Engineering Geology
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Lecture - 20
Stress-Strain Behavior of Soil and Rock

Hello everyone and welcome back. Today, we are going to learn about stress strain behavior of soil and rock, but before we take on the subject matter of today's lesson, we are going to look at the question set of the previous lesson.

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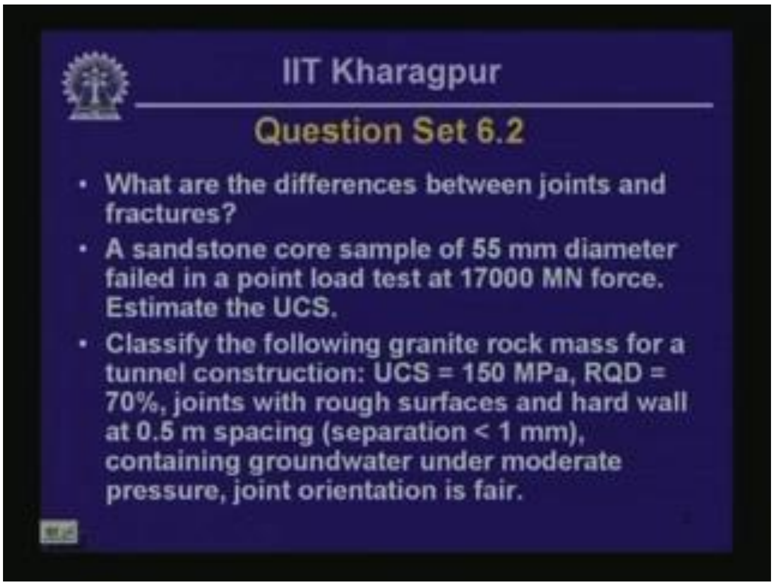

This is the question set. The first question was what are the differences between joints and fractures? So, by joints what I meant was there is no dynamic movement responsible for the discontinuity whereas, in case of fractures, there is a relative motion between the blocks on the either side of the discontinuity. So, joints for instance could arise because of the differential cooling of the original rock forming material, semi-solid rock forming material, or magma and because of that there might have been cracking within the body of the semi-solid object, and these things remain in the rock mass and these features we are going to call by the keyword joint, whereas fractures develops later on during the life time of the rock mass because of relative movement between different portions within the rock mass. So, that is the essential difference between joints and fractures. Then, the second question that I asked was a sandstone core sample of 55 millimeter diameter failed in a point load test at 17000 mega Newton force. Estimate the UCS.

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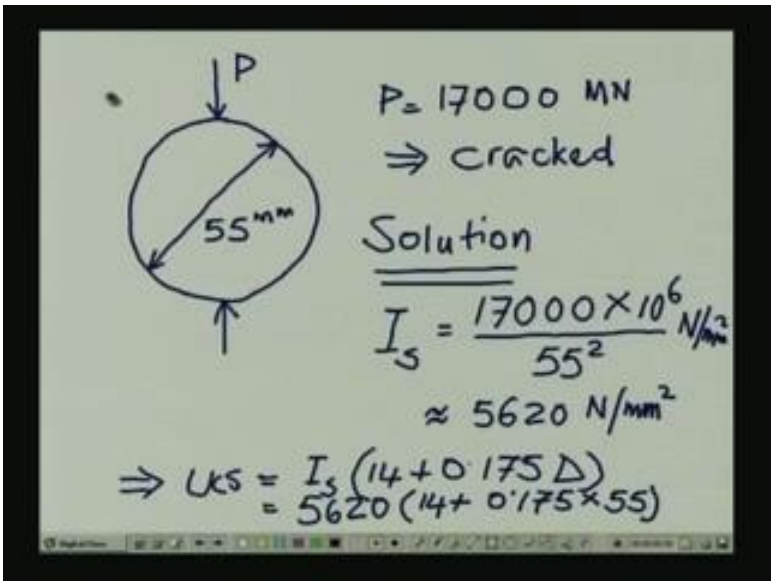

So, the problem here is like this. We have got a core sample and that core sample has got a diameter of 55 millimeter, and this particular core sample was subjected to a point load test and when p became equal to 17000 mega Newton, this particular core cracked. So, this is the problem and based on this data set, we are going to try to find out the UCS on confined compressive strength of this intact rock specimen.

So, first of all, in the solution what we are going to do is to find out the point load index I subscript s, and you recall from your notes from the last lesson I subscript s in this case is going to be equal to 17000 multiplied by 10 to the power of 6 divided by 55 or the diameter of the core specimen square and the unit of it is going to be Newton per millimeter square. That is equal to approximately 5620 Newton per square millimeter or MPA. Actually this unit is the same as MPA.

So, then what we get from this one is UCS. Actually UCS is going to be equal to I subscript s and this expression was again given in the previous days lesson multiplied by 14 plus 0.175 times the diameter of the specimen, and that in turn is going to be equal to 5620 multiplied by 14 plus 0.175 times 55. Okay, let me get rid of little bit of the material from the tablet.

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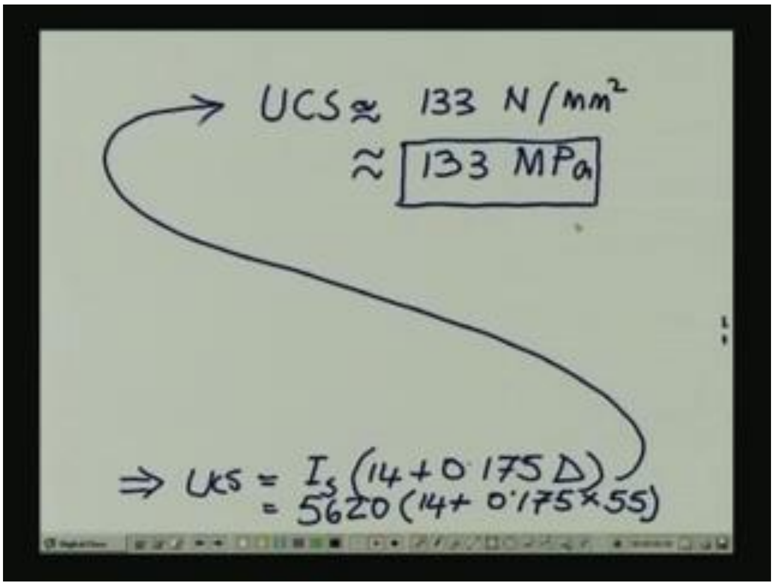

So, then what we get is, let us begin from here UCS will be equal to about approximately 133 Newton per square millimeter, or the same thing restated is 133 mega Pascal. So, that is your answer. So, the estimated unconfined compressive strength of the intact specimen of this type of rock will be then 133 mega Pascal. So, that settles the second problem.

