

Strength of Materials
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Lecture - 7
Analysis of Strain - I

Welcome to the first lesson of the second module which is on analysis of strain. In the first module we discussed about stresses and we have seen various aspects of stresses. Now we are going to discuss certain aspects of strain.

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Specific Instructional Objectives

- After completing this lesson one will be able to:
- Understand the concept of axial / normal Strain and strain at a point in a stressed body.
- Understand the relationship between stress and strain.
- Understand the concept of modulus of elasticity.

After completion of this particular lesson one will be able to understand the concept of axial strain, the concept of normal strain and strain at a point in a stressed body. One will be able to understand the relation between stress and strain, which is very important when we talk about the strength of material, is not only the stresses but the relation between the stress and the strain is important. One will be able to understand the concept modulus of elasticity which we need for the analysis as we go along.

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Scope

- This lesson includes:
 - Recapitulation of previous lesson.
 - Evaluation of Normal strain and strain at a point.
 - Stress-strain relationship and Hooke's law.
 - Different terms and Modulus of elasticity.

We will evaluate normal strain and strain at a point. We look into the relationship between stress and strain and thereby the Hooke's law which are defined for the elastic

body and then the different terms which we get in the stress and strain relationship and the modulus of elasticity.

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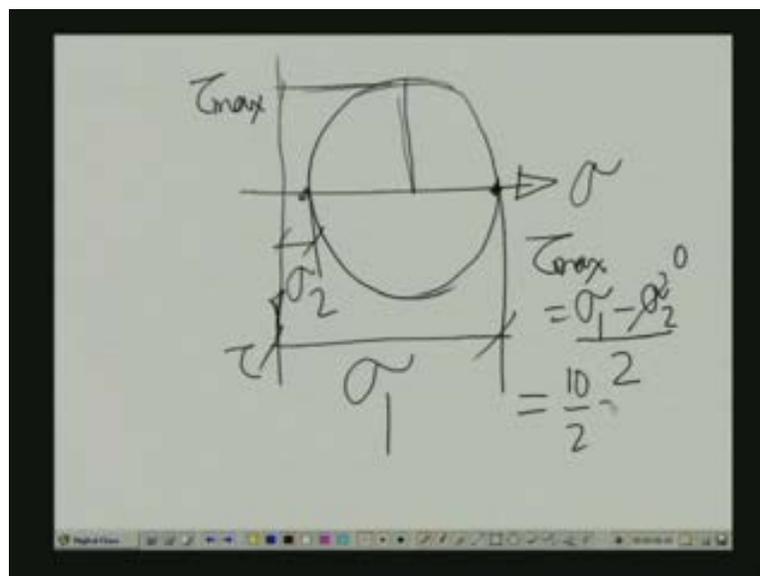
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Answers to Question Set 6.0

- What are the equations of equilibrium in cartesian co-ordinate system of a stressed body?
- What are the equations of equilibrium in polar co-ordinate system of a stressed body?
- What is the value of maximum shear stress if $\sigma_1 = 10 \text{ MPa}$ & $\sigma_2 = 0$?

Before we start looking into the aspect of the strain let us look into the questions. The first question is what are the equations of Cartesian co-ordinate system in a stressed body?

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Equations of equilibrium in cartesian co-ordinate system

- $\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0$
- $\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + Y = 0$

The equations of equilibrium in Cartesian co-ordinate system are $\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X = 0$ and $\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + Y = 0$ where X and Y are the body force components.

Now as you know σ_x and σ_y are the normal stresses to the x and y direction and τ_{xy} is the shearing stress component. So these two define the equations of the equilibrium in two dimensional system.

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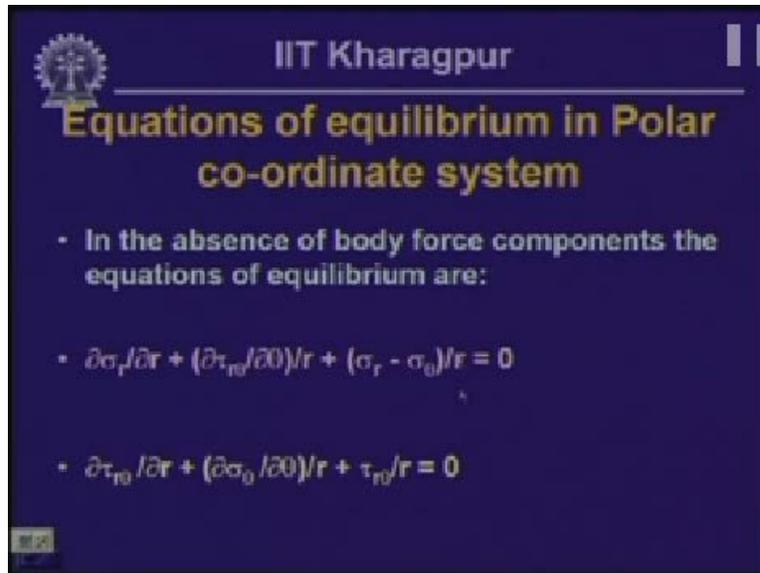
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Answers to Question Set 6.0

- What are the equations of equilibrium in cartesian co-ordinate system of a stressed body?
- What are the equations of equilibrium in polar co-ordinate system of a stressed body?
- What is the value of maximum shear stress if $\sigma_1 = 10 \text{ MPa}$ & $\sigma_2 = 0$?

