Strength of Materials
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Lecture - 3
Analysis of Stress - II

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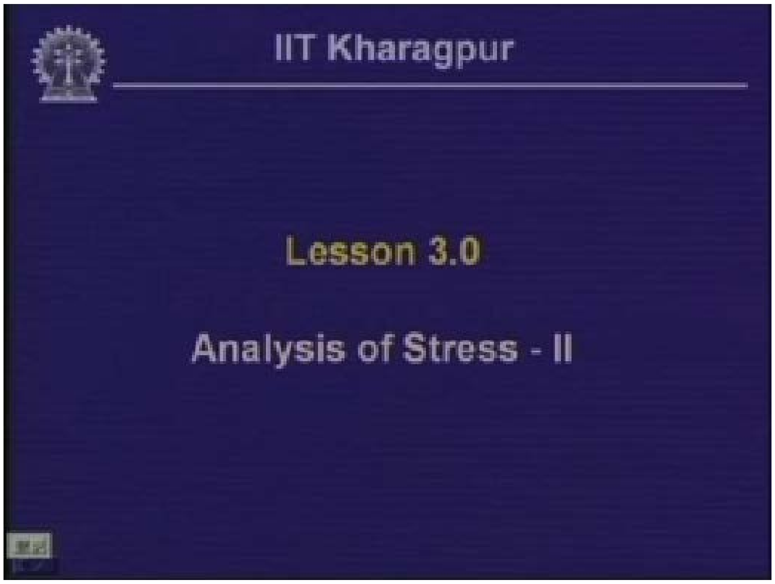

Welcome to lesson 3 on the course Strength of Materials. We are going to discuss about the analysis of the stress. We have already looked into some aspects of stress analysis.

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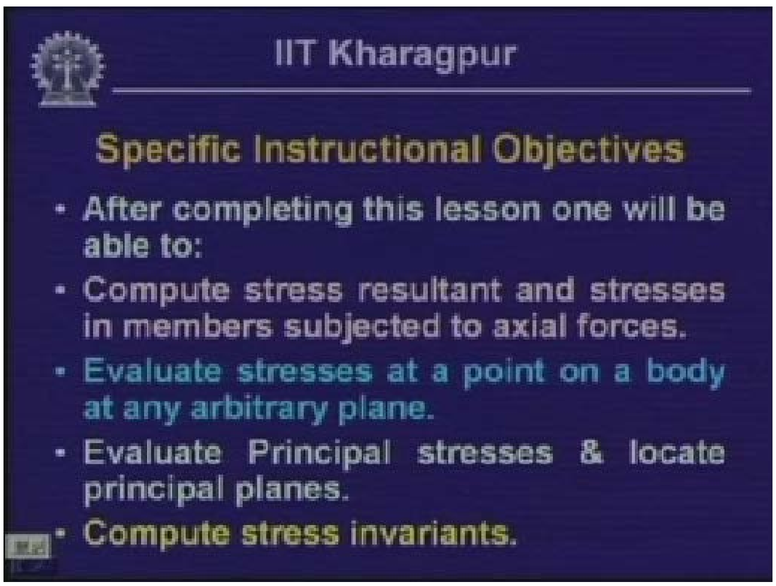

Now it is expected that once this particular lesson is completed one will able to compute stress resultant and stresses in members subjected to axial forces, evaluate stresses at a point on a body at any arbitrary plane, evaluate principal stresses and locate principal planes and also compute stress invariants.

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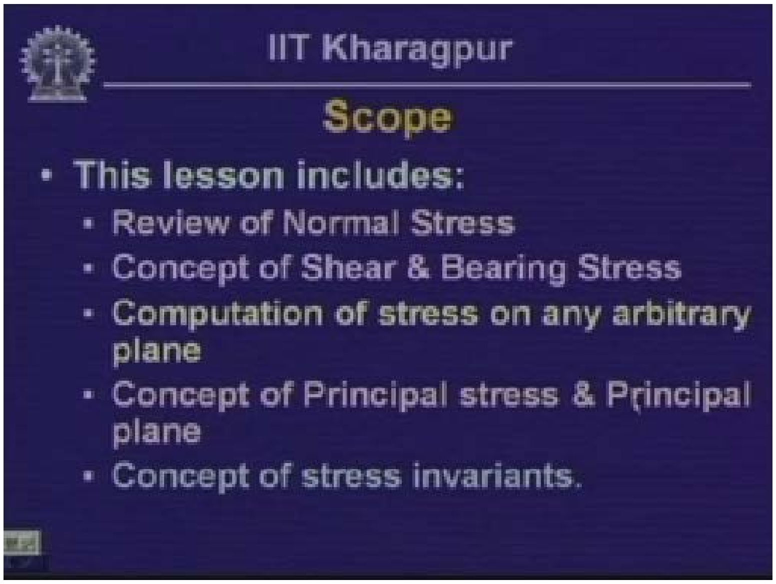

Hence the scope of this particular lesson includes: review of normal stress, concept of shear and bearing stress, computation of stress on any arbitrary plane, concept of principal stress and principal plane and concept of stress invariants. We discussed the types of stress and specifically about the normal stress.

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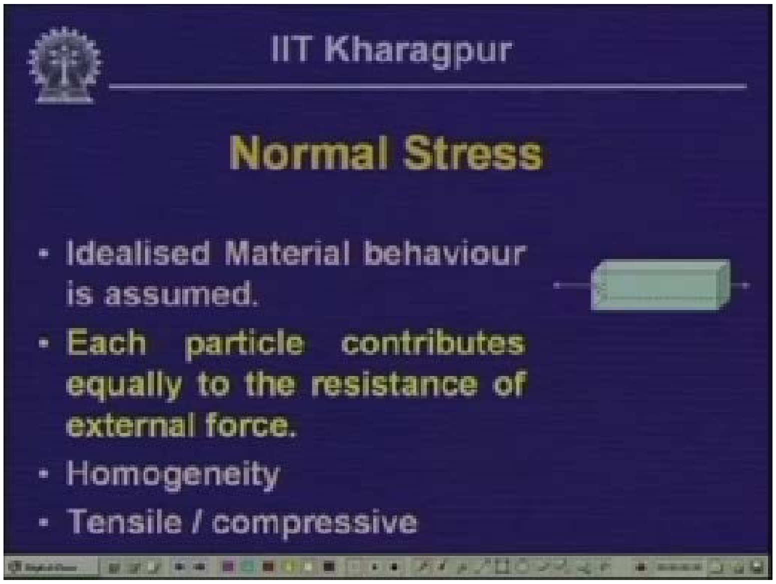

And we had noticed that if we take a body which is axially loaded by a force P and if we take a section and draw the free body diagram, this body is under the action of external force P so at the chord section say “aa” there will be resulting stress component which we call as stress resultant or the force which is resisting P. At every point there will be a stress component and the normal stress multiplied by the area will give the force. So integral of stress multiplied by the small area integrated over this surface will give the stress resultant which is equal to P.