The third problem was, classify the following granite rock mass for a tunnel construction. The UCS for this, an intact specimen of this type of rock was 150 MPa, RQD was 70 percent joints were with rough surfaces and hard wall. The spacing of joints was 0.5 meter and joint separation was less than 1 millimeter. It had groundwater unit, the rock mass had groundwater unit under moderate pressure, and joint orientation was fair with respect to the tunnel alignment. So, our problem then is to find out the RMR. So, again you should recall your notes from the last lesson.

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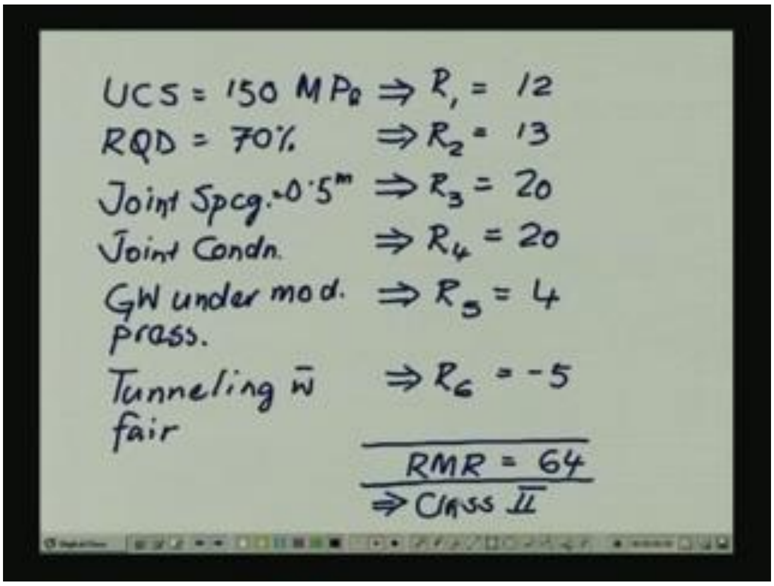

UCS of 150 millimeter 150 MPa will give us rating of rating R 1 of 12, then RQD of 70 percent will give us parameter rating R 2 of 13, then joint spacing of 0.5 meter will give us R 3 equal to 20, then joint condition will give us R 4 of 20. You should notice if you have already dug out your last days notes, then you should notice that this value of R 4 is just a little bit lower than the best joint condition which was tabulated in the previous days lesson that number was 25, but the joint condition in this case was slightly poorer in comparison.

As a result, we are using R 4 of 20, then we have got groundwater under moderate pressure that gives us R 5 of 4 and then, we are doing the rating for tunneling with fair joint condition viz. tunnel alignment. So, that gives us R 6 of minus 5 and then, we add all these numbers and we calculate the RMR of 64 and again, you look at the relevant table in your previous days, previous lessons notes and you will see that this value of rock mass rating will give us a classification of this. So, the rock here is classified as a clause two rock for the purpose of this tunneling work. Now, before I quit this example, let me take few minutes to illustrate actually what I meant by a fair joint condition.

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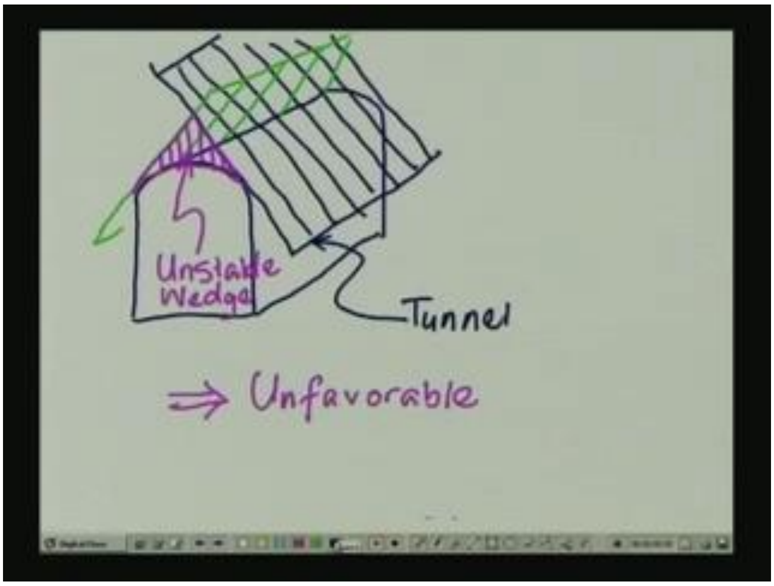

Let us say you have got a tunnel alignment like this. So, the tunnel actually goes like this we are drawing. In this case, it is pretty easy to understand that we are drawing an isometric of the tunnel and let us say the joint orientation in this case is like these joint sets are oriented with respect to the tunnel alignment in this manner. So, this one here is one of the joints, and there is another one. So, the other joint set is oriented in this manner. This is just an example. So, here you have got this type of joint.

Now, you can see that this particular joint configuration is going to be bad for this tunnel construction because this much of wedge of rock, just above the ceiling of the tunnel will always be prone to failure or collapse within the tunnel volume. So, this particular alignment, this is the unstable wedge in this case. So, this is going to be classified as unfavorable condition.