The second question posed was, what are the equations of equilibrium in a polar coordinate system of a stressed body?

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The slide features the IIT Kharagpur logo in the top left corner. The title "Equations of equilibrium in Polar co-ordinate system" is centered in a yellow font. Below the title, there are two bullet points. The first bullet point states: "In the absence of body force components the equations of equilibrium are:". The second bullet point is a mathematical equation: $\frac{\partial \sigma_r}{\partial r} + \frac{(\partial \tau_{r\theta} / \partial \theta)}{r} + (\sigma_r - \sigma_\theta) / r = 0$. The third bullet point is another mathematical equation: $\frac{\partial \tau_{r\theta}}{\partial r} + \frac{(\partial \sigma_\theta / \partial \theta)}{r} + \tau_{r\theta} / r = 0$. The slide has a dark blue background with white and yellow text.

the equations of equilibrium in polar co-ordinate system, this particular aspect was discussed in the sixth lesson of the module one, wherein we defined stress components, the radial stress, sigma r, the circumferential stress sigma theta and we have seen how to derive this equations of equilibrium.

The radial directional stress is sigma r, the circumferential stress sigma theta and consequently we are sharing stress component which are $\tau_{r\theta}$. And in a polar coordinate form these are the equations of equilibrium $\frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0$. $\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial \sigma_\theta}{\partial \theta} + \frac{\tau_{r\theta}}{r} = 0$. These are the two equations which defines the equations of equilibrium in polar co-ordinate system for two dimensional stress analysis.

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Answers to Question Set 6.0

- What are the equations of equilibrium in cartesian co-ordinate system of a stressed body?
- What are the equations of equilibrium in polar co-ordinate system of a stressed body?
- What is the value of maximum shear stress if $\sigma_1 = 10$ MPa & $\sigma_2 = 0$?

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The third question was what is the value of maximum shear stress if σ_1 is equal to 10 MPa and σ_2 is equal to 0?

Let us look into, if you remember the Mohr's circle of stress in which we had sigma axis and the tow axis. Now in the Mohr us circle, the point on the maximum stress is σ_1 , and the minimum stress is σ_2 . Now radius of this circle is defined as the maximum value of the shearing stress, which is tow max. And tow max is given by σ_1 minus σ_2 by 2. In the given problem, σ_2 is 0, and σ_1 is 10. So there by maximum shearing stress is 10 by 2 is 5 MPa. These are the three questions which were posed and the answers for these were defined. Let us look forward into strain analysis.

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Axial / Normal Strain

- Strain associated with Normal stress is called Normal strain and is defined as the elongation per unit length.
- Units



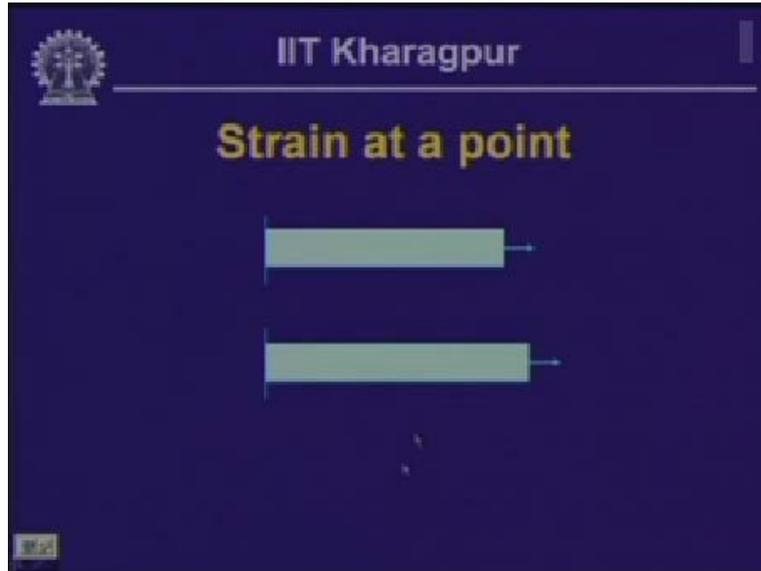
Now we are going to look in to axial normal strain. Now let us assume we have a body which is acted on by force P , length of the body let us assume as L . When a body is subjected to either change in the temperature or subjected to the forces, it undergoes deformations, and in strength of material we are interested to look in to this deformation and we try to define a quantity in terms of this deformation.

Let us assume after deformation, the length is L plus delta. Thereby delta is the deformation or the extension of the body. Now we will define a quantity, which is the ratio of this deformation to the original in the length and is generally designated as epsilon. Epsilon as equals to delta by L and this is known as strain. The load, the force which is acting on the body is in the axial direction and we assume that the deformation is uniform along the length of the body. And thereby everywhere the strain is same.

Also, as we looked in to earlier, when the body is subjected to axial force, we get a stress which we call as normal stress which is the stress normal to the cross section of the body. In line with that, we define that the strain or the deformation which is along the axis has the axial strain, since to compare with the normal stress we call this strain as normal strain. So the strain associated with the normal stress is called normal strain and we define this as elongation per unit length. Now as this is the ratio of two lengths, basically this is a dimensionless number quantity and thereby there are no units as such.

However, it is customary to define strain in terms of the ratio of the lengths, say meter by meter or millimeter by millimeter for a body. Accordingly some times we say the units of strain as millimeter by millimeter. And this strain is defined in terms of a number and sometimes we define this in terms of percentage as well.

