative x direction 10 kilo Newton is acting in the negative x direction so total force that is acting in the negative x direction is 45 kilo Newton and the positive x direction 15 kilo Newton and 30 kilo Newton to forces erecting so thereby this body is in equilibrium under the applications of the forces in the bar.

Last time when we had solved a similar example if you remember what we need to do is we have also from this point to this point let us say this point is A, this is B, this is C, this is D. Now from A to B just a little below B the effect of force which is existent is this 35 kilo Newton so if I take a cut here and make a free body of this of the part A B just tried to the application of this 15 kilo Newton load, then this is 35 kilo Newton. So, to keep this bar in equilibrium we will have to have the resisting or the reactive load which is 35 kilo Newton of this application of the load, it extends or elongates the bar so the extension of this bar under the application of 35 kilo Newton will be delta is equal to PL by AE is equal to P here is 35 kilo Newton so 35 into 10 cube so much of Newton L of this segment is 0.8 meter which is 800 mm divided by the cross sectional area. The cross sectional area of the segment is 0.60 mm square so 160 into E. E for this particular material is 70 GPa is 70 into 10 to the power 9. If you compute it comes to as 2.5 into 10 to the power minus 6 mm. Now this particular bar since it is undergoing axial pull, the tensile pull so this is a positive elongation.

Now let us look into the free body of this particular segment. Just after the application of this force if we cut the bar then we have a forcing system something like this, here we have 35 kilo Newton at this point we have 15 kilo Newton. Now 15 kilo Newton is acting in the positive x direction 35 kilo Newton is acting in the negative x direction so the balance is 20 kilo Newton on this side. So to make this part in equilibrium look to 20 kilo Newton often on this side so this will make the body of this particular free body in equilibrium.

So this particular segment, the part of the body immediately after application of this load to the point prior to the application of this load will have a force system which is something like this. Since 20 is acting in this direction so the redistrict 20 will be acting in this and so for balancing this it has to act in this form thereby this bar will undergo stretching because of this 20 kilo Newton load.

For the third segment for this particular segment if I take a free body here at this point then we have the segment in which the force here is 35 Kilo Newton at this point, it is 15 kilo Newton at this point, it is 30 kilo Newton so we have 30 plus 15 is equal to 45 is acting in this direction, 35 which is acting in this direction so to balance this we will have to have 35 and the 10 acting in this particular direction so in this particular body turn the Resistive force would be turned in this direction. This particular segment is segment 2.

In the first segment we have seen that has undergone an extension of 2.5 into 10 to the power minus 6 mm, the second segment which is under the application of 20 kilo Newton extension is delta for the second part equal to P which is 20 kilo Newton into 10 cube into L which is 1 meter. This is 100 mm divided by the cross sectional area which is 160 into E which is 70 into 10 to the power 9 and this if we compute it comes as 1.785 into 10 to the power minus 6 mm. So this is again an extension because it is a tensile pull which is acting in this particular segment so this is going to give the extension of the bar. Lastly, this particular segment which is under the action of 10 kilo Newton compression will cause the deformation which is in the negative direction is the contraction which we will have in the bar.

Now if we compute the deformation corresponding to this delta 3 is equal to 10 kilo Newton into 10 cube into segmental length here is 0.6 m which is 600 by 160 into 70 into 10 to the power 9 and this if you compute it comes as 0.535 into 10 to the power minus 6 mm and this is negative because it is a compressive force acting in this particular segment and thereby it is undergoing a contraction.

Finally the total elongation of the bar in the first segment, it is undergoing elongation and in the second segment it is undergoing elongation and in the third segment it is undergoing contraction. So the final delta will be some of this three segmental deltas is equal to delta1 plus delta2 plus delta3 and if you compute this will give you the final extension in the bar.