While making this kind of assumption that every where state of the stress exists if the force acts through the centriod of the section we assume that the particle of the material at every point contributes to the resistance of this external force and thereby we assume the homogeneity of this material.

We assume that at every point the same state of stress exists. When a body is subjected to external forces which are trying to cause traction in the member or trying to pull the member we call these kinds of forces and thereby stresses as tensile stresses whereas when the external forces are acting on the member trying to push the member we call this kind of external forces and the stresses as compressive stresses.

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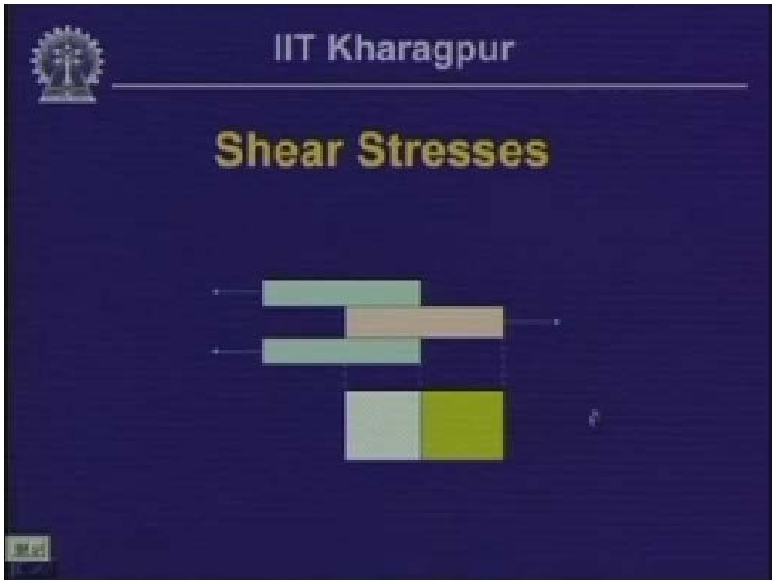

Let us look into the aspects of shear stresses. Let us assume that this particular body is subjected to the action of external forces P and the resistive forces thereby will be P by 2 and P by 2.

Now if we take the free body diagram of this particular member if we cut it over here then we will have this body which is acted upon by external force P by 2 hence the resistive internal force will also be P by 2. If we take the free body of the other part of the cut the resistive internal force is P by 2 and this force will be resisted at this inter phase and this also will be P by 2. Though, this force is eccentric with respect to the centre line of this body but this thickness being smaller we neglect the other effects because of these forces. This force will try to cause stress at the contact between these two elements and this contact area is so near which is the plan. When we look from the top the top view of this body looks like this and this shaded part indicates the contact area between these two pieces of material.

Now if we consider that this particular length as “a” and this as “b” then you can define the shear stress which is designated as tau equals to the force which is acting at the inter phase which is P by 2 by a(b). So a(b) is the contact area over which the shearing stress exits.

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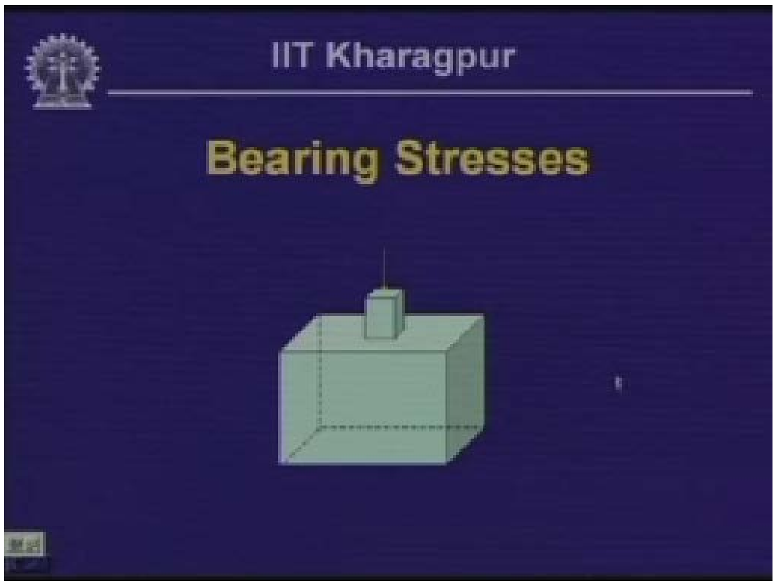

Many a times we come across situations where some blocks are resting over another block and transferring the forces from external sources. For example, if we have a body like this in which, this is a block which is resting on a bigger block and the smaller block is subjected to external load P. If this block is placed, that is, the centroid of the top block is placed on the centroid of the bottom block then at the interface between these two blocks and at this inter phase there will be normal stresses generated and these normal stresses we designate as bearing stress. By the term bearing we mean that the bottom block is bearing the load of the top block. So, if the contact area between the smaller block and the bigger block, this equals to Ac then the bearing stress sigma bearing can be written as the external force P by Ac which is the contact area. So this is called as bearing stress.

Here is another example which is the concept of the bearing stress. We have a rigid bar resting on two supports and this is subjected to external force P. Now if we take the free body diagram of this particular bar, the external force is P so the resistive force at the support point, assuming this P is acting to the centroid of this body is P by 2 and P by 2. This reactive force will be in turn transmitted to the support. This is the support and resistive force which is getting transmitted on and depending on the contact area we have if this is length “a” and the width of this body is “b” then this force P by 2 has contact area which is a(b). Hence the bearing stress that is sigma bearing is equal to P by 2 as the reacting force which is getting transmitted on this support divided by a(b).

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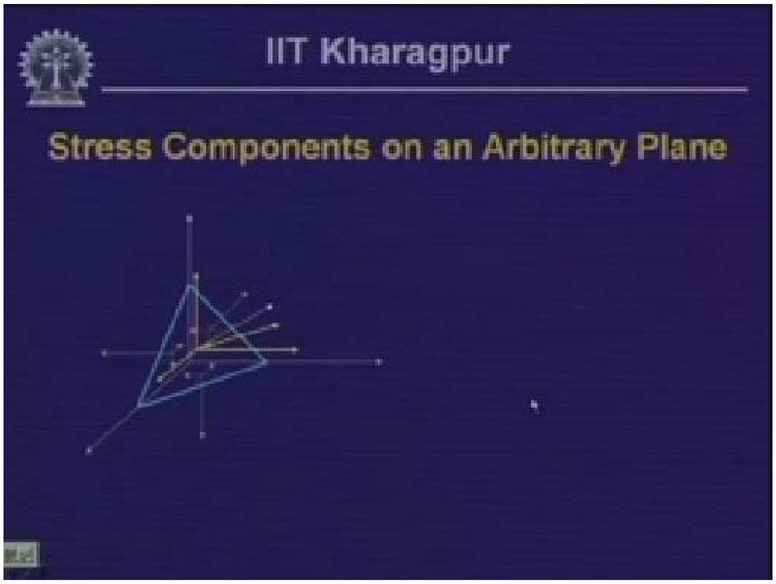

Here are the different stress components that act on any arbitrary plane. Let us consider that A, B, C are any arbitrary plane and o, x, y, and z is the reference axis system. As we have noticed earlier the plane normal to which it coincides with the axis we designate that in the name of the particular axis. Likewise this particular plane is the x plane on which the normal stress $sigma_x$ acts.