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Now the strain when we are calculating for the deformation Δ , we are assuming that this uniform deformation in the entire length of the bar. If the deformation is not uniform, if we compute the strain from Δ by L , then we are assuming the strain on an average sense over the entire body.

Now if the deformation is not same everywhere and if the deformation varies along the length of a bar then the way we have computed stress at a point in a stress body, we compute strain also in a stress body. Now let us look into this particular figure where a bar which is fixed at one end is subjected to a pull P . Now let us assume that we are interested find out the strain at a point, within this body where this is A which is at a distance of x from the fixed end.

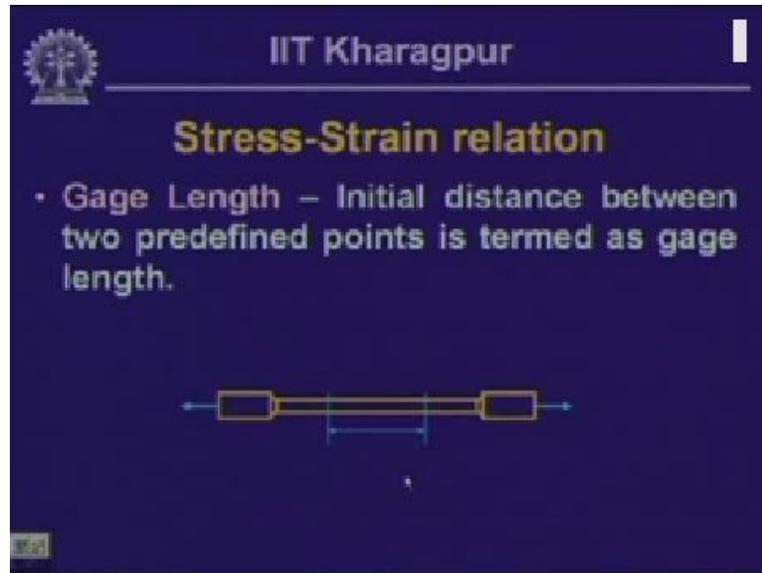
To evaluate strain at a point in a stress body, what we do is consider an imaginary fiber, let us say AB , which is of length dx or Δx . This bar when it is subjected to pull P the fiber also is stretched or deformed and let us assume this is the stretched fiber which was originally Δx , let us say this as $A'B'$ and the length of this stretched fiber is $\Delta x + \Delta \Delta$. Thereby the extension of this fiber is $\Delta \Delta$.

And as we have defined, the strain ϵ for this particular fiber is $\Delta \Delta$ by Δx . And this strain at this particular point A you can define as; strain is equal to $\Delta x \rightarrow 0$, this is $\Delta \Delta$ by Δx . This we can write as $d\Delta$ by dx . Hence $d\Delta$ is equal to ϵdx . So what is the length L if this is the length of the member which is defined as L , then over the entire length the deformation Δ which is $\int_0^L d\Delta$ is equal to $\int_0^L \epsilon dx$ and this is the deformation if the strain varies along the length of the bar.

If we have uniform strain everywhere then this is equal to $\epsilon \int_0^L dx$ which is L and ϵ we get as Δ by L as we have seen earlier. If the strain is

constant everywhere then it is uniform and we get delta by L. If it is not then delta is integral 0 to L epsilon dx. That is how we compute the strain at a point.

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In Strength of Materials, we have seen how to calculate the stress at any point in a stressed body and we have seen various components of the stresses.

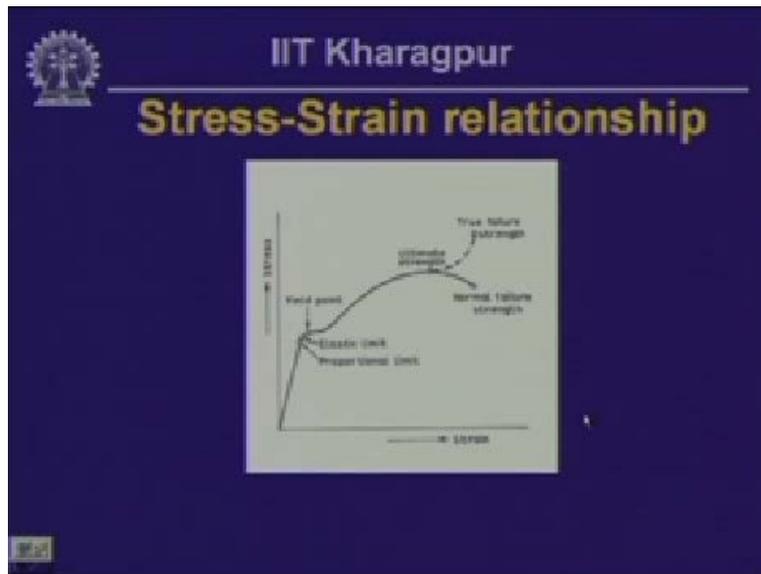
Now we have defined a quantity which we call as strain, strain at a point or strain on an average sense, if it is uniform over the length of the bar. Now what we need to do which is of relevance in Strength of Materials is that the relationship between stress and the strain. Now, for evaluating the relation between the stress and strain, take a body, apply a tensile pull in the body, and we apply this load gradually over the bar, for each increment of the load, we try to find out how much deformation the body undergoes.