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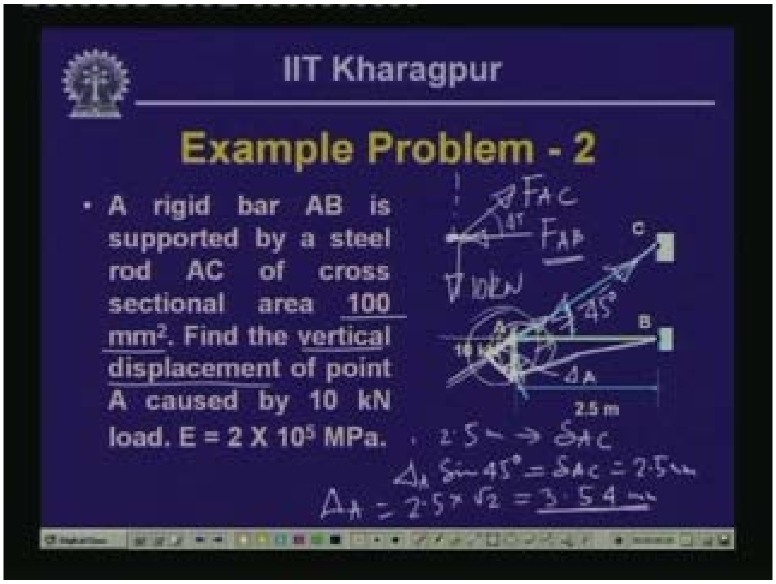

Now let us look into another example problem which is quite interesting. Here we have a rigid bar AB which is being supported by a steel rod AC which is fixed at or used at this particular point and cross sectional area of this particular steel rod is 100 mm square. Now what we are interested in is to find out that how much vertical displacement this particular point undergoes because of the application of this load 10 kilo Newton, this angle is 45 degrees and this length of the rigid bar is given as 2.5m. Now this particular bar being a rigid one because of the application of this load, this bar, the steel rod will undergo extension or contraction which will cause the movement of this particular point from which you need to find out the vertical displacement.

To know how much deformation the steel rod undergoes we need to take a free body of this particular joint and compute the forces. Now if we take the free body of this joint A we have we have the 10 kilo Newton force which is acting. We have the bar AB and we have the steel there. Let us call this force as F_{AC} and this as F_{AB} and this angle is 45 degrees, now if we resolve this F_{AC} in the vertical direction taking the summation of vertical forces has equals to zero that will give us the F_{AC} sin 45 degrees is equal to 10 Kilo Newton thereby F_{AC} is equal to 10 square root of 2 Kilo Newton.

If you take the component of this or if you take the summation of horizontal force as 0 if you take the summation of horizontal force as zero then F_{AC} cos 45 degrees is equal to F_{AB} and F_{AC} is 10 root 2 cos 45 degrees is 1 by square root of 2 so 10 square root of 2 into 1 by square root of 2 is F_{AB} and thereby F_{AB} is equal to 10 kilo Newton and the each of this forces which are acting in the bar.

In this bar it will be F_{AB} is a compressive force and F_{AC} in the joint the forces acting in this direction so in the bar it is acting in this form so it is a tensile force so this is tensile pull in the bar F_{AC}, because of this strain which has been applied on this bar the body is expected to undergo elongation now our job is to evaluate this elongation because of the application of this tensile pull which is tend to group to kilo Newton. So let us compute that how much elongation this bar AC undergoes now the deltaA_C is equal to P L by AE the extension P just now we have computed which is equal to 10 root 2 Kilo Newton into 10 cube N.

We will have to compute the length AC, now we know length AB we know this angle so AC length AC into cos 45 degrees is equal to 2.5 m so L_{AC} is equal to 2.5 square root of 2m so L into 2.5 square root of 2 into 10 cube mm divided by the cross sectional area which is 100 mm square into E which is 2 into 10 to the power 5 MPa Mega Pascal. This gives us a value of square root of 2, this 2 gets canceled with square root of 2 10 to the power 5, 10 to the power 7 this is 10 to the power 6 so this is 25 so this 10 goes up 106 10 to the power 7 so we have 2.5 so deltaA_C is equal to 2.5 mm so the extension of this particular steel rod is equal to 2.5 mm.

Now the interesting part is that these particular bar A B being rigid, now when this rod extends it will try to pull this particular joint or displace this particular joint from this position to some other position and this bar being rigid this will rotate considering these particular point as the center of rotation.