Likewise the plane oBC is normal to that coincides with y direction which is y-plane and the normal stress $sigma_y$ on this plane. And the normal stress on the plane Aoc is the z-plane is $sigma_z$. Also, in those planes there are shearing stresses x-plane in the y-direction will have the stress which we call as tau_{xy}, which we call as tau to the power XZ; on the x-plane, in the y-direction, so tauxy. The component which is in the z-direction is designated as tau to the power XZ. Likewise shear stress component in the y-plane acting in the x-direction we call that as tau_{yx} and tau_{yz}. Similarly this is tau_{zx} and tau_{zy}.

Let us assume that this arbitrary plane has a normal which is outward drawn normal is n. This unit vector can be designated with reference to xyz-plane. Let us assume that this is the reference axis xyz. The unit normal is drawn here. If this vector makes an angle of α with x-axis, beta with y-axis, and gamma with z-axis then we define the cosine components in the x-direction as n_x which is cosine α; n_y is the cosine beta; and n_z as cosine gamma. Thereby the unit vector this distance here can be represented by (n_x) whole square plus (n_z) whole square; (n_z) whole square plus (n_y) whole square will give this unit. So we have in effect (n_x) whole square plus (n_y) whole square plus (n_z) whole square is equal to 1.

Let us assume that on this arbitrary plane we have the resulting stress vector as R and the component of this resultant stress on this plane in the x, y, and z-direction be R_x, R_y, and R_z. Also let us assume that the area of the arbitrary plane is dA which is the area of the plane ABC. Now, if we take the projection of this area on x-plane which is AOB; so area of AOB is dA into n_x which is cosine of this area ABC on AOB. Area BOC is the projection on the area ABC on y plane which is dA into n_y; and area AOC is the projection of ABC on the z-plane which is dA into n_z.

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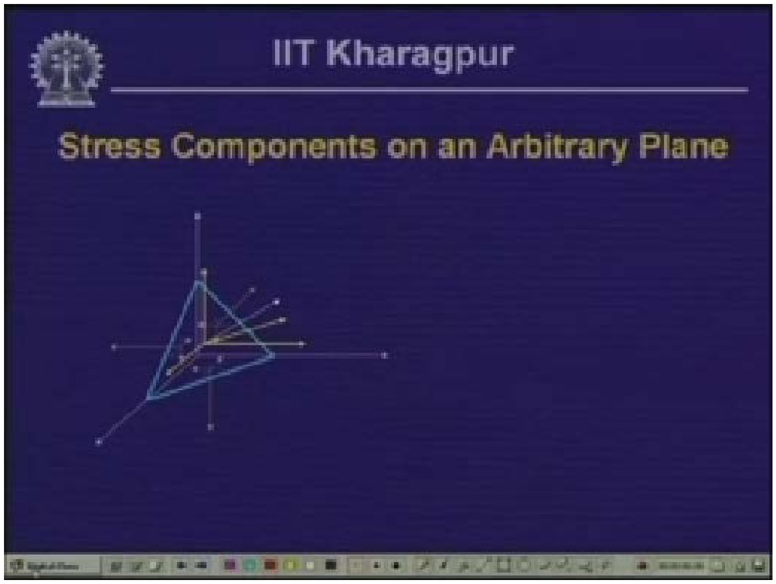

Now if we take the summation of all forces in the x-direction where in the stress components involved will be sigmatau$_y$ in the x-direction, and tauz$_x$ in the x-direction and R_x then we can write down the equilibrium equation in x-the direction. So equilibrium equation in the x-direction will be R_x into area dA; which is a force, stress resultant multiplied by area minus sigma$_x$ dA n_x the area of x-plane minus tau$_{yx}$ which is acting in the y-plane times the area of y-plane dA n_y minus tau$_{zx}$ which is the z-plane acting in x-direction multiplied by the area dA n_z is equal to 0. So the equation is as follows: R_x dA minus sigma$_x$ dA n_x minus tau$_{yx}$ dA n_y minus tau$_{zx}$ dA n_z is equal to 0.

If we divide the whole equation by dA or in a limiting situation we get R_x is equal to sigma$_x$ n_x plus tau$_{yx}$ n_y plus tau$_{zx}$ n_z. Similarly, if we take the equations in the y and z-directions and write down the equilibrium we will get R_y is equal to tau$_{xy}$ n_x plus sigma$_y$ n_y plus tau$_{zy}$ n_z $R_{y.}$ And R_z the resulting stress in the z-direction is equal to tau$_{xz}$ (plane in the z-direction as) n_x plus tau$_{yz}$ (the y-plane in the z-direction) n_y plus sigma$_z$ n_z. That is R_z is equal to tau$_{xz}$ n_x plus tau$_{yz}$ n_y plus sigma$_z$ $n_{z.}$ These are the three resulting stress components. So the stress components on the arbitrary plane which are acting in the x, y and z-directions are represented in terms of the stress components in the rectangular co-ordinate system. This set of equations is normally designated as Cauchy’s stress formula.

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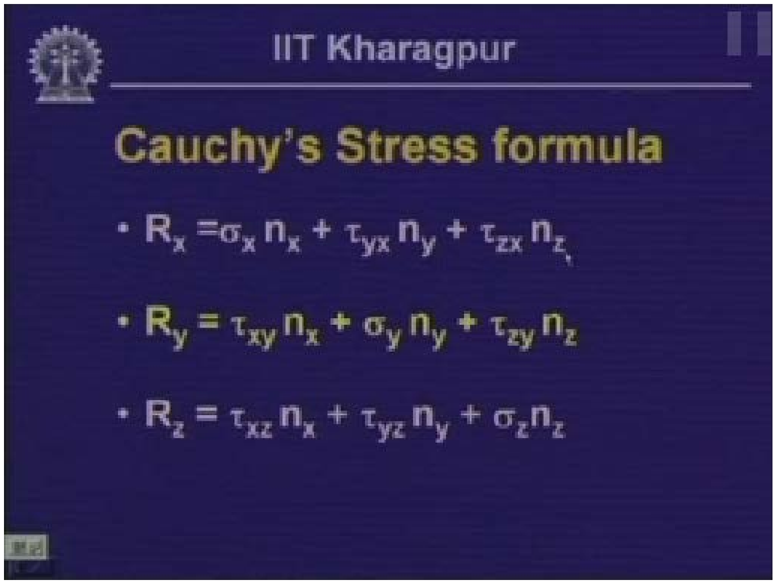

So Cauchy's stress formula is R_x is equal to sigma_x n_x plus tau_{yx} n_y plus tau_{zx} n_z; R_y is equal to tau_{xy} n_x plus sigma_y n_y plus tau_{zy} n_z and Rz is equal to tau $_{xz}$ n_x plus tau_{yz} n_y plus sigma_z n_z.