Here, if we look, if this is a bar we have formed a section in different cross sectional forms. This part is called the grip which is inserted in the tensile stress equipment and the whole bar is pulled. Now on this bar we fix up a length which is in between this bar, which is little away from this grip, so that the length on which we focus our attention is not affected by the variation of the load in the grip zone. And this length on which we focus our attention we call this as the gauge length. So this is the initial distance between the two predefined bars and this we call as gauge length.

And as I said this length we consider between the bar and little away from the grip zone so that this particular zone is not affected by the force distribution or the stress distribution in the grip zone. Now if we apply a pull on this bar gradually, as we have seen the bar will under go deformation and there by there will be change in the gauge length and if we measure that increment or the deformation, then we can compute the strength.

And as we have seen earlier the stress for a body which is subjected to an axial pull, the normal stress is the axial pull divided by the cross sectional area. So for each increment of the load, we can compute the stress, we can compute the correspondingly the strain and then we can plot a graph to establish the relationship between the stress and the strain.

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Now if look in to the plot between the stress and the strain, the plot is something like this. In this particular figure, this axis represents the strain and the y-axis represents the stress. Now plot each increment of the load, we compute the deformation, there by we get the strain. And for several such points we plot it, on a steel bar this is the profile we get a different stage of loadings.

Now for this particular figure there are various terms indicated with it. One we have called as proportional limit, this we have called it elastic limit, this particular point we have called it a yield point, this is the ultimate strain or ultimate stress, this particular point, actually this is wrong. This is not normal, this should be nominal and this is the nominal failure strength. Now here if you note it that we compute the stresses here with reference to the original cross section of the bar which we have computed.

If you can visualize, when we are pulling this bar or when we are applying a tensile pull, as it deforms the cross sectional area of the bar reduce, now the stress a we know is stress divided by the cross sectional area, the cross sectional area as we take the original cross sectional area, and accordingly we get the profile of this curve. If we take the actual area, load divided by the actual area then the stress value will be different from this. In fact from this particular point configuration is some thing like this. Now the stress when you compute reference to the original cross sectional area of the bar, we call those stresses as the nominal stresses and correspondingly the nominal strain. Otherwise if we compute the stresses with reference to the change cross sectional area of the bar, we call that stress

as true stress and the corresponding strain as true strain. So here what we have plotted is the nominal stress and the strain.

Now, the meaning of this proportional limit is, up to the level of proportional limit, the stress is proportional to the strain. So we say the stress, σ is proportional to the strain ϵ . Now this particular point elastic limit is the point, up to which if the load is applied on the body and if it is released the body comes back to its original state and that we call as the elastic limit of the body. But beyond elastic limit, if we apply load and go beyond the elastic limit, and if we release the load the specimen will not come back to its original place and some amount of deformation is permanently set in the body and that we call as permanent state. And yield point is the point, when the body starts yielding. It goes beyond the elastic limit, the plasticity starts forming in the section.

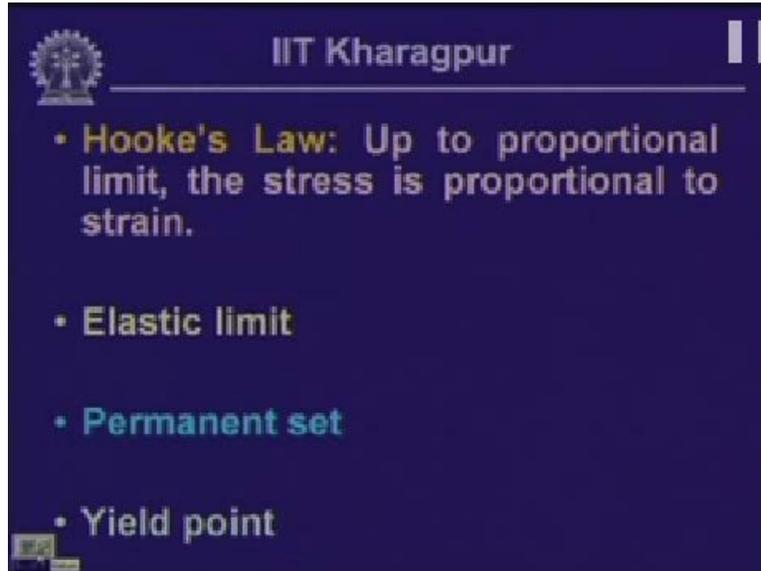
And if we keep on applying the load at that particular point in fact increasing the load, the bar deforms and the extension, the deformation becomes excessive and reaches to the stress which we call as loading down on the load when the bar fails and that how we get failure stress which is less than the ultimate stress.

What is the maximum stress a bar can attain?

We call that as ultimate stress and this particular point is the failure stress. Now in the bar these two points proportional limit and the elastic limit is vary difficult to distinguish and we consider that up to this level, this up to the elastic limit, the stress is proportional to the strain. If we remove this proportionality constant, σ is written as constant times E , which is constant for that material and this is what we know as Hooke's law.

Up to the proportionality limit or up to the elastic limit, the stress is proportional to the strain. Or stress equal to E times strain, where E is called as the constant of proportionality or the modulus of the elasticity, which is an important parameter in the Strength of Materials and this is constant for the material which we are considering during the evaluation of the stress and the strain known as Hooke's law which is at the elastic limit, the stress is proportional to the strain or σ is equal to E times ϵ .

