Strength of Materials
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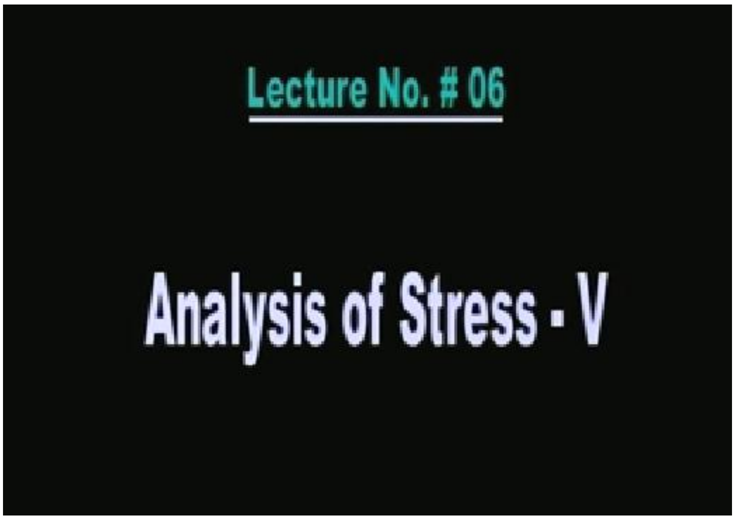
Lecture No #6
Analysis of Stress V



Welcome to the Lesson Six of the course on Strength of Materials. In this particular lesson we are going to discuss certain aspects of analysis of stress.

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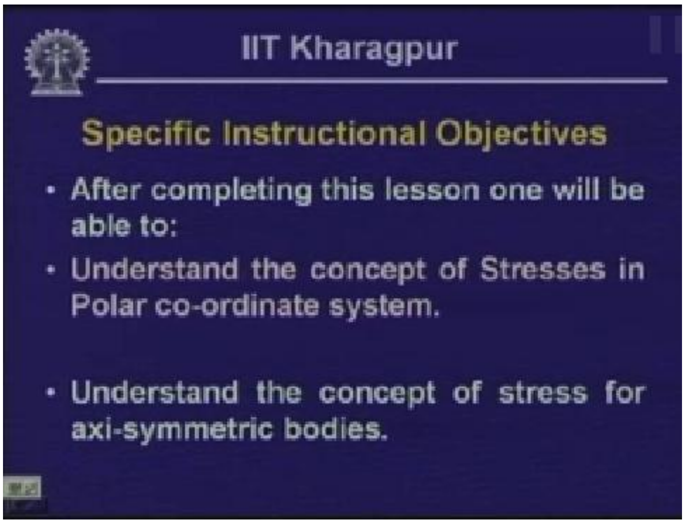
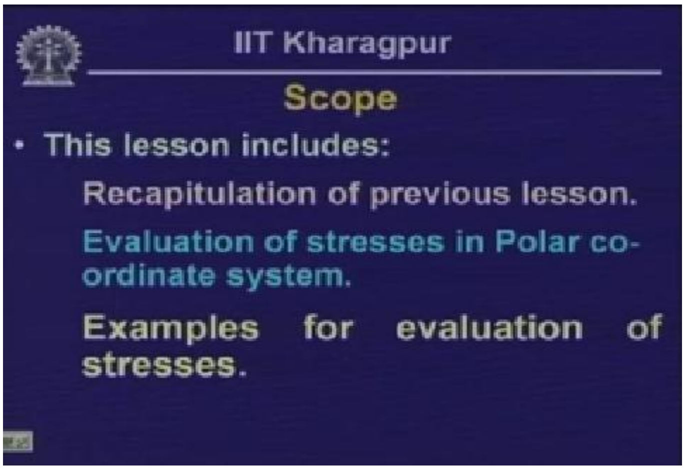
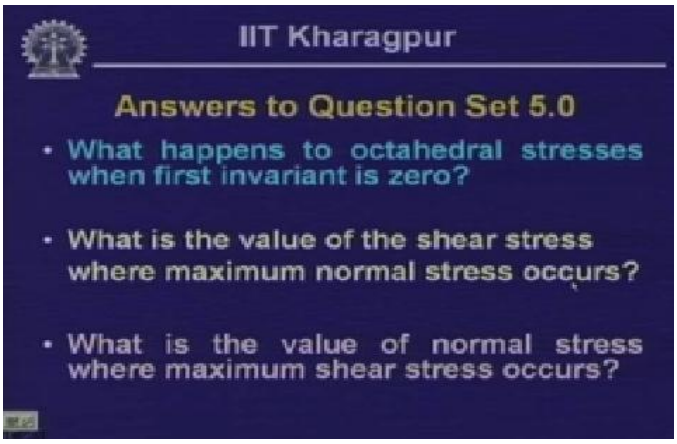


Slide 1-1:0



Once this particular lesson is completed one should be able to understand the concept of the stresses in polar coordinate system, you will be able to understand the concept of the stress for axi-symmetric bodies which eventually can be derived from this polar coordinate system of stresses. We will also look into how to evaluate stresses at different points.

Slide 2-1:35

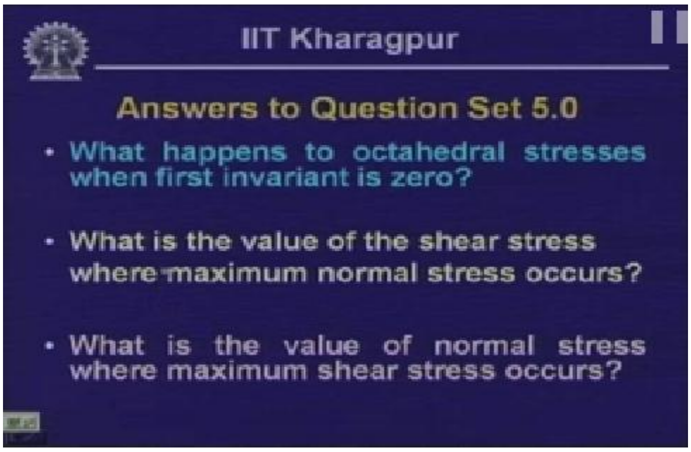


This particular lesson includes the recapitulation of the lessons we discussed already such as evaluation of stresses in polar coordinate system and examples for evaluation of stresses at particular point in the stress body.

Some questions to be answered: What happens to octahedral stresses when first invariant is 0?

Now let us look into octahedral stresses. The normal stresses on the octahedral planes which we had calculated σ octahedral = 1/3 ($\sigma_1 + \sigma_2 + \sigma_3$). We had defined the octahedral planes as the planes which are equally inclined with the principal axis reference system. And thereby the stresses which are acting σ_1, σ_2 and σ_3 in the rectangular stress system and the octahedral stress as defined is 1/3 $(\sigma_1+\sigma_2+\sigma_3)$ which is the summation of the normal stresses in three dimensional stress system, this is called as first invariant.