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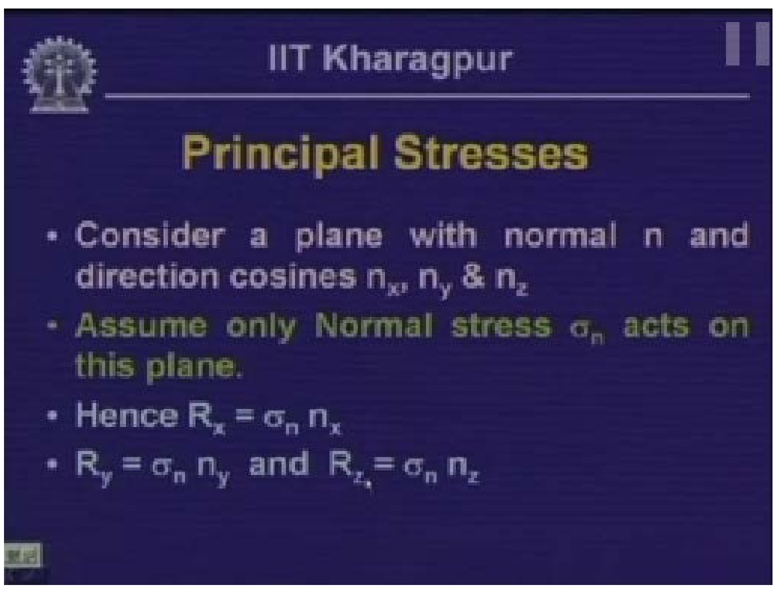

Now let us look into, if we consider a plane which has normal n and the direction cosines for the normal are n_x, n_y and n_z. Also, we assume that on this particular plane only the normal stress acts in the direction of the normal to the plane. Hence if we take the components of this in the x, y, and z direction then as we have designated before R_x as the resulting stress in the x-direction; R_y as the resulting stress in the y-direction and R_z as the resulting stress in the z-direction they can be written in the direction cosines as: R_x as sigma_n n_x; Ry as sigma_n n_y; and R_z as sigma_n n_z.

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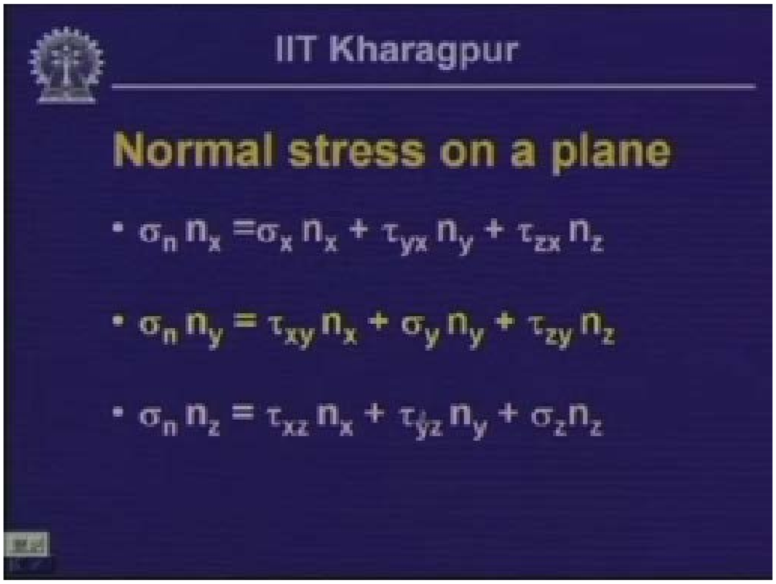

Exactly in the same form the way we have evaluated Cauchy's stress formula taking the equilibrium equations in the x, y and z-direction we can compute the resulting forces in x, y, and z-direction in terms of sigma_n.

In the previous case we had forces in the x direction as R_x dA is equal to sigma_x dA n_x plus tau_{yx} dA n_y plus tau_{zx} dA n_z and by dividing the whole equation by dA we had R_x is equal to sigma_x n_x plus tau_{yx} n_y plus tau_{zx} n_z. Exactly in the same form in place of R_x now we have sigma_n n_x the resulting stress in the x-direction and this multiplied by area gives the force in the x-direction, this equals to sigma_x dA n_x plus tau_{yx} dA n_y plus tau_{zx} dA n_z. Hence from this we can write sigma_n n_x is equal to sigma_x n_x plus tau_{yx} n_y plus tau_{zx} n_z.

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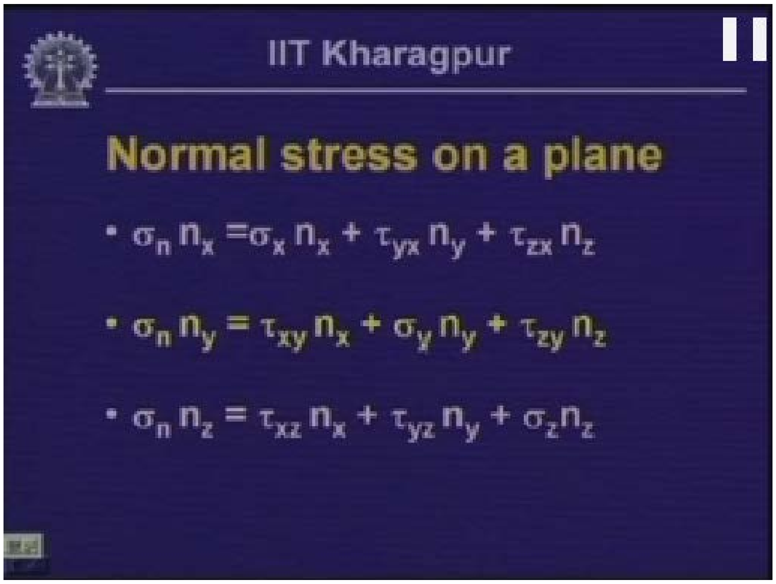

This is what is represented here the equations of equilibrium in the three directions x, y, and z. Now these equations can be rearranged and can be written as (sigma$_x$ minus sigma$_{n)}$ n_x plus tau$_{yx}$ n_y plus tau$_{zx}$ n_z is equal to 0; tau$_{xy}$ n_x plus (sigma$_y$ minus sigma$_n$) n_y plus tau$_{zy}$ n_z is equal to 0; tau$_{xz}$ n_x plus tau$_{yz}$ n_y plus (sigma$_z$ minus sigma$_{n)}$ n_z is equal to 0.

We have already seen that tau$_{yx}$ is equal to tau$_{xy;}$ tau$_{zx}$ is equal to tau$_{xz}$ al; tau$_{zy}$ is equal to tau$_{yz}$. Now these three equations can be thought of as simultaneous equations containing n_x, n_y and n_z and we can evaluate the values of n_x, n_y and n_z. Now, if we expand this particular equation we will get a cubical equation in sigma$_n$.