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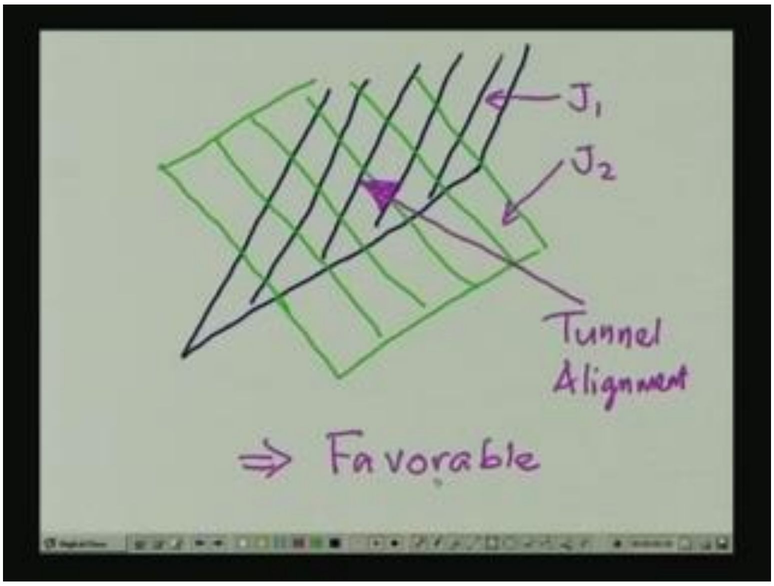

On the other hand, if we had the tunnel passing through an orthogonal direction, that means, everything else was to be the same. Let us say the tunnel alignment, an alternative tunnel alignment was like this were one joint set was oriented in this manner, and the other one was oriented like this exactly in the similar way as in the previous illustration. So, these are the two joint sets that we are dealing with in this particular problem, but here the tunnel is moving rather than being parallel to the strike of the joints. It is moving perpendicular in this case. The problem of rock fall is going to be much more limited. So, this one here is the tunnel alignment, and this one is the joint one and that plain, there is joint 2. So, this is going to be called a favorable condition. So, that illustrates this particular aspect. Finally, before I wrap up the discussion and get into the subject matter of today’s presentation, I want to take up a house keeping point.

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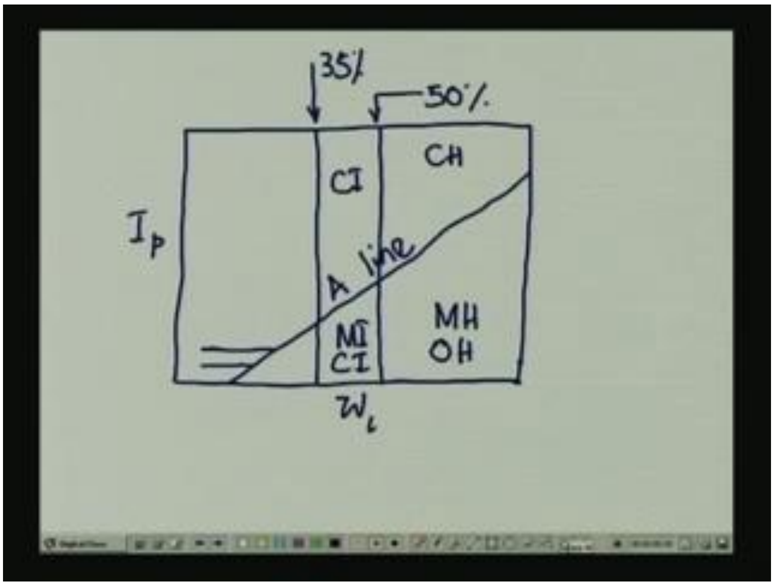

This is something to do with the classification of fine grain soil that I was talking about at the beginning of the last lesson, and there what I said is that we have got a line. So, this one is a line and what we are doing is talking the plasticity index against the liquid limit, and I said that in case of the code, you have got two vertical lines at w l of 35 percent and w l of 50 percent, and you recall that this, here this zone here is MH OH whereas, this one is CH and so on and so forth. So, these areas, there are several different classifications depending on which zone you are falling in.

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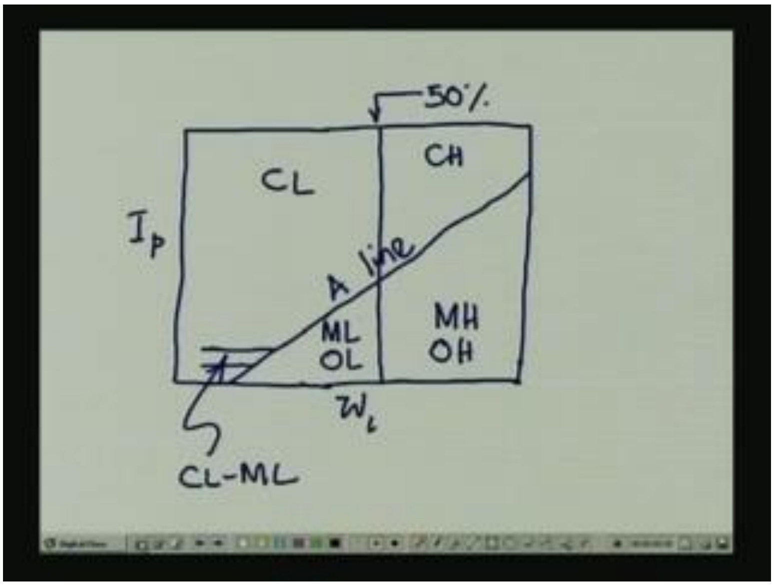

Now, in case of the ASTM code, this particular line is not there and consequently, you also do not have soil types that are classified as of intermediate plasticity or intermediate compressibility. So, in case of ASTM, you could only have CL or CH or CL ML and then, here you could have ML or OL. So, that is as per ASTM whereas, in case of code classification, classification is a little bit more involved. So that takes care of the point that I wanted to discuss.