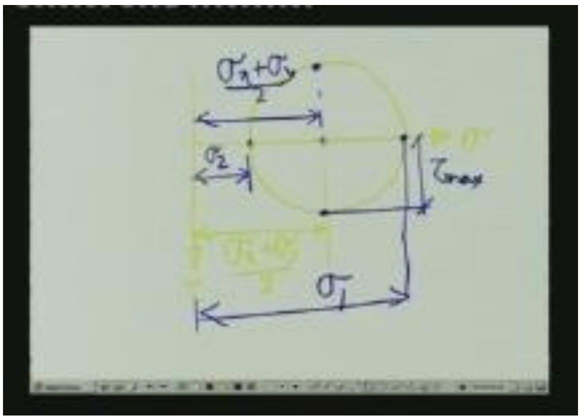
Slide 3-1:57



If you remember $\tau_{oct}^2 = 2/9(\sigma_1+\sigma_2+\sigma_3)^2 - 6/9\,(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$.

Hence as it has been asked if $(\sigma_1 + \sigma_2 + \sigma_3) = 0$ then eventually the normal stress on octahedral plane, $\sigma_{octahedral} = 0$. So, if the first invariant is 0 then the octahedral normal stress is equal to 0, only shear stress will exist on the octahedral plane.

Slide 3-4:17

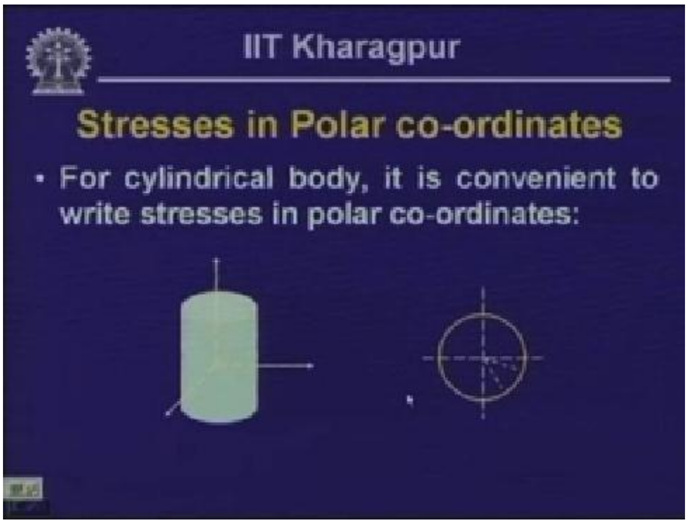


Now the second question which was posed was: What is the value of the shear stress where maximum normal stress occurs?

The third question is: What the value of normal stress is where maximum shear stress occurs?

Probably these two questions can be answered through the same diagram. If you remember the last time, we had shown how to plot the Mohr's circle. This is the σ-axis and this is the τ -axis. Now, if we draw the Mohr's circle of stress, the centre of the Mohr's circle from τ -axis is given as (σ_x + σ_y)/2 and the maximum value of the normal stress at this point in this particular plane we normally designate as σ_1, and the minimum value of the normal stress is σ_2. So, if you note in these two planes where the maximum and minimum normal stress acts the value of the shear stresses are 0. So the plane where the maximum normal stress acts there the value of the shear stresses are 0 and these planes are called as principal planes. If you note the maximum shear the value of the shear stress is that of the radius of the Mohr's circle is τ max. If you note here that in this plane we have the normal stress which is equal to this particular magnitude (σ_x + σ_y)/2. So, from this diagram itself you can answer both the questions. The planes where the normal stresses are at maximum the shear stresses are 0 and the planes where shear stresses are maximum there normal stresses exist and the value of normal stresses are (σ_x + σ_y)/2.

Slide 6-6:57



Let us look into aspects of how to evaluate stresses in polar coordinates?

So far we have discussed about the rectangular stresses in a body where we have assumed that the boundaries are straight boundaries.

Now there are several cases where other than the straight boundaries we get problems, we get structural elements where the surfaces are curved and to represent the stresses on those curved surfaces. It is ideal to represent them in terms of a coordinate system which can be expressed in terms of radius and the rotational angle θ which we call as cylindrical axis or polar reference axis.

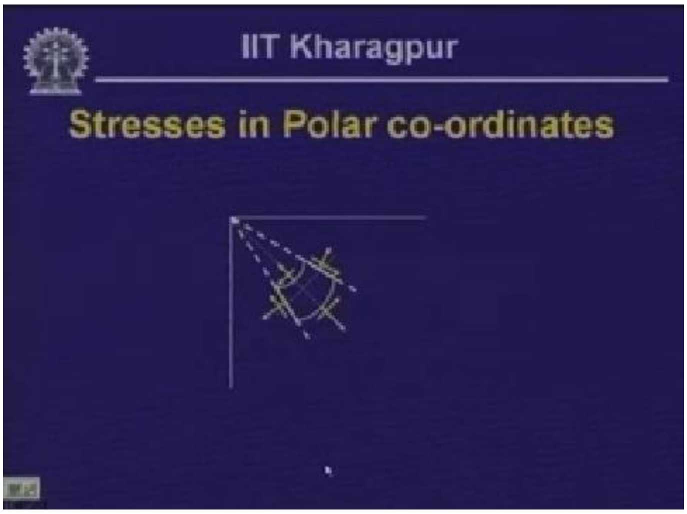
If we have cylindrical body of this particular form, in this we have earlier seen the reference axis system as x, y, and z. Now if we select a point here on this body, let us say this is P, the coordinate of this particular point can be described by these coordinates x, y and z. Also, this particular point can be represented through another reference system which if we project this point on this xz-plane and draw a line over here and if we define this particular angle as θ and its projected length as vector r and this distance as y then the coordinate of this particular point can be expressed as a function of r, θ, and y. This is with reference to the Cartesian system x, y, and z and this particular reference is with reference to the polar coordinate system which is r, θ and y. Now, if we look into the plan of this or the cross section of this, then if we draw two radial lines from the centre, let us say that this particular radial line is at an angle θ, then these two radial lines make a small angle of dθ.