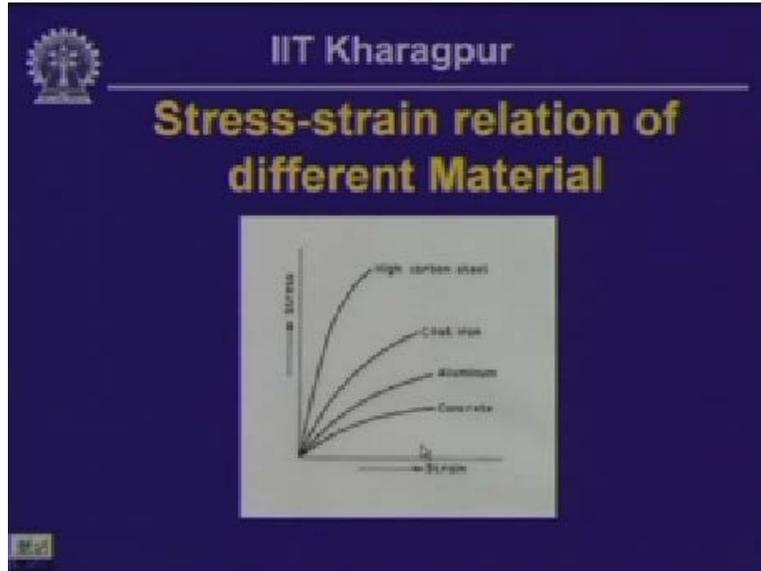
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So we have seen the Hooke's law which is up to the proportionality limit, the stress is proportional to the strain and there by we get $\sigma = E \epsilon$. We have seen the elastic limit, if the bar is loaded and is allowed to extend and if the bar is within the elastic limit, if we release the load, the bar is expected to come back to its original state.

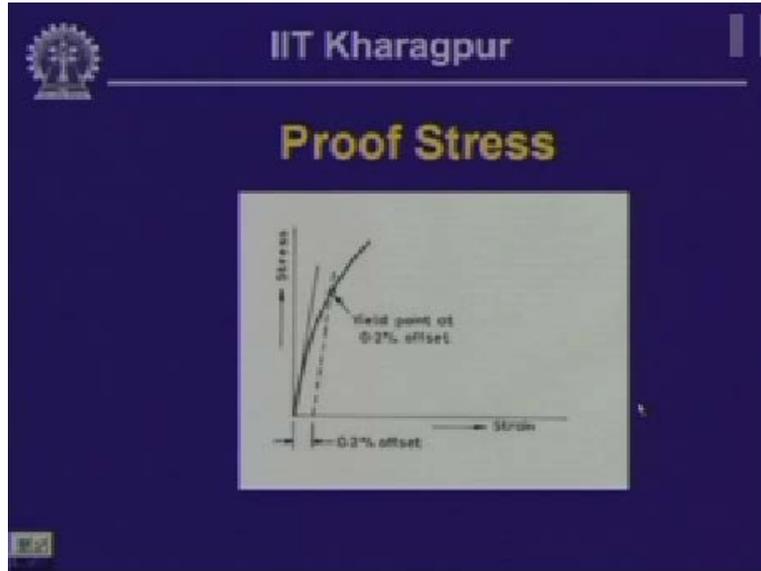
The permanent set is, as the member yields as it reaches to yield stress then plasticity forms in the section, then if we release the load, as we have seen in the case of elastic material when the bar is still within its elastic limit the bar comes back to its original position but once it starts yielding, then if we release the load, the bar does not come back to its original position and some amount of deformation sets permanently in the body and that is what we call as permanent set. Yield point again is the point where the material starts yielding or it goes beyond the elastic limit and plasticity starts setting in the member.

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Well, these are some of the stress strain relationship for different materials. If we take concrete specimen, apply tensile pull in tensile stress equipment, and then we get the profile similar to this, which is the stress strain relationship. If we take materials like aluminum, cast iron or high carbon steel we get these kind of profile and these are necessary to know the relationship between stress and strain by the modulus state of elasticity. So that we can compute the stress and can compute the strain and we can establish the relationship between stress and strain in a body when the material is used either for some equipment or the machine part or in structural body where we interested to find stress and strain, we need to know the relationship between stress and strain material that with which the machine part or the structure is composed of.

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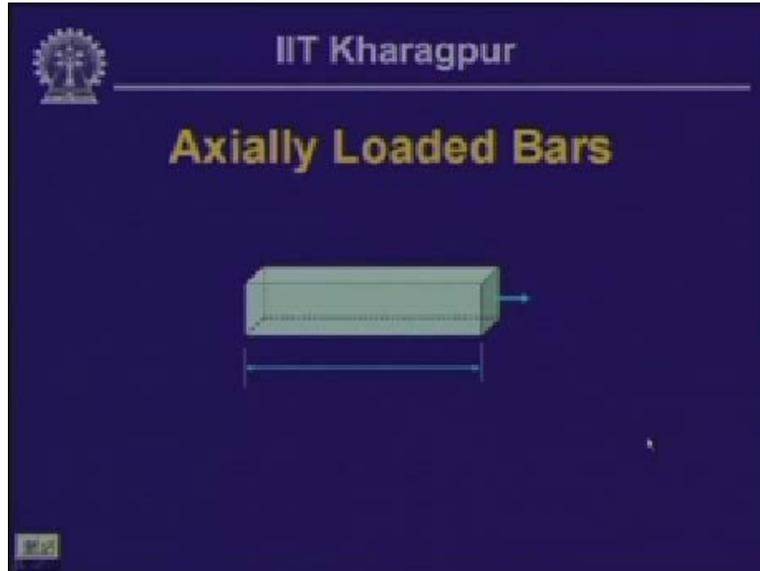


Now as we have seen in the previous figure, in the first figure, in this particular figure as you have seen that we have defined yield point. In this particular zone, this particular curve shows that we have defined yield point. So corresponding to this stress we know that this is the yield stress we defined as σ_y , but as we have seen in the subsequent figure that we do not have any defined yield point and in strength of material when we deal within the elastic limit we need to know the what is the value of the yield stress beyond which the member will start yielding. So we try to limit our stress when we deal with elastic level of analysis.