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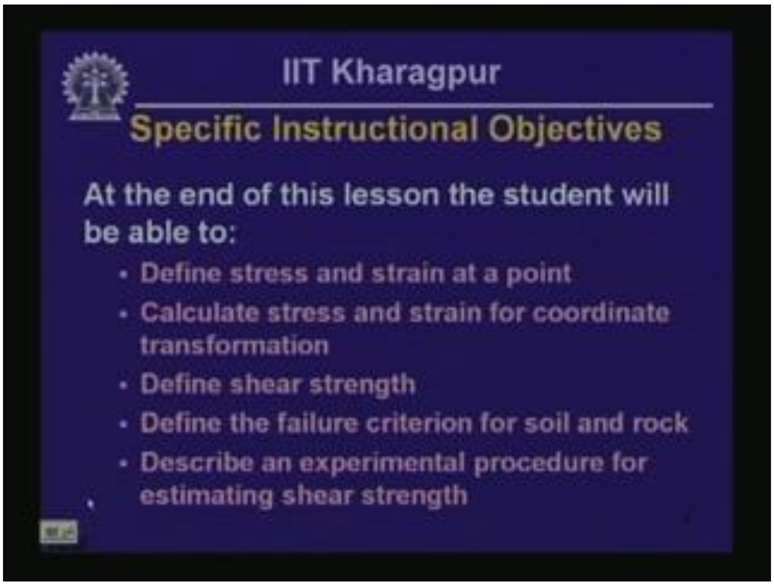

Now, we get on with today's subject matter. So, what we want to learn at in this lesson? We want to be able to define stress or strain at a point, and then we would be able to calculate stress and strain for coordinate transformation. We are going to define shear strength of the soil and rock, we are going to define a simple failure criterion for soil and rock, and describe finally an experimental procedure for estimating the shear strength of soil or rock. So, those are the objectives.

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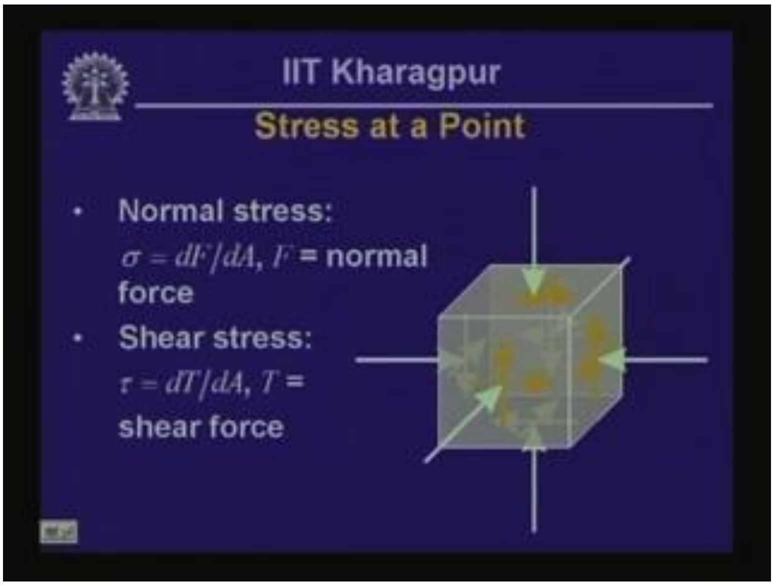

Then, let us begin with the depth with what we mean by stress at a point. Now, you all know from your previous curriculum that stress is given by force per unit area. Now, that force could be a normal force. In that case, the stress is going to be a normal stress or here, what we are using the symbol. We are using symbol sigma to denote normal stress, or if the force is parallel to one of the planes, then the concerned relevant stress is going to be called shear stress and for this type of stress, we are going to use symbol tau. So, then we have got two types of stresses really. One is normal stress, and the other one is shear stress.

Now, a graphical way of denoting or illustrating the stress at a point is by considering a small element. Small element, usually cubical element within the soil or rock mass and that element is shown on the right side of this particular slide. What we have here is a normal stress aligned in the vertical direction. So, that is the only normal stress here on this particular element.

Now, we can have normal stress in other directions as well. So, here we have got one of the pairs of normal stresses acting in the horizontal direction, and this is the second pair of normal stress in the horizontal direction. So, in general for a three-dimensional problem, you are going to have three normal stresses aligned in three orthogonal coordinate directions, and then you are going to have shear stresses. So, we are beginning here with shear stresses acting on the vertical plane on the right side of this particular element, and if you have got that kind of shear stress, in order to maintain equilibrium of this particular element, we are also going to have shear stress arising on horizontal planes at the top and bottom of the element acting in clockwise direction in an opposite sense to those to the shear stresses that we are operating on the vertical planes on the corresponding vertical planes.

So, you should notice here that all the four shear stresses will act together in order to maintain equilibrium. All of them has to occur, has to arise together or they are not going to arise at all. So, the magnitudes of each of these four stresses, four shear stresses shown on this particular element are going to be the same.

Similarly, we are going to have four more shear stresses here. We are considering the vertical plane that is facing us as well as the horizontal planes near the top, and the bottom of the element. So, all these four shear stresses are again going to be of equal magnitude, although the magnitudes of these shear stresses could be different from the previous four shear stresses that we considered, and finally, we are going to have four more shear stresses and now, our description of the state of stress of this particular point enclosed by the small infinitesimal cubical element is complete. What we have here are three components of normal stresses, and what we also have is three more components of shear stresses and their complimentary counter parts.

So, in general, then for a three-dimensional state of stress, we are going to have three normal stresses oriented in three orthogonal directions within the body within the rock mass or soil volume, and we are also going to have three more shear stresses on planes on three orthogonal planes.