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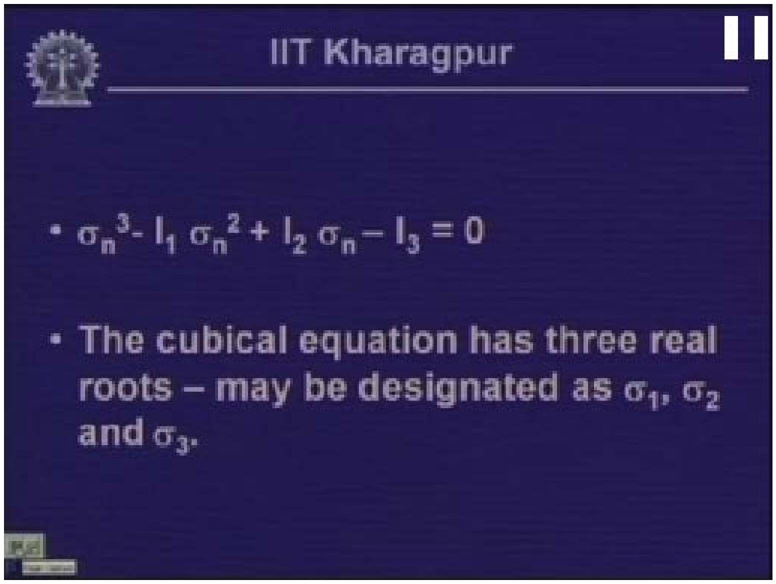

This is the cubical equation in $sigma_n$: ($sigma_n$) whole cube minus l_1 ($sigma_n$) whole square plus l_2 $sigma_n$ minus l_3 is equal to 0. And once we solve this cubical equation we are expected to get three roots which eventually will turn out be real and we designate those roots as $sigma_1$, $sigma_2$ and $sigma_3$. And corresponding to each values of sigma we will get the values of n_x, n_y and n_z.

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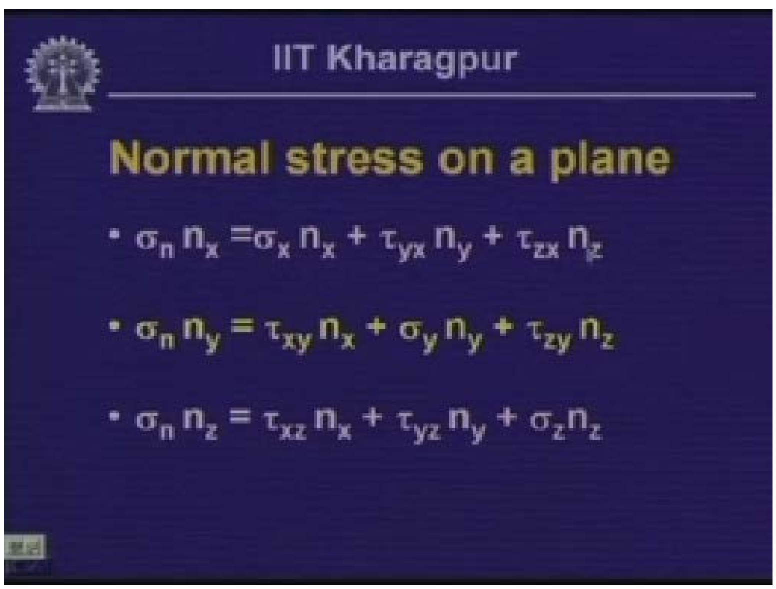

So here we have the equations on the arbitrary plane on which the stress in absolutely normal and we have observed that we can get simultaneous equations n_x, n_y and n_z in terms of $sigma_x$ tau_{xy}, tau_{yx} and $tau_{yz;}$ and $sigma_x$, $sigma_y$ and $sigma_z$.

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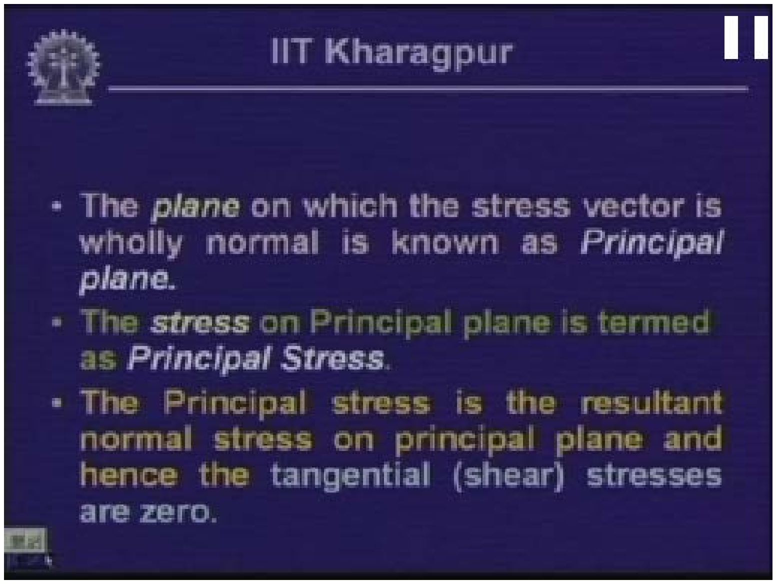

This is the cubical equation in the terms of I_1, I_2, and I_3 and for each of these sigma values if we are interested to compute the values of n_x, n_y, and n_z we can do that using these equations along with the expression we have n_x square plus n_y square plus n_z square is equal to 1. So, for a trivial solution n_x, n_y, n_z is equal to 0 which is not really going to give us the solution because n_x square plus n_y square plus n_z square is equal to 1 now for a trivial solution of the simultaneous solution we said that the determinant of the coefficients of n_x, n_y, and n_z is equal to 0. And if we do that, we have (sigma$_x$ minus sigma$_n$) tau$_{xy}$ tau$_{xz}$ tau$_{xy}$ (sigma$_y$ minus sigma$_n$) tau$_{yz}$ tau$_{xz}$ tau$_{yz}$ (sigma$_z$ - sigma$_n$) is equal to 0.

So if we say that the determinant of this equals to zero for trivial solution we get the cubical equation in sigma$_n$ cube minus I_1 sigma$_n$ square plus I_2 sigma$_n$ minus I_3 where the values of I_1 is equal to sigmax plus sigma$_y$ plus sigma$_z$; I_2 is equal to determinant sigmax sigma$_y$ tau$_{xy}$ plus sigma$_y$ sigma$_z$ tau$_{yz}$ plus sigma$_z$ sigma$_x$ tau$_{xz}$; I_3 is equal to sigma$_x$sigma$_y$sigma$_z$ plus tau$_{xy}$ tau$_{xz}$ tau$_{yz\text{ and}}$ this is symmetrical tau$_{xy}$, tau$_{xz}$, tau$_{yz}$ these are the three coefficients I_1, I_2, and I_3.