We like to limit over self to the elastic level, so we need to know what is the value of the yield stress?

But from this kind of stress strain distribution, it is difficult to know the value of the yield stress. Now, to compute the value of yield stress we do is to observe that the strain corresponding to the yield stress value is the order of point two percent, this is point 0, 0 two is the strain. Now, if we draw a tangent to the curve at this particular point, which we call it as initial tangent and if we draw a line at point two percent strain and if draw a line parallel to initial of the tangent, the point were it cuts the curve, the stress strain curve, the corresponding stress we call as yield stress and this yield stress is normally we designate as proof stress. So for the material where we apply a tensile pull and plot a stress strain curve and corresponding to that stress strain curve if we do not get a defined yield point, corresponding to which we are interested to find yield stress then we compute the yield stress in the direct way corresponding to point two percent of strain and this stress we call as proof stress or the yield stress of that particular material.

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Well as we have seen earlier, in case of the stresses when a bar is subjected to axial pull we have computed the stress which we have defined as the normal stress, the stress which is normal to the cross section of the bar and the load is acting perpendicular to the cross section through the axis of the bar.

Now for an axially loaded bar, if we are interested to compute the strain, let us assume that this particular bar say is fixed at this end, the length of this bar is L and is subjected to pull P , thereby if the δ is the extension then the strain is equal to δ by L and we consider that for this particular bar a this analysis you carry out is within the elastic limit of the bar. And we have seen right now in the stress strain relationship that within the elastic limit of the bar is the stress is proportional to the strain. And we write σ equals to $E(\epsilon)$. So, in place of ϵ , you can write $E(\delta$ by $L)$. Hence from this equation we can write the deformation δ as σ times L by E . As we know that this is a bar which is axially loaded and the load is acting through the axis of the bar, any cross section.

Normal stress σ P is divided by the cross sectional area. So σ is equal to P divided by A . So if we substitute the value of σ in this we get, δ is equal to PL by AE . So for an axially loaded bar if we know the axial pull, if we know the cross sectional area, if we know the length and if we know the material with which this bar is made of, for which we know the modulus of elasticity then we can compute what will be the deformation in the bar.

Considering that every where the strain is same, however the strain is not everywhere, if there is the variation of the strain everywhere, in the previous calculation that the strain or the deformation, δ as we have seen at deformation δ is equal to the integral 0 to L ϵ dx and as we have seen in the stress strain relationship, σ is equal to E times ϵ so this is equal to integral 0 to L , in case of ϵ , if you write this is σ by

$E dx$ and σ if we write as P by A then we have ΔL equals to $\int_0^L \frac{P}{AE} dx$. Now if P and cross sectional area are different then we get deformation different at different points. But if the axial load P and the cross sectional area A they remains same, if there is no change then we get the same expression which is $\frac{PL}{AE}$, if the $\frac{P}{AE}$ is constant then $\int dx$ will be L . So ΔL by $\frac{PL}{AE}$ is constant for the cross sectional area A .

However, if there is variation in the P , area A and the length of the bar then we get different deformations at different points and correspondingly the strain value will be different. That is how we calculate deformation in the axially loaded bar.

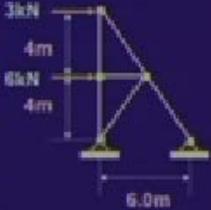
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Example Problem - 1

• A billboard supporting steel truss is shown in the figure. The cross sectional area of all members is 100 mm^2 . Calculate the stress in each member due to horizontal forces as indicated. The truss joints are pinned.



Having defined this strain we have looked in to the concepts of stress in the module one, now we defined the quantity called strain, we have established the relationship between stress and strain and within elastic limit we have seen in the axially loaded bar, how we can evaluate the deformation if we know the load and if the strain is uniform and what is the relation between deformation and the load corresponding to the cross sectional area of the member and in terms of the modulus of elasticity of the material and if there is the variation of the load, if there is the variation in the cross sectional area, then it is expected that the deformation to be different and accordingly the strain will be different at different point. Let us once again look at an example related to evaluation of stresses at different points.

Now this is the truss member in which, which is supporting the bill board. Billboard is supported by two which are stress and cross sectional area of all the members. Call this as A , this point as B this is then C and this is D and this as E . Now we are interested to know the stress in each member when the particular board is subjected to a load and the area, they are the forces which are acting at 3 Kilo Newton here and 6 Kilo Newton here,

so what we need to do is to evaluate first the forces in each member and once we compute the forces in each member, so when the supports are pin jointed.

We have the axis, axial force in each member and this actional force at each member area will give out the stress, that us what we are interested in calculate the stress in each members. Now from this triangular configuration this length is six meter, this is given as 4m and this is given as 4m. So the distance BC is also going equals to this divided by this is equal to the force divided by A which is $\frac{1}{2}$, so this is 3m.