Now if we take a small element over here and try to look into the stresses that will act then we will have two planes, this particular plane normal to this plane is in the direction of the radius; we call this as the r-plane. Over these we will have the stresses which are acting as the normal stress and so is this, which we call as σ_r. The stress normal to this surface acting along the circumferential direction is call that as σ_θ.Also we will have the tangential stress on this plane as well as on this plane and we will have tangential stress in the radial direction as well. (Please refer to the lecture slides)

This tangential stress which we defined as the shearing stresses is τ on the r-plane acting in the direction of θ, we call this as $\tau_{r\theta}$ and the other tangential stress on the θ-plane is $\tau_{\theta r}$, eventually $\tau_{r\theta} = \tau_{\theta r}$. So we define the state of stress on this particular body at a particular point is equal to σ_r the radial stress, the tangential stress is σ_θ, and the shearing stresses $\tau_{r\theta}$.

If a particular body at a point is subjected to these stresses then how we arrive at the equations for equilibrium.

Slide 7-11:45



Here is the representation of state of stress at a particular point in a stress body, and as we have designated that this particular stress which is acting normal to the r-plane, let us call that as σ_r . Let us assume that this particular distance is dr. The first radial line let us call, this is at distance of θ, and the small angle made by these two radial line is dθ. The radial distance from the origin to the first part of the element, let us call that as r. Hence the stresses which are acting σ_r is the normal stress, the tangential stress is $\tau_{r\theta}$, and the circumferential normal stress as σ_θ.

So the stress which is acting on this surface, since it is at a distance dr, while deriving the equilibrium equations at the particular point with reference to the rectangular Cartesian axis system if you try to find out the stress at two different points then there is incremental stresses which is, $\sigma_r + (\partial\sigma_r/\partial r)dr$. Likewise, the tangential stress which is the shearing stress is equal to $\tau_{r\theta} + (\partial\tau_{r\theta}/\partial r)dr$. On this we have σ_θ, so on this particular surface the normal circumferential stress is equal to $\sigma_\theta + (\partial\sigma_\theta/\partial\theta)d\theta$ and the shearing stress which is acting on this surface is $\tau_{r\theta} = \tau_{r\theta} + (\partial\tau_{r\theta}/\partial\theta)d\theta$. Also, if we draw a tangent at this particular point please note that normal stress on this surface makes an angle of dθ/2.

If we take the equilibrium of forces in the radial direction and in the tangential direction we can get the equations of equilibrium. Now on this particular plane, the area on which this particular stress acts is (r + dr) dθ and rdθ. If we assume the thickness of this particular element perpendicular to the plane of this board as unit, then area of this particular surface is equals to (r + dr) dθ × 1 and this multiplied by the stress will give the force in this particular plane. Similarly the force on this particular plane is σ_r × r dθ × 1. Also we have the tangential stresses on this surface which are in the radial direction and σ_θ will have the component in the tangential direction and also in the radial direction and the component in the radial direction will be σ_θ (sin dθ/2) and dθ being small we can approximate sin dθ/2 = dθ/2. (Please refer to the lecture slides)

So, if we write down the equations of equilibrium in the radial direction then the equations we get are:

$(\sigma_r + (\partial\sigma_r/\partial r)\,dr)\,(r+dr)\,d\theta - \sigma_r\, r\, d\theta + (\tau_{r\theta} + (\partial\tau_{r\theta}/\partial\theta)\,d\theta)\,dr - \tau_{r\theta}\,dr - (\sigma_\theta + (\partial\sigma_\theta/\partial\theta)\,d\theta)\,dr \times d\theta/2 - \sigma_\theta\,dr\,d\theta/2 = 0$

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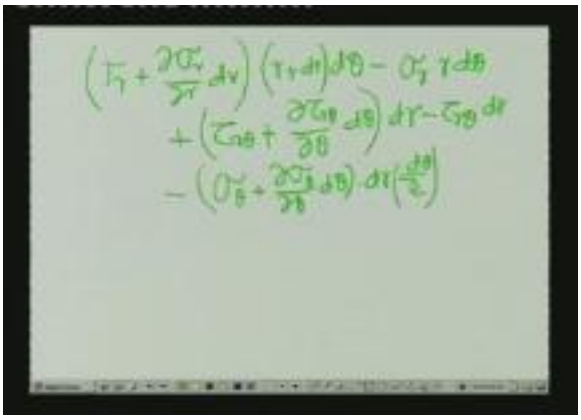

Hence on simplification this gives,

$\sigma_r\, r\, d\theta + (\partial\sigma_r/\partial r)\,dr \times r\,d\theta + \sigma_r\,dr\,d\theta - \sigma_r\,r d\theta + \tau_{r\theta}\,dr + (\partial\tau_{r\theta}/\partial\theta)\,d\theta\,dr - \tau_{r\theta}\,dr - \sigma_\theta\,dr\,(d\theta/2) - \sigma_\theta\,dr\,d\theta/2 = 0$

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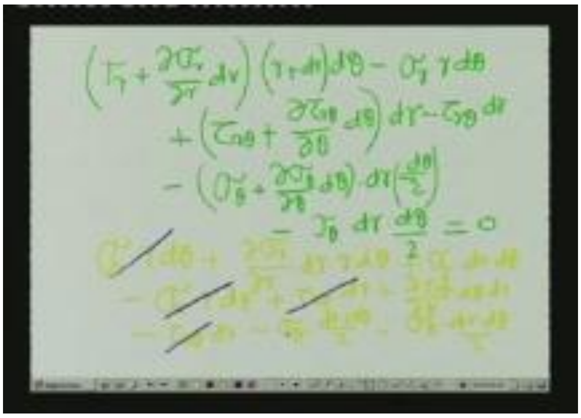


If we simplify this equation further and divide the whole by dr dθ then what is left out is,

$\partial\sigma_r/\partial r + (1/r)(\partial\tau_{r\theta}/\partial\theta) + (\sigma_r - \sigma_\theta)/r = 0$

This is the equilibrium equation in the radial direction.

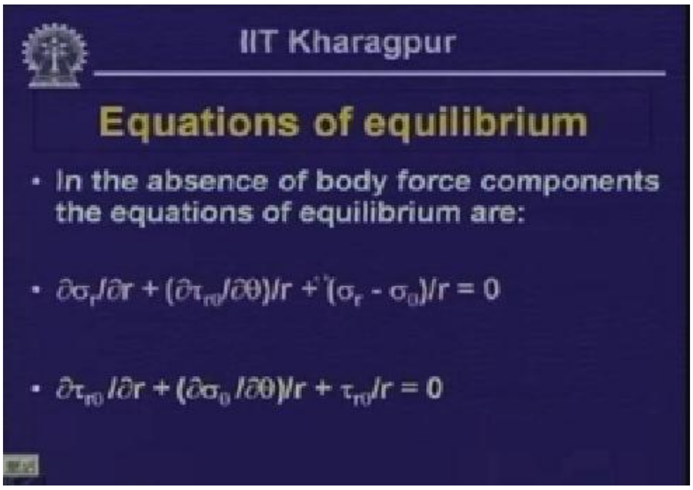
Similarly, if we take, equilibrium of the forces in the circumferential direction then we can write down the equation as

$(\sigma_\theta + (\partial\sigma_\theta/\partial\theta)\,d\theta)dr - \sigma_\theta dr + (\tau_{r\theta} + (\partial\tau_{r\theta}/\partial r)dr)(r + dr)\,d\theta - \tau_{r\theta}\, r\, d\theta = 0.$

This gives the expression finally after simplification as,

$\partial\tau_{r\theta}/\partial r + (1/r)(\partial\sigma_\theta/\partial\theta) + \tau_{r\theta}/r = 0$

Slide 10-24:21



These are the equations of equilibrium and in this particular case we have not accounted for the components of the body forces, both in the radial and circumferential direction we have neglected the body forces.