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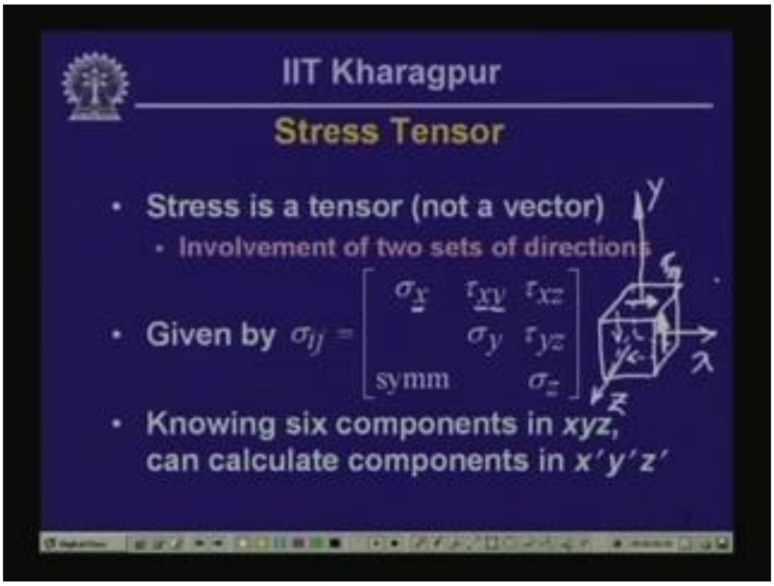

So, then stress tensor within a point within a mass of rock or volume of soil is going to be given by six independent components, and you should notice that the stress we are calling a tensor, and this is not a scalar nor is it a vector because if you recall that in case of a scalar, we do not have any orientation to worry about whereas, in case of vectors, we had to worry about only one orientation whereas, in case of stress, we have to worry about two orientations. One of those orientations are going to be the directions in which the forces are acting, and the other one is going to be giving the orientation of the planes on which these forces are acting.

For example, we considered a normal stress and that normal stress arose because of a force acting on infinitesimal area a. So, we have to worry about the direction of the force as well as the direction of the outward normal to this infinitesimal area. So, we have to worry about two different directions in this case, and what we have in this process is not a scalar, not a vector, but we have got a quantity that is called a tensor and any such quantity transforms in a particular mathematical way when we have to transform this system of coordinates from let say x, y and z set of coordinates to x prime, y prime or z prime set of coordinate system, and we have got a mathematical rule that we can drive which actually allows as to compute the six components in x prime, y prime, z prime systems from the six components of the stress tensor given in x, y, z coordinate system.

Now, here you should notice getting back to the matrix. Here you should notice that incase of normal stress, we are having one subscript and this is the subscript that defines the direction of the normal stress. In this case, the direction of the application of the force will be coincidental with the direction of the outward normal to this particular infinitesimal area on which the normal force is acting whereas, in case of shear stress, we have got two subscripts and the first subscript in this case denotes the direction of the force that is acting on the infinitesimal area, and the second subscript in this case denotes the outward normal on the area on which the force is acting.

So, let us illustrate with a small sketch here. Let us say you have got an infinitesimal element like this, and let us say in this case our x or y coordinate system is aligned in this manner, and the x coordinate system is aligned in this manner and the z coordinate system is aligned in this manner. We consider the top plane, the top horizontal plane of this particular element and let us say we have to name the shear stress that is acting in that manner. You notice that as soon as shear stress was occurring on the horizontal top plane, we introduced another shear stress occurring on the right vertical plane and similarly, you are going to have two more shear stresses on the two planes that are not visible from the front.

So, we are going to have other shear stress acting leftward on the bottom horizontal plane, and one more shear stress that is going to act downward on the vertical plane that is towards the left end of this particular element. So, we have our problem now is to name this particular shear stress. Now, you realize that this shear stress on the top horizontal plane arose because of an action of a force that was acting along the x direction. So, the first subscript which is going to go with this particular shear stress is going to be x, and since this shear stress is occurring on a plane for which the outward normal is aligned with the positive y direction, the second subscript is going to be given by y. So, these shear stresses are going to be called tau x y.

Similarly, you could have tau x z and tau y z when you consider the other planes. Now, you should also notice that all the four shear stresses in this case is going to be called tau x y in spite of the fact that shear stress acting on the bottom horizontal plane which is acting leftward, that component is also going to be called tau x y, and that is because the outward normal on that particular plane is acting in negative y direction because of which this particular shear stress arose is acting on the negative x direction as well. So, if both are negative, then we are not going to consider the sign and if both are positive, then also we are not going to consider the sign of these things. So, tau x y will be the same as tau minus x minus y.

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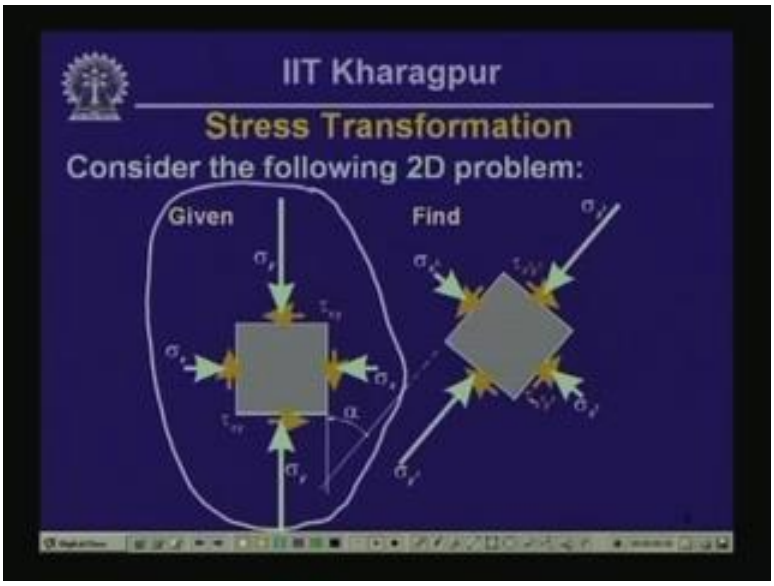

Then, let us consider the transformation in a two-dimensional state of stress. What is a two-dimensional state of stress? It is a state of stress in which we do not need to consider the third coordinate direction. So, here for instance, we have got only confined within a plain element, not a cubical element as we were considering so far and the directions we are going to consider in this case are x and y. So, what we are going to try to do in this case is, we are given a general state of stress on a two-dimensional element in x y coordinate system. If we have got a transform coordinate system x prime y prime, we want to be able to calculate what are the stresses occurring on that particular plane element.