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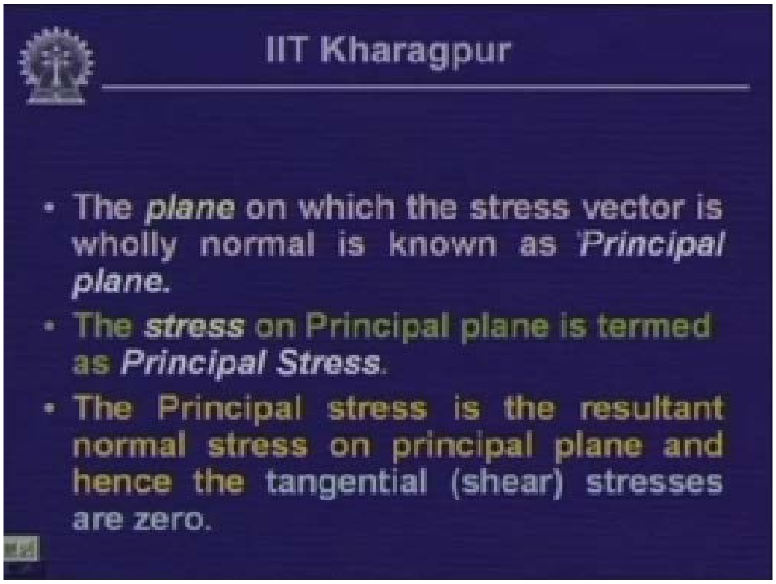

Now the plane on which this stress vector is fully normal is known as principal plane. This is what we considered in the previous situation where we had taken the arbitrary plane and we considered the stress vector which is along the normal and thereby we obtained the cubical equations in sigma$_n$ and from which we obtained the three roots sigma sigma$_1$, sigma$_2$ and sigma$_3$.

In this particular plane the normal stress is acting along the normal of the plane. So the plane on which this stress vector is wholly normal is called as principal plane. The stress on this principal plane which is absolutely normal is called as principal stress. And since the stress acting is in the normal direction there are no tangential stresses, the principal stress is the resultant normal stress so on a principal plane there are no tangential or shearing stresses, they are 0.

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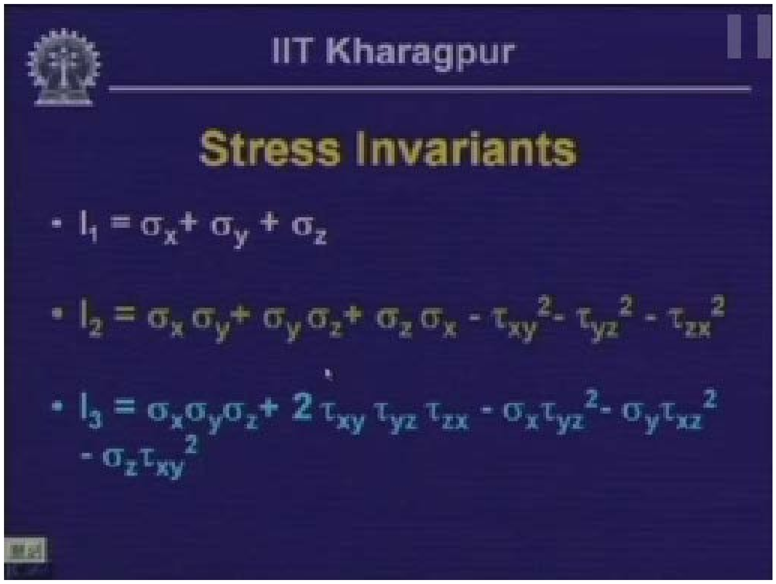

Now we have seen the cubical equation in sigma$_n$ from which we have obtained three roots sigma$_1$, sigma$_2$ and sigma$_3$. And we have looked into three coefficients I_1, I_2 and I_3 I_1 is equal to sigma$_x$ plus sigma$_y$ plus sigma$_z$; I_2 if we expand determinants I_2 is equal to sigma$_x$ sigma$_y$ (tau$_{xy}$) whole square plus sigma$_y$ sigma$_z$ (tau$_{yz}$) whole square plus sigma$_z$ sigma$_x$ (tau$_{zx}$) whole square; and I_3 is equal to sigma$_x$ sigma$_y$ sigma$_z$ plus 2 tau$_{xy}$ tau$_{yz}$ tau$_{zx}$ minus sigma$_x$ tau$_{yz}$ square minus sigma$_y$ tau$_{xz}$ square minus sigma$_z$ taux$_y$ square. We call these as invariants. It is to be noted that the principal stress which we have calculated at a particular point remains the same irrespective of the reference axis system we take.

In this particular case we have taken x, y and z as the rectangular axis system. Supposing at that particular point if we take different axis system which is represented as x prime, y prime and z prime if we write down corresponding normal stresses and shearing stresses as sigma$_x$ prime, sigma$_y$ prime and sigma$_z$ prime then we can observe that the values of I_1, I_2 and I_3 which are represented in terms of the normal stresses and shearing stresses they remain the same because the principal stresses at that particular point remains unchanged.

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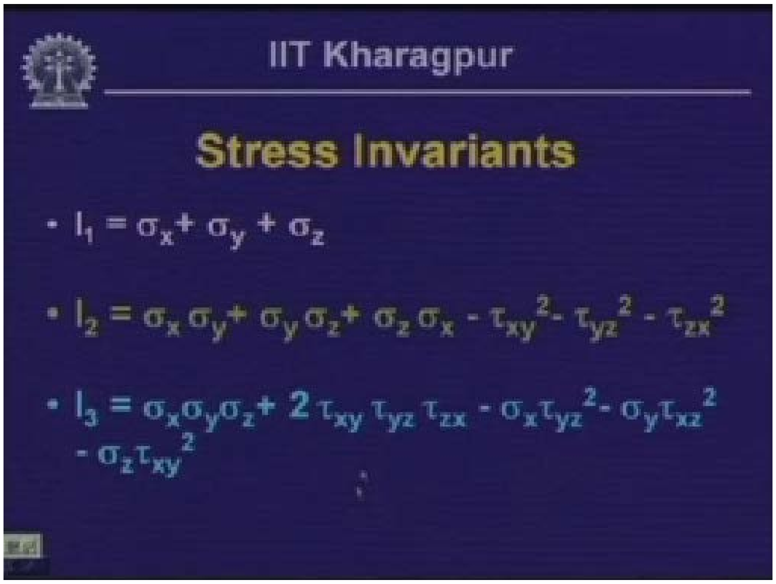

That is why these coefficients I_1, I_2 and I_3 are known as stress invariants. Let us see how to solve these stresses in some physical problems? In this particular example here there are two plates which are connected together by a bolt and it is subjected to a pull - external force P. This is the plan of the two plates and if we take a section we cut the plate here and cut the plate here and view from this side then the section looks like this.

Therefore the width of the plate is 200 mm and the thickness of the plate is given as 10 mm and the tensile pull that the plate is subjected to is 50kN. Now we will have to compute average normal stress at a section where there are no holes for the bolts. So, if we cut the section here and draw the free body diagram.