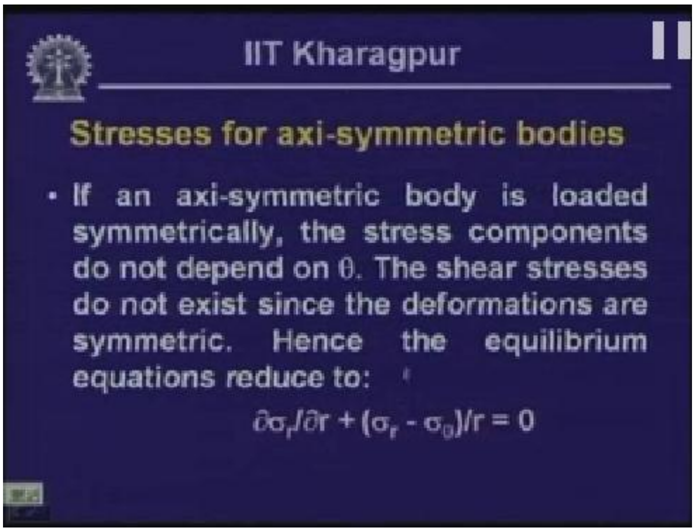
Hence we have,

$\partial\sigma_r/\partial r + (1/r)(\tau_{r\theta}/\partial\theta\ \sigma_r) + (\sigma_r - \sigma_\theta)/r = 0$……………………….(1)

$\partial\tau_{r\theta}/\partial r + (1/r)(\partial\sigma_\theta/\partial\theta) + \tau_{r\theta}/r = 0$……..…………………………(2)

These are the two equations of equilibrium which are explained in reference to the polar coordinate system.

Slide 11-25:19



Thereby from this we can find out the stresses that are referred in axi-symmetric body. We encounter several kinds of structural elements where the stresses or the boundaries may not be perfectly straight; you can have curved boundary over which there could be stress which can be radial stress or which can be described by the stress σ_r and σ_θ and if the loading on such body is symmetrical then we have a perfectly symmetrical body or loading is perfectly symmetrical in its vertical direction.

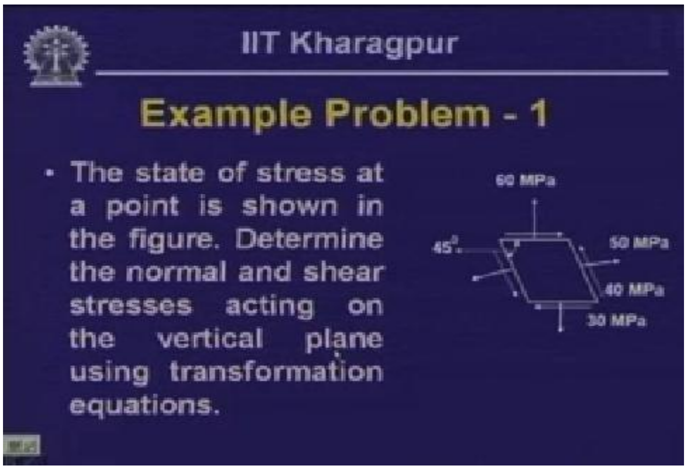
Then if we take any cross section or any longitudinal section for that matter, if we take section through the diameter at every section the level of the stress will be the same. Hence it shows that the stresses at any of these sections are independent on wherever we take the section, so the stresses are independent of θ and these kinds of bodies are called as axi-symmetric bodies. That means these bodies are perfectly symmetrical with reference to the vertical axis. For such bodies if the loading also is vertical and symmetrical then any cross section we take, at each section the same state of stress exists. This kind of stress and the body we call as axi-symmetry bodies.

So the bodies which are perfectly symmetrical referring to its vertical axis we call them as axi-symmetry bodies and for axi-symmetry bodies if they are loaded symmetrically then the stress components do not depend on θ. Therefore, any longitudinal section we take the shear stress components are absent, because we have the symmetric deformation and thereby the shear stress components do not exist.

If we take the absence of the shearing stresses then the equilibrium equation reduces to,

$\partial\sigma_r/\partial r + (\sigma_r - \sigma_\theta)/r = 0$, where only the normal stresses exist and the shearing stresses are absent.

Slide 12-28:09



We have tried to give an outline of the state of stress if we refer them in terms of the polar coordinate system. Earlier in a stress body we have looked into that, if we have the rectangular components of the stresses σ_x, σ_y, and τ_{xy} we looked into how to evaluate the stresses at different points and on different planes. Now, if we try to represent the stresses on any plane in a polar coordinate system where the normal stresses σ_r, σ_θ, and shearing stress $\tau_{r\theta}$ exist we have seen how to write down the equations of equilibrium.

Here if you look in this particular point in the stress body the stresses given are; on a horizontal plane the normal stress is 60 MPa, on a particular plane which is inclined with reference to this horizontal plane is 45°, the normal stresses are 50 MPa; the shearing stress is 40 MPa; and on this horizontal plane we have shearing stress as 30 MPa. What we will have to compute is the normal and shear stresses which are acting on the vertical plane using transformation equations.

Here the given values are,

σ_y = +60 MPa and τ_{xy} = 30 MPa

σ_x' = 50 MPa and $\tau_{x'y'}$ = 40 MPa.

We will have to compute what is the value of σ_x which is acting on the vertical plane and correspondingly what is the shearing stress τ. These are the two values we have to evaluate.

As per transformation equations on any plane,

σ_x' = (σ_x + σ_y)/2 + (σ_x – σ_y)/2 × cos 2θ + τ_{xy} sin 2θ

Now in this particular problem the stress on the inclined plane is given as σ_x' and $\tau_{x'y'}$ and normal to this particular plane is at an angle of 45°.