So, if we call this as theta. Now if we take the cross section area here, then we have the member which is like this. This is theta, and we have the horizontal force is acting here, which is 3 Kilo Newton, now the direction of the force is this and the direction of the force is in this direction. Now if we take the horizontal component of this force, which is FAC, so $FAC \sin \theta$ is equal to 3 Kilo Newton. $FAC \sin \theta$ is equal to 3 Kilo Newton. Now $\sin \theta$ over here is equal to 3 by 5, this is 4 and this is 3 hence this is going to be 5. $\sin \theta$ is equal to 3 by 5. So FAC is equal to 5 Kilo Newton. If we take the vertical equilibrium of the forces so $FAC \cos \theta$ and if we call this force as FAB then $FAC \cos \theta$ is equal to FAB or $FAB \cos \theta$ is equal to minus FAB is equal to 0, that is the vertical forces in the equilibrium. So FAB is equal to $FAC \cos \theta$. And value of $\cos \theta$ is 4 by 5 and FAC being 5, so the value of FAB is 4 Kilo Newton.

Now, if we look in to the direction of the forces assumed over here, this is the force in the joint. Member is subjected to like this, which is compressive in nature. So the member AC the joint force direction in this will be subjected to a compressive force so in the member the force is in this direction and thereby the member AB is on the tension and so this is the tensile force. The member AC and member AB is subjected to a force of 5 Kilo Newton and 4 Kilo Newton so these divided by the cross sectional area, 5 Kilo Newton divided by 100 mm square will give us the stress in FAC and correspondingly 4 Kilo Newton by 100 mm square will give me the stress in FAB.

Likewise we have to compute the forces in this bar, in this bar, this bar and this bar. Supposing if we take or cut the structure from here and take the free body of the upper part of it then we get a configuration which is like this here; we have 3 Kilo Newton, here we have 6 Kilo Newton, his is force call FBD, this is A, this is B, this is C and let us call this as D and this point here we call as E.

So this is ABD member in this direction, which is FCD and here we have force FCE. We can compute forces if we take the moment of all the forces with respect to this then, 3 Kilo Newton is going to distribute the forces three times the moment and FBD will have the moment this times this distance which is equal to 3. So 3 Kilo Newton into 4 which is in anticlockwise moment, if we take moment about C is equal to 0, so 3 into 4 is equal to FBD into 3. This is going to be in anticlockwise direction, so thereby FBD is equal to 4 Kilo Newton. And again this is at a joint at this direction, so the member the force will be in this particular direction so this gives us again a tensile pull, so this is a tensile force. So this force divided by the cross sectional area will give me the stress in the member.

Now this particular angle we have assumed as theta and this is also theta and again we can take the horizontal and the equilibrium forces of all the forces and we can compute the value of FCE and FCD and thereby the value of FCD we will get as equals 5 Kilo Newton and this is a tensile pull and the force FCE we will get as 10 Kilo Newton which is compressive force. And so we have seen AB, BC, BD, CD, CE and the member which is BC.

If we take the equilibrium of this particular joint and we have 6 Kilo Newton here and 6 Kilo Newton at joint B, we have member AB, we have member BD and we have member BC where force will be FBC and value from this will come as 6 Kilo Newton, which is a compressive force.

Once we get force in each of this member, force divided by cross section area will give me the stress in each member. That is how we compute the stresses in the member. Now by stating that the truss joints are pinned means that the forces transmitted in each of these members are purely axial in nature.

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Example Problem - 2

- A steel rod having a cross sectional area of 300 mm^2 and a length of 150 m is suspended vertically from one end. The rod supports a tensile load of 20 kN at its free end. Find the elongation of the rod. Value of $E = 2 \times 10^5 \text{ MPa}$.

Having discussed that particular problem related to the evaluation of the stress as we had discussed in the previous module, how to compute stresses at a point in a stress body, now in this particular lesson we have seen how to compute the deformation in a member for an axially loaded member or there is variation in the load of the cross sectional area, what is going to be deformation in the member along with the length of the member.

Now let us look into some examples; how do you compute these deformations if the forces in the member is known, we know the cross sectional area of the member and we know the material property which is of the elasticity of the material. Now in this example, we have steel rod, which is having cross sectional area of three hundred millimeter square and the length of the bar is 150 m , is suspended vertically from one end.

The rod supports the tensile load of 20 Kilo Newton at the free end we will have to find out the elongation of the rod. Or first we will have to find out the deformation. The value of E is given as 2×10^5 MPa. So it is like you have a bar of length 150m and it is supported at one end, it is hung from the top and at this free end this bar is subjected to a load of 20 Kilo Newton, the length is 150m.

So we are interested to compute that what is going to be the elongation of the bar because of the axial pull? Bar is going to be stretched or it is going to deform, so we are interested to know how much this bar under go deformation because of the axial pull which is acting, one end is fixed and the other end is pulled. Now here as we have seen deformation Δ is equal to $\frac{PL}{AE}$ where P is the axial pull acting in the bar, L is the length of the bar, A is the cross sectional area of the member, which is uniform here and E is the modulus of elasticity of the material. So this is equal to P is given as 20 Kilo Newton, so 20×10^3 Newton, L is 150m, so 150×10^3 so much of millimeter divided by cross sectional area A which is 300 mm square into E which is 2×10^5 which is N by mm square MPa.