So if this particular point rotates, it is expected that it will take circular r. Now what we are interested is to find out the vertical displacement of this particular point. Now assuming that this rigid bar undergoes a movement in a circular path and this displacement the deformation being small this circular r we assume as a straight one.

If we take this deformation if you call as deltaA if we take the component of this deltaA along AC and perpendicular to AC then this perpendicular segment will indicate the rotation of this deformed AC and this particular part will reduce stretching of the bar or the steel rod AC. So this is the extension of the steel rod AC this is the final position and this particular movement indicates the rotation of the bar AC so this particular joint we will finally come down to this point. Now we have already computed that how much extension this particular steel bar AC has undergone which is equal to 2.5 mm.

This is our deltaA_C and what we are interested is the value deltaA now this angle is 45 degrees so this is 45 degrees this angle also is 45 degrees and thereby this angle is also 45 degrees so from this particular triangle where in this arm is deltaA this is deltaA_C so deltaA sine45 degrees will give deltaA_C so deltaA sine 45 degrees is equal to deltaA_C which is equal to 2.5 mm hence the deltaA is equal to 2.5 star root 2 which is equal to 3.54 mm. So the vertical displacement of point A or this joint A is equal to 3.54 mm. Here you can see that how we can apply the expression for extension. Because of the application of the load in our system where two bars are connected a rigid bar is held by an extensible bar which is producing the reflexion of this particular joint.

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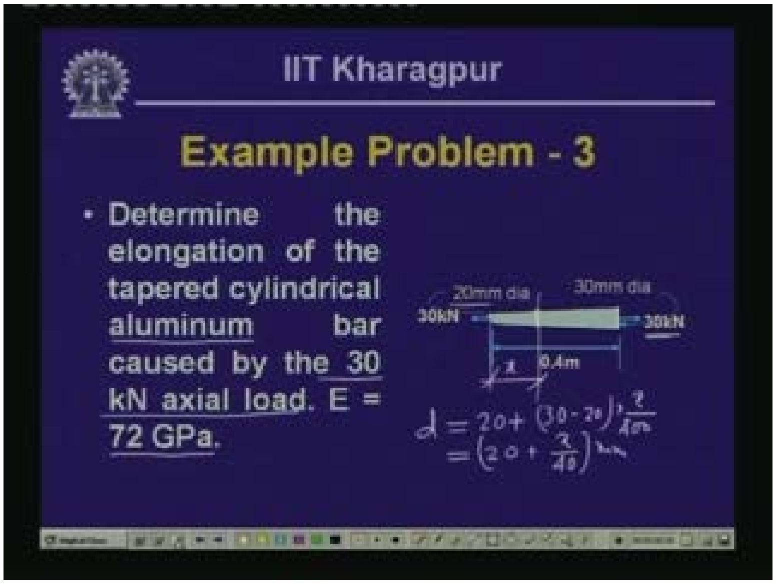

Let us look into another problem where the bar is no longer uniform but it has a variation along the length of that bar on this end the diameter of the bar is given as 20 mm the other end it is given as 30 mm and it is being pulled by a load 30 kilo Newton, what we need to do is determine the elongation of the tapered cylindrical aluminum bar caused by this 30 kilo Newton axial load where E is given as 72 GPa.

We have to discuss on how to compute the elongation in a bar where the cross section is no longer uniform, but it is varying along the length of the bar so applying that concept. Now what we do is that at any cross section which is at a distance of x along the length of the bar we try to find out what is the diameter so the diameter d at this particular point which at a distance of x is equal to (20 plus 30 minus 20) x by L. Here L is equal to 400 mm is equal to 20 plus 10 by 400 so x by 40 mm. This is the diameter which we have at this particular location and since it is varying at every point this diameter is changing as a function of x.