So θ = 45°, thereby 2θ = 90°

Thus, 50 = (σ_x + 60)/2 + (σ_x – 60)/ 2 × 0 + τ_{xy} × 1……………….. (1)

The second equation is $\tau_{x'y'}$ = 40

Thus, 40 = – (σ_x – 60)/2 × 1, or

– σ_x + 60 = 80, or

σ_x = – 20 MPa

if we substitute in this equation (1), this gives,

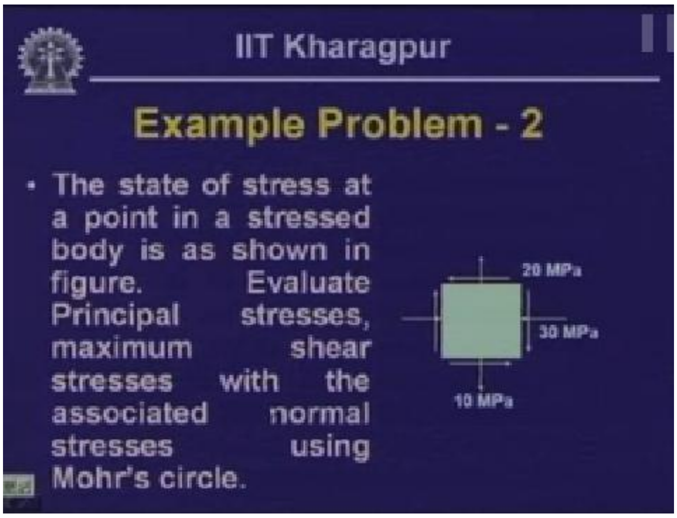
50 = 20 + τ_{xy}, or

τ_{xy} = 30 MPa, which is the shear stress component in the horizontal plane.

So, if we draw the element now on which the stresses act we have on this as σ_y, now we have evaluated the σ_x and also τ_{xy}. On this plane which is at an angle of 45° with the x-plane on which the σ_x' and $\tau_{x'y'}$ acts. These are the values of σ_x; σ_x gives you – 20. Here if you see I have made a mistake that the stress is – 20 so that indicates that the normal stress will be acting in the opposite direction, it will be a compressive stress, whereas σ_y is acting in the positive direction. Here τ_{xy} is positive so the direction of τ_{xy} is in the positive direction of y.

Here is another problem. This particular problem states that these are the stresses acting with the rectangular axis system which are σ_x, σ_y, and τ_{xy} and we will have to evaluate the principal stresses, maximum shear stresses along with the normal stresses. We will have to use Mohr's circle to evaluate these quantities.

Slide 14-35:20



Here if you look into the normal stresses which is acting in the x-plane is compressive in nature having magnitude of 30 MPa.

σ_x = – 30 MPa and σ_y = + 10 MPa

If you note the direction of the shearing stress it is opposite to the positive y-direction. Also this particular shear along with the complimentary shear on the other face is causing rotation in the clockwise direction which according to our sign convention is negative. If you remember in the Mohr's circle, we said that the shearing stresses which are causing anticlockwise rotation is positive, since here this is the clockwise rotation so these are negative shear. So, on this particular surface, since this is causing anticlockwise, this particular shear is a positive shear.

Now if we try to represent these in the Mohr's circle then let us see how it looks like.

This is the reference axis system and this is the σ + and this is –σ, this is + τ and this direction is $-\tau$. Now on the x-plane we have σ_x = – 30MPa which is this direction and τ_{xy} = –20 so this is the shear, so we get the point somewhere here, which is σ_x = –30 and τ = –20. Then we have in the perpendicular plane the y-plane which is 90° with reference to the physical plane, here in the Mohr's plane it will be 180° and we have $+\sigma_y$ = 10 MPa, and we have + τ = 20 MPa.

Now if we join these two points this is where it crosses the σ line, we get the centre of the Mohr's circle. So with this as centre O as the centre and OA as the radius we draw the circle.

This gives us the Mohr's circle of which the centre is this particular point and as you know the distance of the centre form the τ-axis ($\sigma_x + \sigma_y$)/2.

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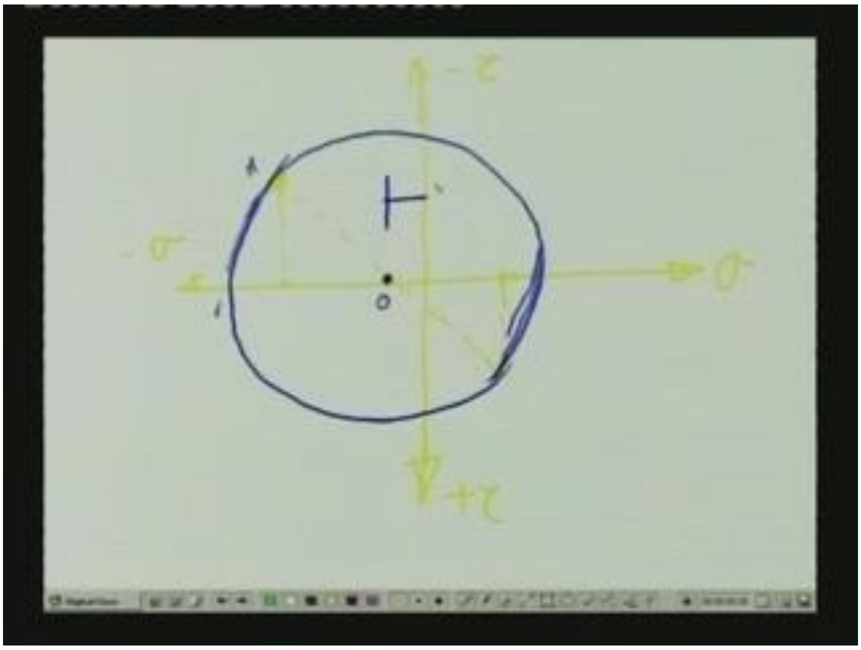

Now here

σ_x = – 30

σ_y = + 10

So centre will be at (– 30 + 10)/2 = –10 MPa with respect to origin.

As we have evaluated earlier that distance OO'= ($\sigma_x - \sigma_y$)/2 = (– 30 – 10)/2 = – 20

So from here to here is 20, and this is τ_{xy} which is also 20. So the radius or this particular distance OA = $\sqrt{20^2 + 20^2}$ = $\sqrt{400 + 400}$ = 28.28

Now, we are going to evaluate the principal stresses.