So this 10 cube, 10 cube cancels with 10 to the power 5 and 0, 6. These two 0s cancel. This 2 and this 2 gets canceled. So we have 15 is equal to 3 into 15 is equal to 50 mm. So the amount of elongation, this particular bar will be under going because of this load.

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Example Problem - 3

- An elastic steel bar of variable cross section is subjected to axial loads as shown in the figure. The cross sectional areas of segments AB, BC & CD are 1000 mm^2 , 2000 mm^2 and 1000 mm^2 respectively. Evaluate the elongation of the bar. $E = 2 \times 10^5 \text{ MPa}$.

The diagram shows a horizontal bar divided into three segments: AB (2m), BC (1m), and CD (1m). Axial loads are applied at the ends: 200 kN at A (pointing right), 400 kN at B (pointing left), 100 kN at C (pointing right), and 100 kN at D (pointing left).

Now let us look into another example problem, this is interesting, because here we have a bar, a plastic bar is in the elastic limit, and in this we have variable cross section. The bar cross section is not uniform as in the previous case. Here for this particular stretch of the bar, here one cross sectional area, let us call this as A and this as B and this is C and this is D, here part AB, the segment AB is having area of 1000 mm^2 , segment BC is having 1000 mm^2 and segment CD is of 1000 mm^2 and this particular bar is

subjected to pull here of 200 Kilo Newton, here it is 400 Kilo Newton, here it is 100 Kilo Newton and here it is 100 Kilo Newton.

Now for such kind of system, we need to look into whether the whole bar is under the action of equilibrium of the forces or not. We look into a bar which is subjected to the action of axial force of 400 Kilo Newton in the positive x-direction. In the negative x-direction which is 100 plus 100 plus 200 is equal to 400 so this is in equilibrium. Now if we take the free body of different part of this particular bar, then we can find out that what is the amount of forces that is each member is subjected to.

Now let us take a section somewhere here, let us call section one-one and if you take free body of this, this particular segment is subjected to pull here, 200 Kilo Newton, and as we have seen the free body in our module one, the opposing force will also be 200 Kilo Newton. This particular member is subjected to pull of 200 Kilo Newton. If we take section here, section two-two and if you take the free body part of it, then here we have force 200 Kilo Newton, here we have a force which is 400 Kilo Newton acting in the other direction, so the resulting force 200 Kilo Newton acting in the opposite direction, if we balance this has to be 200 Kilo Newton.

So this is being pulled which has compressive force of two hundred Kilo Newton and if we take the free body of this part, then we have the free body part is like this, this is 200, this is 400 and this is 100, so 100 plus 200 on this side and 400 on this side, so it has to be balanced by hundred on this side. So this particular member is subjected to 100 here and 100 here which is under compression. So the first part, this part is under tensile pull of 100 Kilo Newton and central part is under compression of two hundred Kilo Newton, the third part is in the compression of 100 Kilo Newton.

Now for these if we compute, if we calculate the deformation for the three segments we can find out the total delta as the sum of summation of $P L$ by $A E$. As in each of these three segments the cross section area over these stretch, over this cross sectional area of particular section remains same. For this particular cross sectional area the cross sectional area is same so we can write this equation as, for the segment AB as the $P_1 L_1$ by $A_1 E$ plus $P_2 L_2$ by $A_2 E$ plus $P_3 L_3$ by $A_3 E$ for the three segments. Now for the second segment this P_2 is negative, this is compressive, if we call P_1 as tensile as positive, for the third segment P_3 is compressive again negative.

Now if we substitute the values as we know P , we know the value of L_1 , we know the value of L_1 , correspondingly P_2 , L_2 and A_2 and we know the value of P_3 , L_3 and A_3 and if we calculate that we can compute the value of the deformation and this delta is equal to 200 and 10 cube into 2000 by 1000 (2 into 10 to the power 5), second one will be minus is equal to 100 minus this 100 into 10 cube into 1000 by 1000 (10 to the power 5) is equal to 1 mm.

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Example Problem - 4

- An aluminium bar with a cross sectional area of 160 mm^2 carries the axial loads as shown in figure. Compute the total change in length of the bar. $E = 70 \text{ GPa}$.



The diagram shows a horizontal bar divided into three segments. From left to right, the segments have lengths of 0.8m, 1.0m, and 0.6m. Four axial loads are applied: 25kN (tension) at the left end, 15kN (compression) at the junction of the first and second segments, 30kN (tension) at the junction of the second and third segments, and 10kN (tension) at the right end.

Here is another problem which is aluminium bar with cross sectional area of 160 mm^2 square carries the axial loads, you have to compute the values of deformation to compute.

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Summary

This lesson included:

- Concept of Strain at a point and axial / normal strain.
- Stress-strain relationship and relevance of different points.
- Examples to demonstrate the evaluation of strain in a stressed body.

Now, to summarize; this particular lesson included concept of strain at a point and axial and normal strain, stress-strain relationship and the relevance of different point in the stress strain curve, example to demonstrate how to evaluate the strain and the deformation stress body.

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Question Set 2.1

- What is meant by elastic limit?
- What is the difference between Nominal stress and true stress?
- How will you evaluate strain in a bar with gradually varying cross section?
- Answers will be provided in the next lesson

1/2

Here are some questions:

What is meant by elastic limit?

What is the difference between nominal stress and true stress?

How will you evaluate strain in a bar with gradually varying cross section?