So, graphically stated what we know in this case is sigma x, sigma y and tau x y. So, all these things here are known. We also know the coordinate transformation law. That means, how we are going to transform x y coordinate system to x prime y prime coordinate system. That means, we know this quantity alpha, this angle of rotation alpha and our job here is to find out the quantities sigma x prime, sigma y prime and tau x prime y prime. That is our objective.

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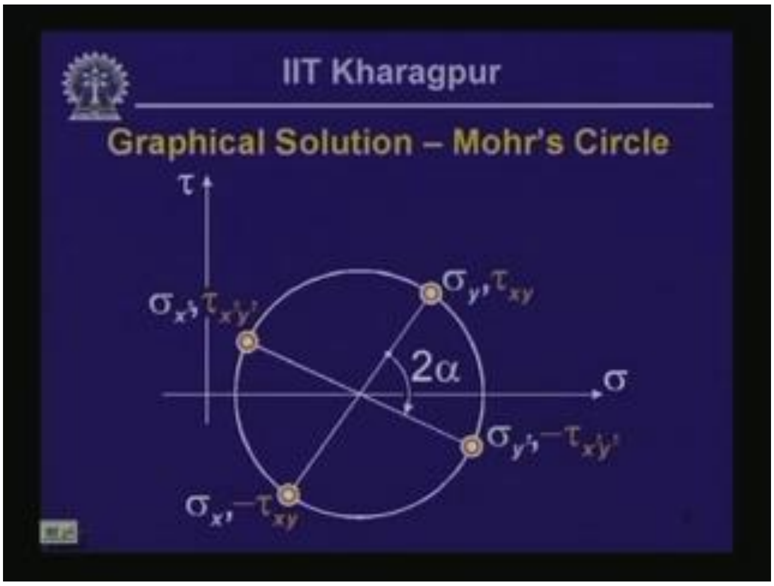

So, what we do actually is, we used a graphical construction. In this case you could do the same thing analytically, but what I am illustrating here is the graphical process and the tool here is called Mohr's circle. So, what we do here? We plot the states of stress on two orthogonal planes on tau was a sigma plot. Tau is plotted on the vertical axis and sigma in this case is plotted on the horizontal axis. So, what we had there if you recall the element is sigma x and tau x y, and sigma y and tau x y.

Now, here you should notice that with sigma x, we are plotting negative of tau x y whereas, with sigma y, we are plotting positive tau x y. That is because here we have to use another sign convention that counter clockwise shear stress that wants to rotate an element in a counter clockwise direction is going to be considered positive, and shear stress that is going to rotate an element in the clockwise direction is going to be called negative.

So, now we have got two points, and these two points represent the state of stress on two orthogonal planes. One is sigma y gamma tau x y, and other one is sigma x gamma tau minus of tau x y and if we draw a circle with the center on tau is equal to zero axis, then the points on this particular circle are going to represent the states of stress on any other two-dimensional element surrounding the point in question in case of two-dimensional problem. So, then what we do? Our problem is to find out the states of stress on axis rotated by an angle alpha in the clockwise direction.

So, what we do then is to find out two diametrically opposite points which are separated by an angular distance of two alpha from the first two set up points, and the points on the second set of coordinates is called are going to be given by, in this case sigma x prime tau x prime y prime and sigma y prime and negative of tau x prime y prime. So, that is the construction. It is a very convenient construction and this circle here is called the Mohr's circle. Now, before I quit this discussion, I want to introduce two more objects, two more interesting aspects. The first one is the major principle stress and minor stress is the notion of major principle stress and minor principle stress.

So, if you consider two points on Mohr circle representing the state of stress at a point within two-dimensional element, then the point that is on the right end of this two-dimensional element is going to be called the major principle stress, and you should notice that on this particular plane, the shear stress is zero and the one on the left most extremity of the Mohr circle represents a state of stress and that is called the minor principle stress, and the shear stress on this point is also 0.

Generally, the major principle stress is denoted using symbol sigma 1 and the minor principle stress is denoted using symbol sigma 3, and you may wonder what happened to sigma 2, and that is really when you have got a three-dimensional problem. You have got one more principle stress and that is called the intermediate principle stress, and for that we reserved the symbol sigma 2. Now, I should also mention here which you have already noticed possibly that incase of normal stress, we are using the convention of compression positive and tension negative, and that is because rock and soil, they are very weak in tension. So, it is convenient to consider compression positive in this case.

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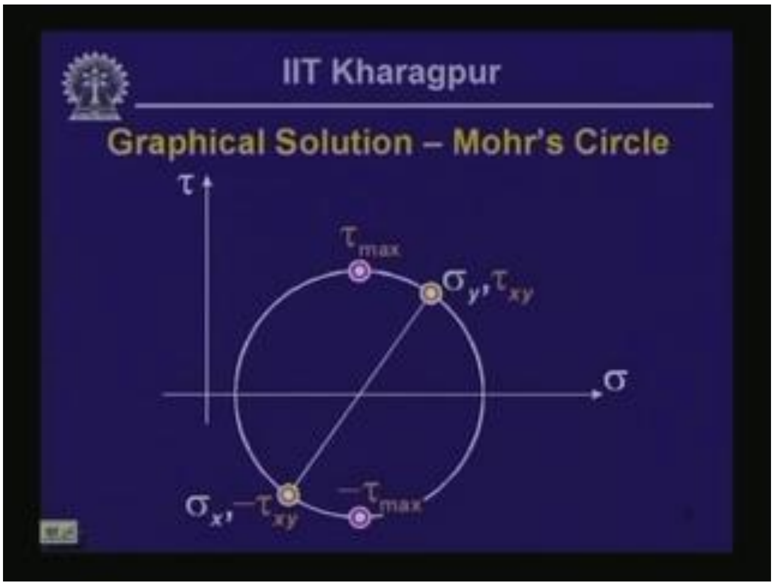

We also have got one more concept here and that is the concept of maximum shear stress, and these are at the crown and at the bottom of Mohr circle and in this case, the one at the top is going to be tau max and the one at the bottom is going to be called negative of tau max. Tau max is really half of the difference between the major principle stress and the minor principle stress.