$\sigma_1 = (\sigma_x + \sigma_y)/2 + \sqrt{((\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2}$ = –10 + 28.28 = 18.28 MPa

$\sigma_2 = (\sigma_x + \sigma_y)/2 - \sqrt{((\sigma_x - \sigma_y)/2)^2 + \tau_{xy}^2}$ = –10 – 28.28 = –38.28 MPa

From this particular diagram, if we look into that, this is the value of the radius and this is the plane, where we get the value of maximum value of the shear stress and this is the plane where you get the minimum value of the shear stress. Now at this particular point, the value of the normal stress is this which is $(\sigma_x + \sigma_y)/2 = -10$ MPa in this particular problem. Therefore the maximum value of shear stress = radius = 28.28 MPa.

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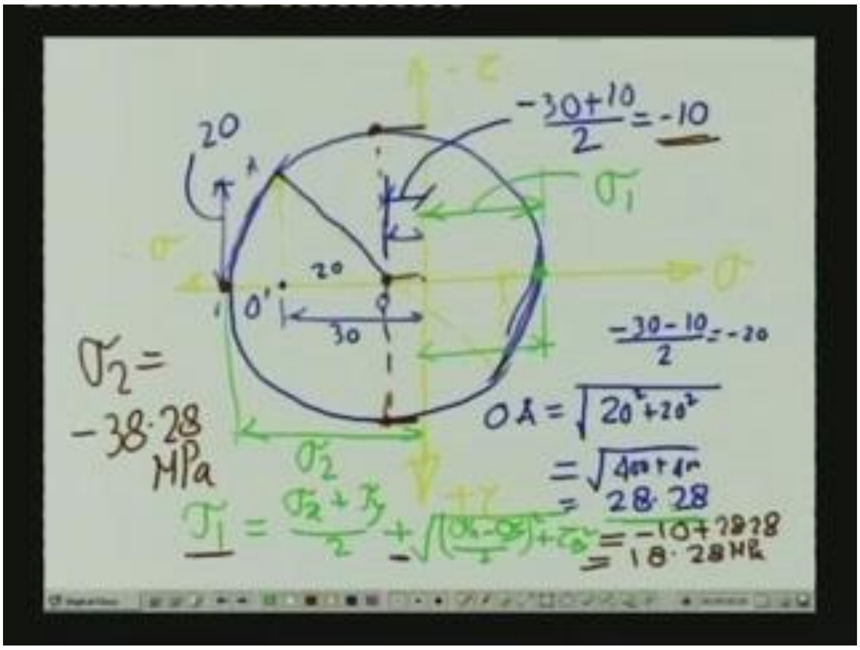

So τ_{max} = 28.28 MPa and the corresponding normal stress on this particular planes where the maximum and minimum share stress occurs, that is equals to the $(\sigma_x + \sigma_y)/2 = -10$ MPa. So these are the values which we get, they are:

The maximum principal stress σ_1 = 18.28 MPa;

The minimum principal stress σ_2 is = – 38.28 MPa;

The maximum shear stresses τ_{max} = 28.28 MPa;

The normal stress which acts on the maximum and the minimum shear stress = –10 MPa.

We can evaluate the position of these planes, the maximum, and minimum normal stresses with reference to the plane. Now this is the plane which is representing the vertical plane, normal to the plane which coincides with the x-axis, which you call as x-plane.

Now, with reference to this particular plane, if we go in the anticlockwise direction this particular angle will give us the value half of which is the orientation in the physical plane which locates the maximum principal stress and perpendicular to that is the plane where in this minimum normal stress acts. These are the values and that is how we can compute the stresses using Mohr's circle.

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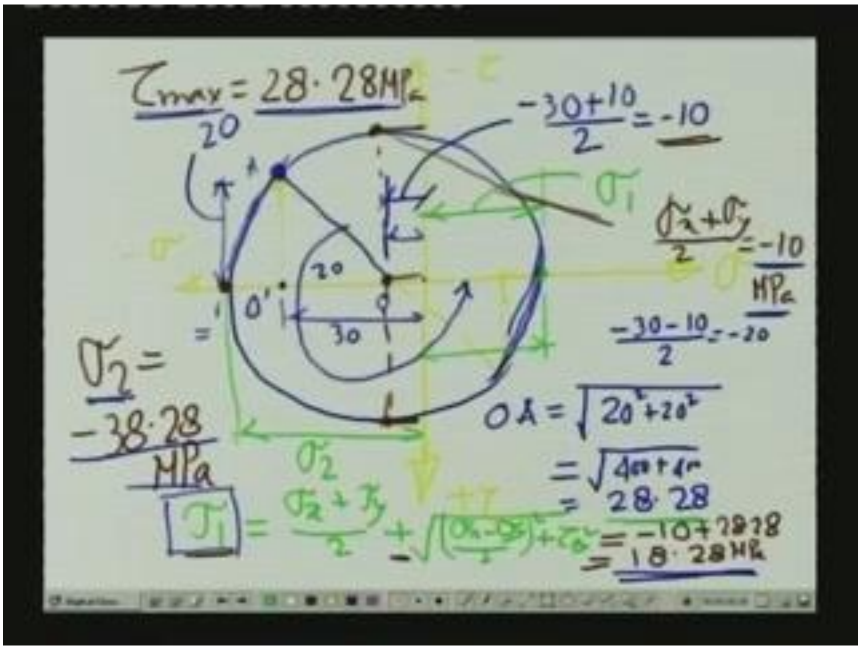
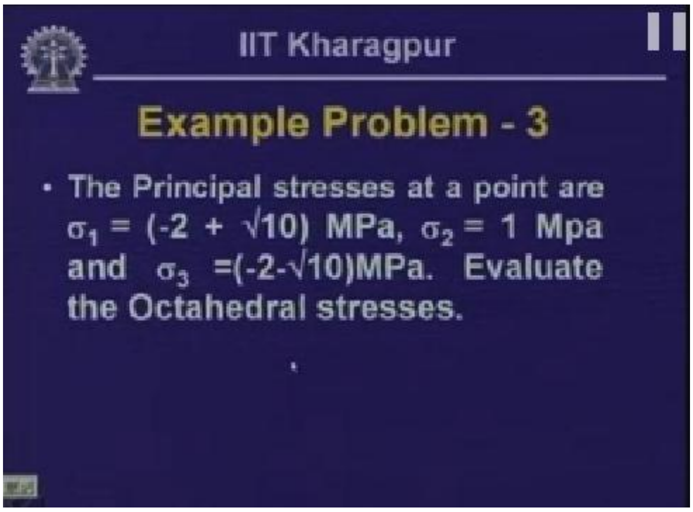


Slide 16-46:50



Here is another problem. We already know that the octahedral stresses are the stresses which acts on the octahedral plane and octahedral plane are the planes are equally inclined with respect to the principal axes, σ_1, σ_2,and σ_3 axes system.