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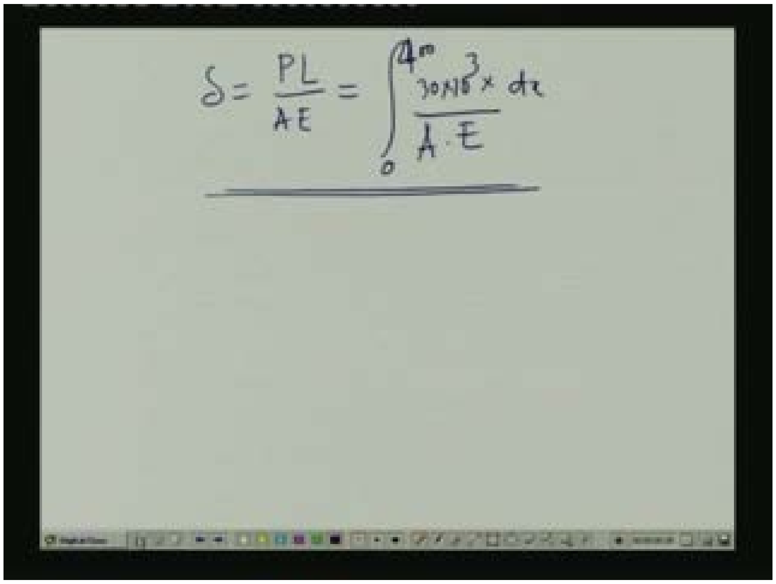

So we compute delta is equal to P L by A E now, here this bar is undergoing an axial pull with a load of 30 kilo Newton so we have restricted this over the length L which is 0 to 400 P is 30 kilo Newton. Since we are taking the diameter of small segment which is dx so A is a function of x and we have a value of E. This if we integrate then we get the extension of the bar.

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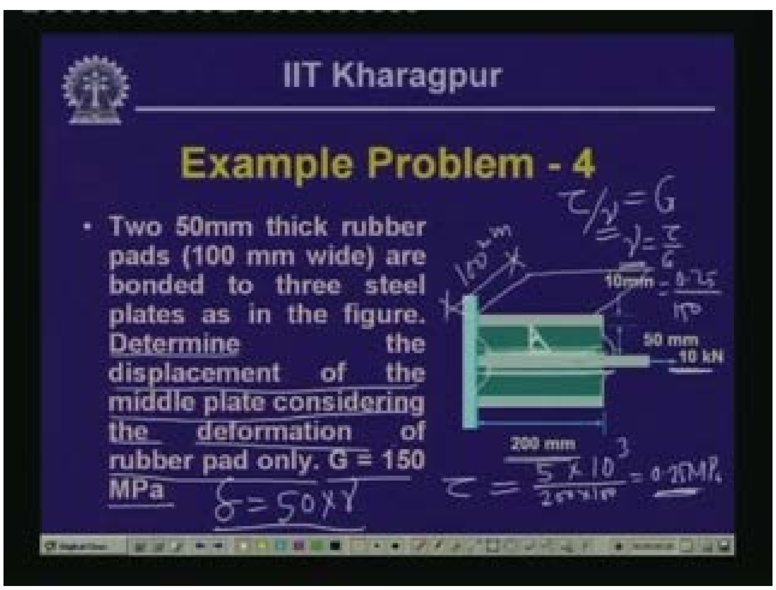

I have another problem which is the applications of the concept of shearing strain. There are 2 rubber pairs, there are 3 steel plates of 10 mm thickness and 2 rubber pairs are attached to this steel plates in between the steel plates which is of the rubber pair thickness is 50 mm and the length of these plates and the rubber pairs are 200 mm the width of this rubber pairs which we look into in the third dimension this distance is 100 mm.

Here when this central bar is being pulled the rubber is undergoing deformation. Here it has been told that, determine the displacement of the metal plate considering the deformation of rubber pad only. Since this rubber pad is connected with this when this is being pulled the pad is undergoing deformation and thereby this will try to be pulled in this form.

Now add these interfaces between the steel plate and the rubber pad, there will be a shearing force which would be acting on this plane which will try to cause. If we take a line here this particular line we will try to undergo a movement which will cause a shearing strength in the body. Now 10 kilo Newton force is acting on this steel plate thereby we have two surfaces on which this can the shearing force can get distributed so half the force will be acting on this surface and half of the force will be acting on this surface and thereby there will be a strain angle and if we can compute this strain angle then we can compute this deformation.