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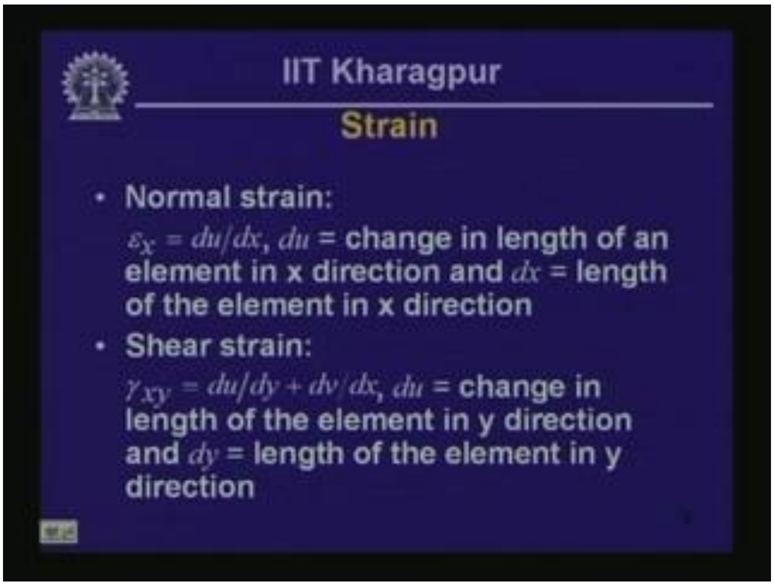

Then, we want to introduce the notion of strain within an element and strain again could be a normal strain or shear strain. Normal strain is given by change in length of an element in a particular direction divided by the length of the element in that direction whereas, shear strain represents the change of angle within the element and the expressions are given as stated on this particular slide. You should note these expressions. We are going to draw a sketch to illustrate these expressions.

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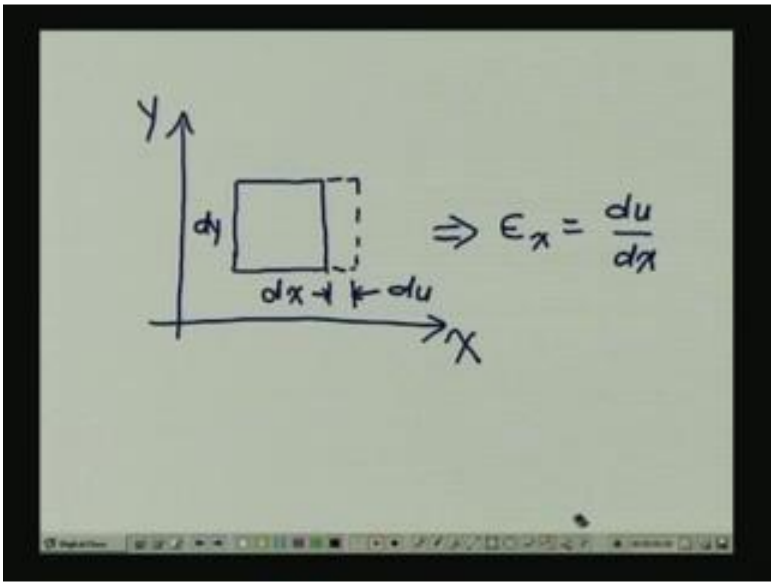

Let us say we have got a two-dimensional element like this. This is our x direction and that is our y direction. Let us consider two-dimensional element and the x dimension of the two-dimensional element is d x and the y dimension is d y. So, first of all, let us consider that by some means we stretch the element in the x direction, and this amount of stretching is given by d u. It is small infinitesimal quantity and this definition is only going to work when the deformation is small. So, that will give us the normal strain in the x direction as d u d x was given in this slide before. Now, let us consider another deformation pattern.

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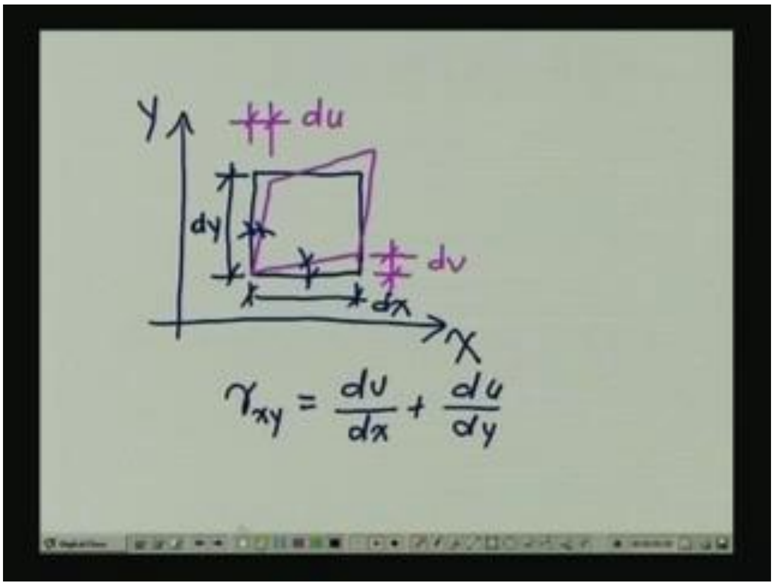

In this case, what we do is, we have got an element like this and the element gets deformed in this manner. So, this is the deformed shape of the element. You should notice that the volume of the element does not change in this case. So, what is happening here is that this quantity is going to be called d v because that is the change in dimension in the y direction, and using the earlier convention, we are going to call this quantity here as d u and as before the undeformed dimensions of the element is d x and d y or d x and d y.

So, shear strain really is the sum total of these two angles and since, these angles are going to be small because the deformations as small. We have got this definition of engineering shear strain for this deformation pattern gamma x y is d v d x plus d u d y, and this was the expression given in the previous slide.