Now, if we know the values of principal stresses at a point, we can compute the values of octahedral stresses. Now,

$\sigma_1 = -2 + \sqrt{10}$ MPa

$\sigma_2 = 1$

$\sigma_3 = -2 - \sqrt{10}$ MPa

And we will have to evaluate, the values of octahedral stresses.

Now as we have seen in the beginning itself, that

$\sigma_{octahedral} = 1/3\,(\sigma_1 + \sigma_2 + \sigma_3) = 1/3\,(-2 + \sqrt{10} + 1 - 2 - \sqrt{10}) = -1$ MPa.

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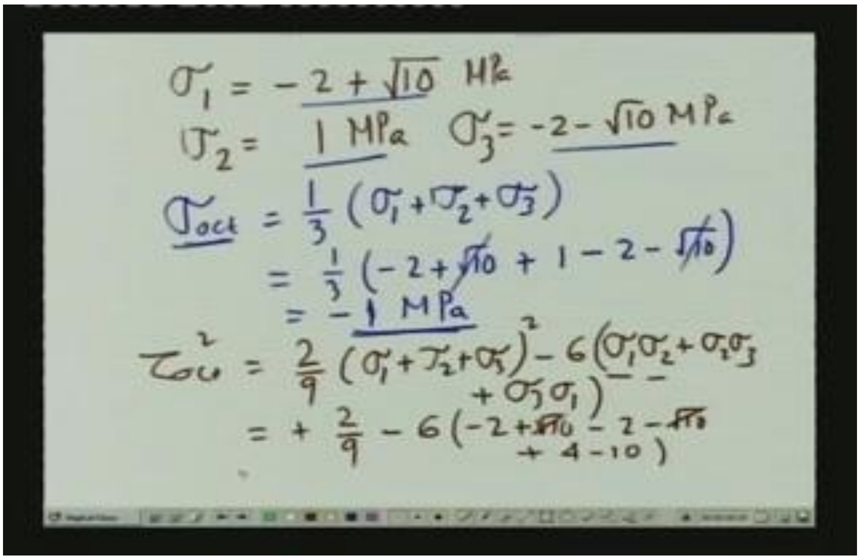


For τ_{oct},

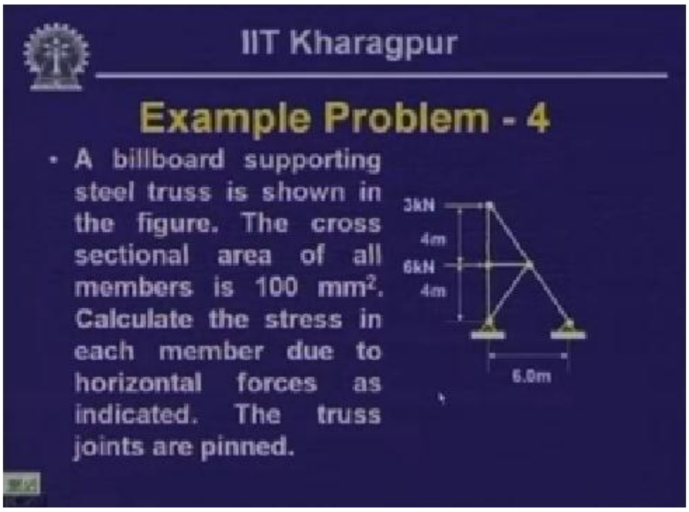
$$9\,(\tau_{oct})^2 = 2\,(\sigma_1 + \sigma_2 + \sigma_3)^2 - 6\,(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)$$

Substituting all the values and on simplification, we get,

$$\tau_{oct} = \sqrt{\frac{78}{3}} = 2.944 \text{ MPa}$$

Here is another problem, this is the supporting structure which supports a billboard, many a times we use boards, sign boards on which the advertisements are put and these boards are supported by some steel structures and these boards are subjected to the wind pressure.

Slide 18-50:55



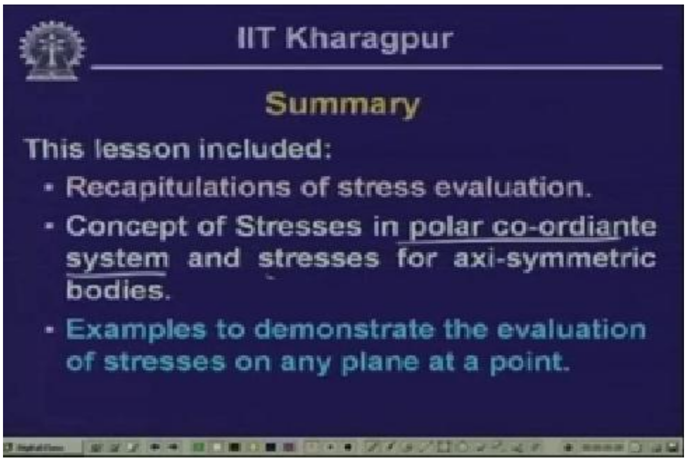
The wind pressure when it comes on the board eventually it transfers the load on to the supporting structure, hence this is one of the supporting structures in which we have framework and all the members are connected in the pin joint and this is the force which is acting on this member. Now our job is to find out the stress in each of these members from this particular force. What we need to do is, first we evaluate forces in each of the members which are arising from these external forces, and thereby we can compute the stress since the cross sectional area of the member is given, force divided by the area will give us the stress.

If we assume this angle as θ, then this being 6 from the similar triangle we get this as 3, so eventually this is also θ, and if we drop a perpendicular this is also θ, and this is also θ and the values of cos θ is 4/5. This particular hypotenuse, this being 3, and this being 4, is 5 so is this, this is also five, so the values of cos θ = 4/5 and value of sin θ = 3/5. Now what we need to do is to draw free body diagram and evaluate the forces.

One section we can take here and draw the free body diagram, and another section we can take here and draw the free body diagram and we can compute the forces. Once we compute the forces, we can find out the stresses. You try to compute the stresses for this particular problem.