The stress which will be acting is equal to the force which is acting in this place divided by the area and the horizontal pull which is acting at the interface is equal to the half of this load which is 5 kilo Newton so the shearing stress tau is equal to 5 star 10 cube Newton divided by the area which is 200 star 100 which is equal to 0.25 MPa.

So this is the shearing stress which is acting and as we know that shearing stress divided by shearing strength is equals to a shear modulus G, a shear modulus G is given over here. So we can compute gamma the shearing stress which is equal to tau by G the tau is 0.25 and G is 150 Mpa. So this gives as a value of shearing strain gamma and once we know gamma, gamma times this thickness will give the movement that it undergoes. So thickness 50 mm into gamma will give us the deformation this bar will undergo because of the deformation in the rubber band.

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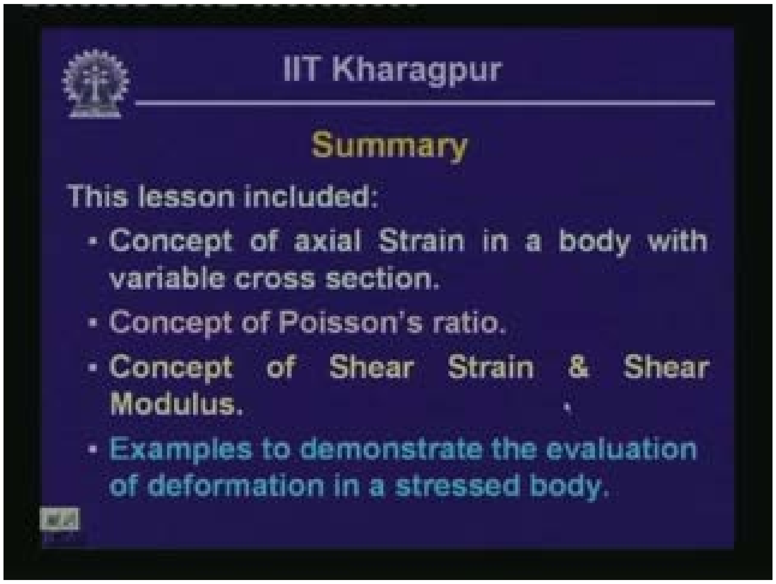

Let us summarize: This particular lesson included the concept of axial strain in a body with variable cross section, the concept of Poisson's ratio and then the concept of shear strain and shear modulus and some examples to demonstrate the evaluation of deformation in a stressed body.

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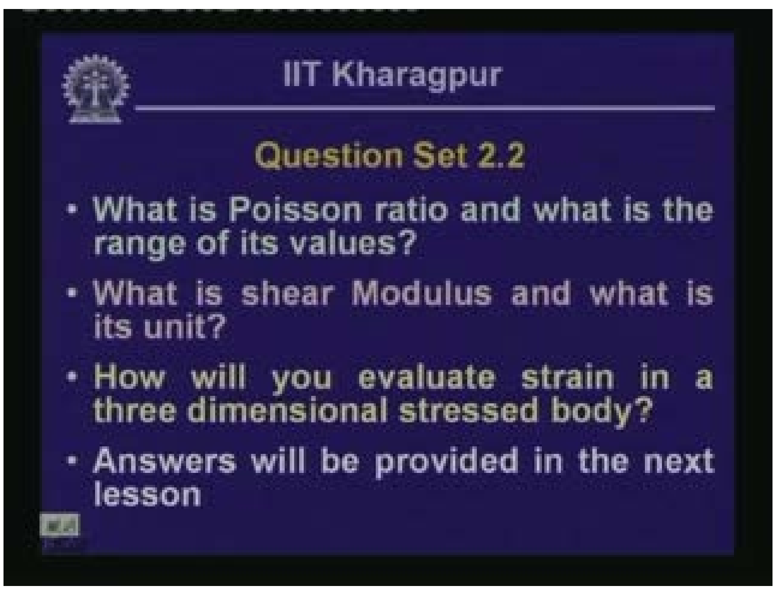

Here are some questions:
What is Poisson's ratio and what is the range of its values?
What is shear modulus and what is its unit?
How will you evaluate strain in a three dimensional stressed body?