Here we have the plate which is put in a three dimensional form and this is acted on by a load 50 kN, the width of the plate is given as 200 mm and the thickness of the plate is given as 10 mm. So, at this particular section the resistive force for this external force 50kN which is acting through the centroid of this plate is R and hence from the equilibrium of these forces R is equal to 50 kN.

Hence the normal stress sigma at this particular cross section where there are no holes is equals to R divided by the cross sectional area sigma is equal to R by A is equal to 50 into 10 cube N by 200 into 10. This gives us the normal stress as 25 N by mm square and 1000 mm is equal to 1 meter so this is equal to 25 into (10) whole power 6 N by m square. Since we designate N by m square as P so it is 25 into (10) whole power 6 P and since (10) whole power 6 P gives the mPa, hence we are going to write this as 25 mPa. Also, we will have to compute the average shear stresses in the bolts.

The force which is acting here gets transmitted from this particular plate to this plate through this connection where these two plates are connected by the bolts. And as a result if we draw free body at this interface the force P will be get transmitted at this interface and the two bolts which are connecting these two plates together will be subjected to this force P. So we have one bolt here and another bolt here of diameter 20 mm and these two bolts will be resisting the force P by 2 and P by 2.

The plate is subjected to load P and this P is transmitted through the top plate through the interconnecting bolt. So the force which will be resisted by these bolts is half of P, hence the stress which is acting in the bolt which is the force on this particular area is called as the shearing stress on the plane of the bolt. So the average shear stress in each bolt which is tau average, tau is equal to P by 2 by pi by 4 (d) whole square (divided by the area of the bolt) and P is equal to 50; 25 into 10 cube N by pi by 4 into 400 is equal to 250 by pi N by m square which is so much of mPa. So this is the average stress in the bolt.

Thirdly, we will have to compute the bearing stress between the bolts and the plates. When the force is getting transmitted from one plate to another it is getting transmitted through these interconnecting bolts.

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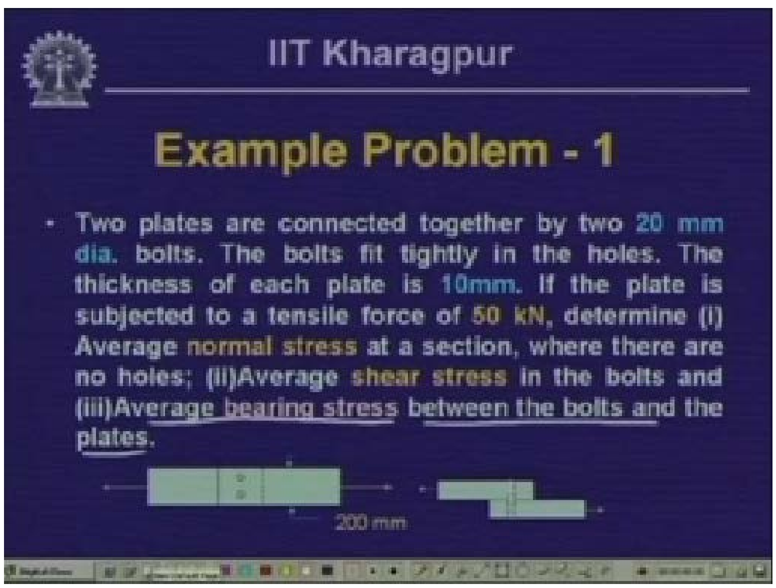

And if we look into the transmission of the force in little greater detail we will see that this is the plate, here two bolts are present and this plate is being pulled by force P. Now the transfer of force from the plate to the bolts which are here when plate is being pulled, this part of the plate, this is the bolt, the hub part of this particular plate comes in contact with this particular bolt and there may be release in the contact between the plate and the bolt surface.

Basically the plate is resting on this surface of the bolt. On an average sense we take the projection of this surface which is the diameter of this bolt and the contact area is here which is diameter times the thickness of the plate. So the contact surface which we get is half the perimeter the projection of which is d and the thickness of the plate at that particular interface. This is the area which is in contact with the plate and the bolt.
The bearing stress is the function of the contact area so the sigma_{bg} bearing is equal to the force P and since we have two bolts each bolt will get half the forces so P by 2 divided by the contact

area which is dt; $sigma_{bg}$ is equal to P by 2 by dt is equal to 25 into 10 cube N by 20 into 10 is equal to 125 MPa.

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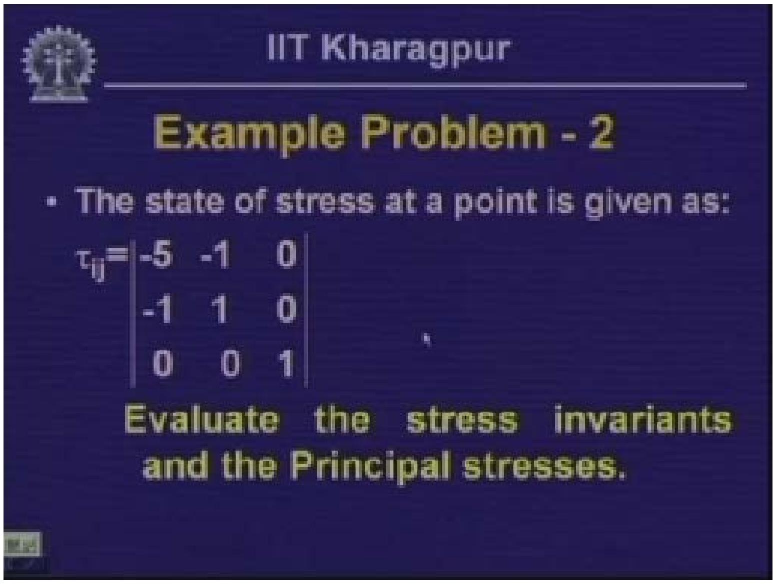

Here is another example where we are interested to evaluate the principal stresses and the stress invariants. We have learnt how to compute the principal stresses from the cubical equations and the stress invariant components. Now the state of stress at a point is given by this, this is the stress tensor where $sigma_x$ is equal to minus 5 units; $sigma_y$ is equal to 1 unit; and $sigma_z$ is equal to 1 unit and tau_{xy} is equal to minus 1; tau_{xz} is equal to 0; and tau_{yz} is equal to 0. So, if we compute the stress invariants, as we have seen, I_1 is equal to $sigma_x$ plus $sigma_y$ plus $sigma_z$; $sigma_x$ is equal to minus 5 plus 1 plus 1, is equal to minus 3. I_2 is equal to sigmax $sigma_y$ (tau_{xy}) whole square plus $sigma_y$ $sigma_z$ (tau_{yz}) whole square plus $sigma_z$ $sigma_x$ (tau_{zx}) whole square is equal to (minus 5 minus 1) plus (1 minus 0) plus(minus 5 minus 0) is equal to minus 10.