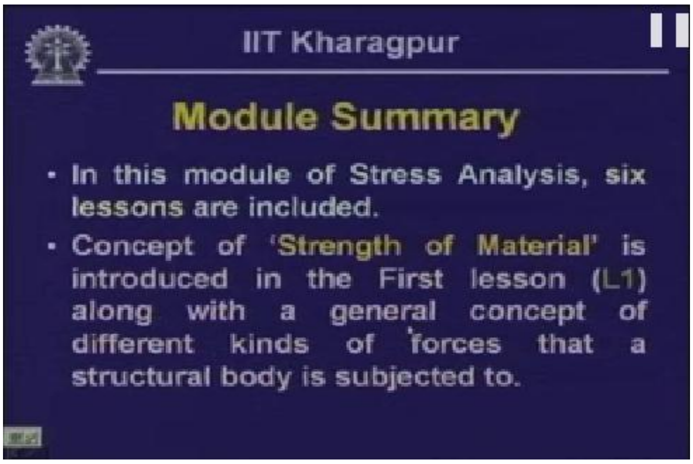
In this particular lesson we tried to look into the stresses which we have evaluated earlier with reference to Cartesian system σ_x, σ_y and τ_{xy}.Also, we have tried to look into that if a body which is does not have the straight boundary and there are curved boundary, how to represent the stresses σ_r, σ_θ and $\tau_{r\theta}$ which are in terms of polar coordinate systems.

Slide 18-54:06



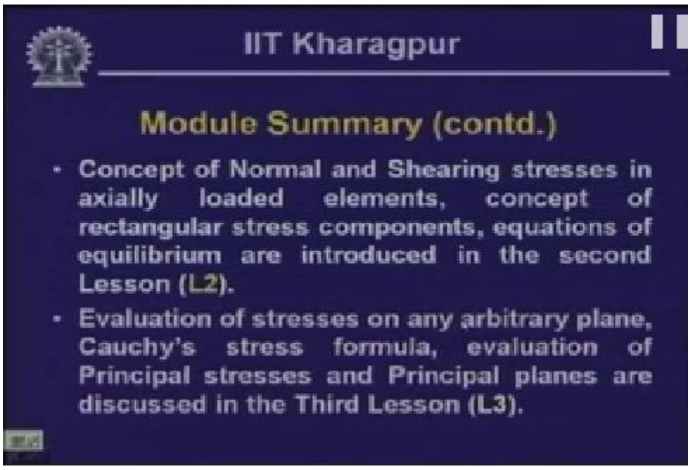
And also from there we have seen how to evaluate stresses for the axi-symmetric bodies and also we looked in to some examples to demonstrate how we can evaluate stresses at any point on the stress body either using transformation equations or by the use of the Mohr's circle. We also tried to see how to compute octahedral stresses on a particular, on a particular stress body; keep in mind this octahedral stress is useful when we talk about the evaluation of the stress in the inelastic when we go beyond the elastic strain.

Slide 19-54;51



This particular lesson was last in the series of the module stress analysis. We have computed or we have looked into the six lessons in the particular module stress analysis. These six lessons if we look into chronologically, in the first lesson I tried to give you the general concept of kinds of forces and what really is the meaning of subject strength of material, so it was introduced to you. Subsequently the second lesson we had I tried to give you the concept of normal and shearing stresses and you know how to evaluate the equations of the equilibrium from the stresses that are acing in the Cartesian system, σ_x, σ_y and τ_{xy}.

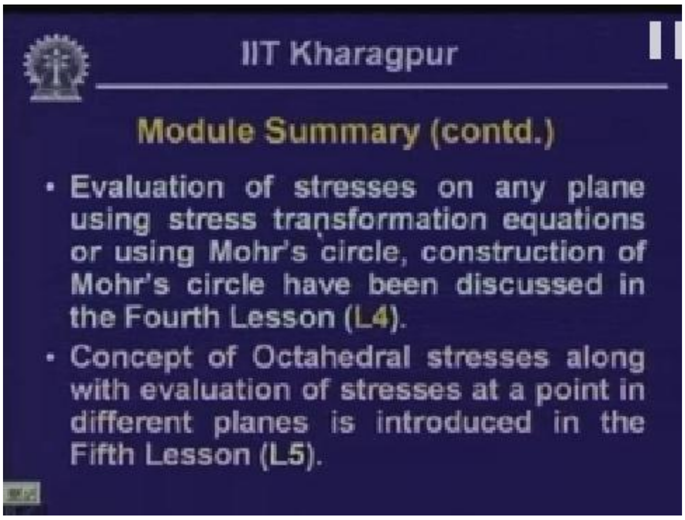
Slide 20-55:32



In the third lesson we have tried to evaluate stresses on any arbitrary plane, if we have any plane which is oriented with the value of θ with reference to x-plane and thereby we arrived at values of the stresses which we have defined as Cauchy's stress formula. We tried to evaluate the maximum normal stresses which we defined it as principal stresses, and these principal stresses which we are acting in the principal planes, we have tried to locate them in this particular lesson.

We have demonstrated that through few examples.

Slide 21-56:39

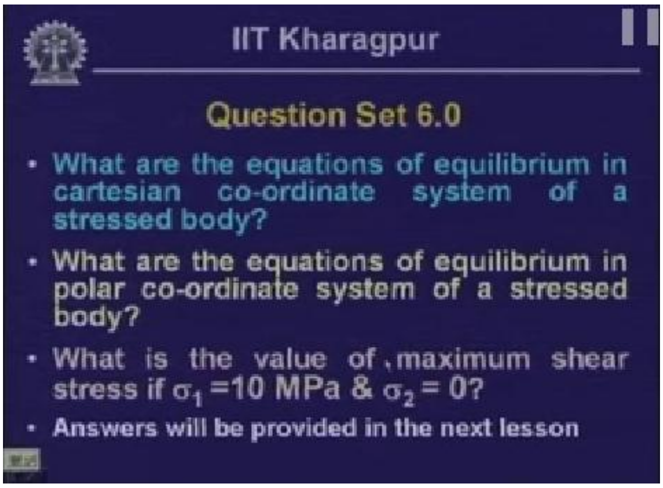


In the fourth lesson we have evaluated the stress on any plane using transformation equations, in fact we have discussed this lesson as well. How to evaluate stress any plane using transformation equations or using Mohr's circle?

In the fourth lesson we have discussed in detail how to construct a Mohr's circle if we know the stresses at a particular point in a body. In the fifth lesson we tried to give you the concept of octahedral stresses and also we looked into how to evaluate the stress at a particular point in the stress body at different planes.

As you know the concept of the octahedral stress, that this particular stress or the stresses which act on the octahedral planes which are inclined equally with the principal axes system and these stresses are useful when we talk about the evaluation of stress in the inelastic stage.

Slide 22-58:04



In this particular lesson, we tried to give you the concept of stresses in the polar coordinate systems that you have already looked into.

Now having looked into these aspects of stresses here are some questions to answer:

- What are the equations of equilibrium in Cartesian coordinate system of a stressed body?
- What are the equations of equilibrium in polar coordinate system of a stressed body?
- What is the value of maximum shear stress if σ_1 = 10MPa, the maximum principal stress, and the minimum principal stress σ_2 = 0?

Slide 23-58:17