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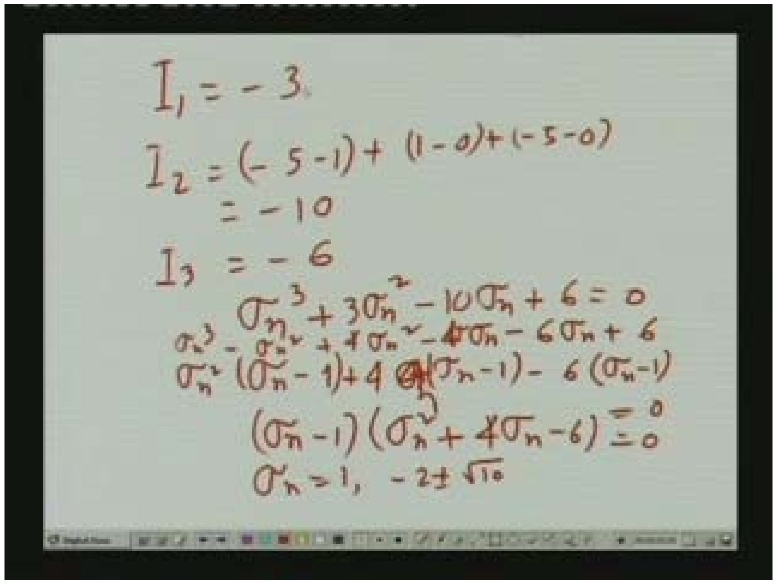

Likewise if you compute I_3 is equal to minus 6. Hence the cubical equation which you get is, sigma$_n$ cube minus I_1 into (sigma$_n$) whole square here I_1 is equal to minus 3 plus 3 (sigma$_n$) whole square, I_2 is equal to minus 10 hence minus 10sigma$_n$ plus 6 is equal to 0.

Now sigma$_n$ is equal to 1 we will make this function as 0, since sigma$_n$ in 1 of the root of the equation, sigma$_n$ minus 1 if we take, sigma$_n$ square so this gives sigma$_n$ cube minus sigma$_n$ square plus 4sigma$_n$ square minus 4sigma$_n$ minus 6sigma$_n$ plus 6; so minus 6 into (sigma$_n$ minus 1) is equal to 0. So we get (sigma$_n$ minus 1) into (sigma$_n$ square plus 4sigma$_n$ minus 6) is equal to 0. From this we will get sigma$_n$ is equal to 1, minus 2 plus or minus square root of 10. These are the roots of this equation. These are the three principal stresses. These are the three invariants I_1, I_2, and I_3. That is how we can compute principal stresses and stress invariants.

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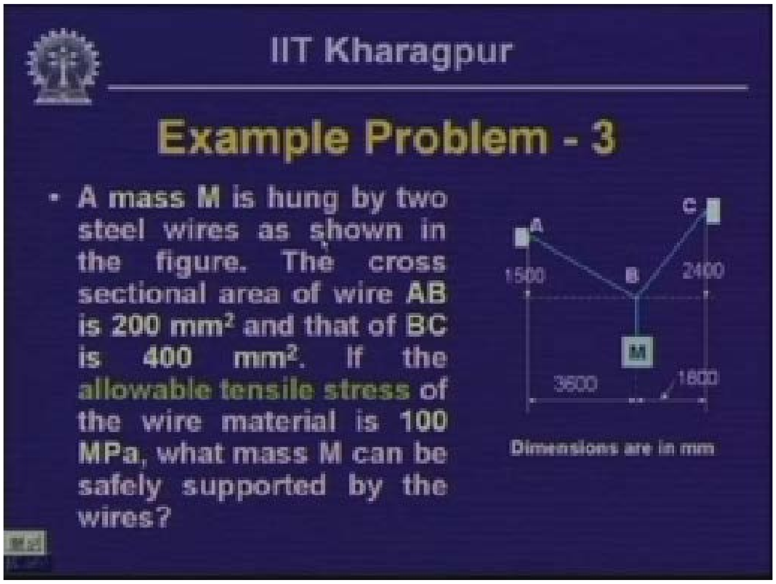

Now in this we have another example problem in which mass is hung by two wires AB and BC and the cross sectional areas of these two wires are given as 200 mm square and 400 mm square. And if the allowable tensile stress of these wire material is limited to 100 MPa then you will have to find out the mass M that can be safely supported by these wires. This particular problem can be solved by taking the free body diagram of this.

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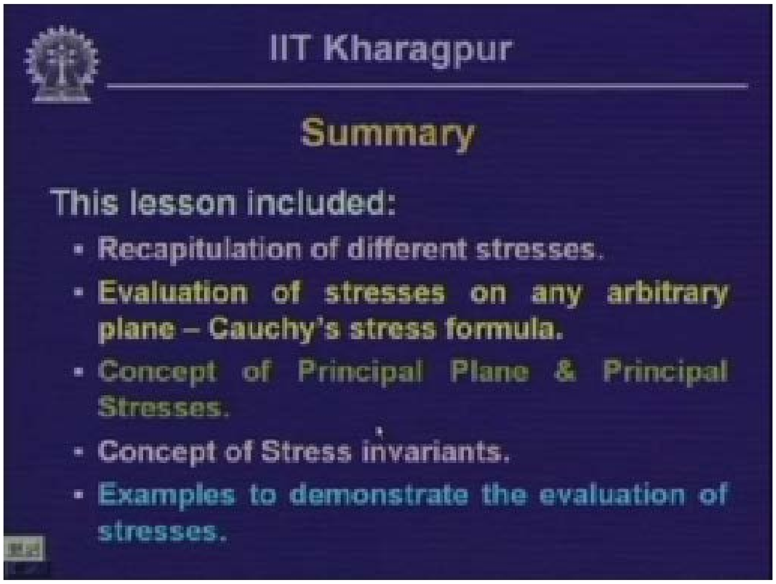

To summarize what we have learnt in this particular lesson; first we recapitulated on different kinds of stresses and those stresses are the normal stress, and the normal stress on an axially loaded bar. What is the maximum normal stress that acts on an axially loaded bar is the axial pull P or compressive force push by cross sectional area and cross sectional area which is minimum is normal to the axis of the bar.

Thereby we have seen the relationship between the normal stress corresponding shearing stresses. We have evaluated the stresses on any arbitrary plane which we generally designate as Cauchy's stress formula. And from this Cauchy's formula we arrived at concepts of principal plane and principal stresses and we have said that the principal plane is the one on which the stress is fully normal and thereby tangential stresses are at 0 and the shearing stresses in the plane is 0. While computing the principal stresses we have seen different coefficients which we designated as stress invariants I_1, I_2 and I_3.

We have noticed that I_1, I_2 and I_3 are the functions of normal stress at a point which are $sigma_x$, $sigma_y$ and $sigma_z$ and the corresponding shearing stresses at the particular point. We have seen some examples to demonstrate how to evaluate stress at different points. We have tried to give you the concept of normal stress, we have computed the normal stress at a particular cross section, we computed the shearing stresses in the bolt, where the normal force which is acting on the plate is transferred into the bolt cross section and then we have computed the bearing stresses, the bearing stresses are acting at the contact area between the plate and the bolt and finally we have computed the principal stresses and stress invariants.

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