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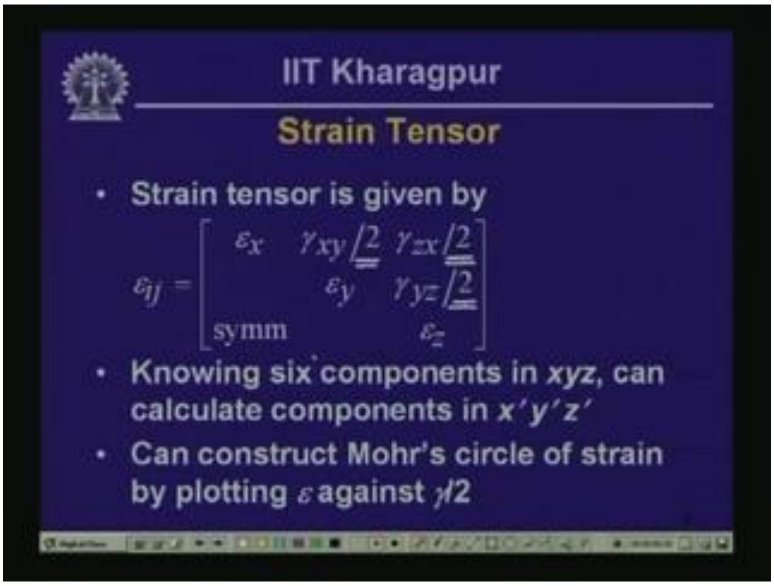

So, similarly you could define the other components of strain. What we what? You end up with a tensor again as was the case with stress like that shown near the top of this particular slide, but you should notice here one change in comparison with the stress tensor is that you have to really divide the shear strain by a factor of two in order to compose the strain tensor. So, you cannot consider gamma x y, gamma z x and gamma y z like you did in case of shear stress tau x y, tau z x and tau y z, but you have to divide the strain quantities by 2 in order to compose the strain tensor.

So, as was the case with stress if you know all the components, all the six components of strain in x, y, z set of coordinates, then you could calculate the components of strain in x prime, y prime and z prime set of coordinates, and it is obvious you could also think about Mohr circle of strain. Only difference is that if you recall incase of stress, we constructed Mohr circle by plotting tau versus gamma, but, sorry tau versus sigma, the shear stress versus shear strain, but here we are going to plot shear strain quantities divided by 2 against the normal strain quantities. So, we are going to plot gamma over 2 versus epsilon in order to construct Mohr's circle for strain.

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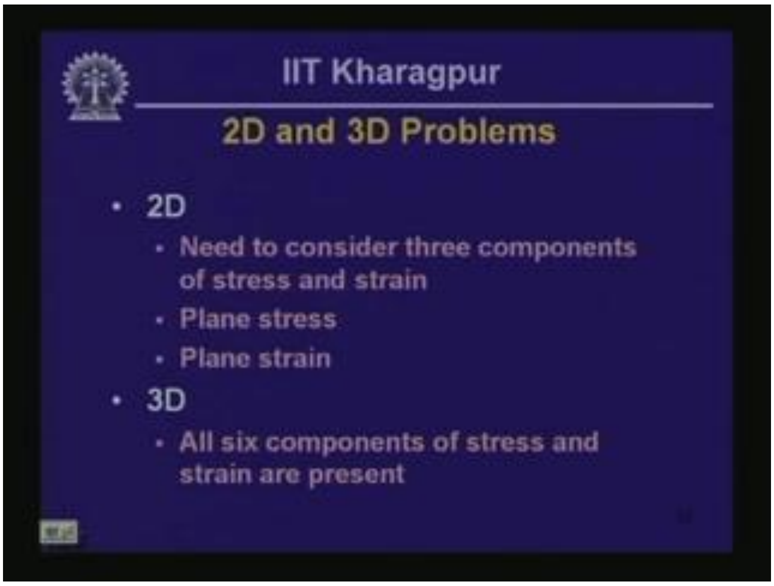

Now, we can have as was evident from the discussion that we had so far a two-dimensional problem or three-dimensional problem. In a two-dimensional problem, we only need to consider three components of stress and strain. The examples of this include a plain stress problem or a plain strain problem.

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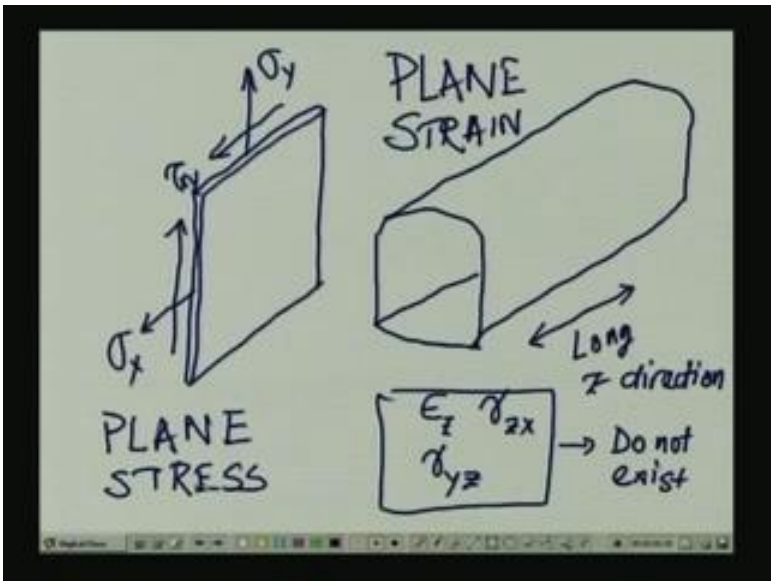

In case of a plain stress problem, really you have got a flat element. So, let us consider a flat plate for instance, and this flat plate is capable of supporting forces only in those directions. Then, we are going to have a two-dimensional problem, you could have actually shear stresses also occurring in this manner. So, this type of problem is called a plain stress problem. So, here let us call this one as sigma y, this one is tau x y, and that value there is sigma x whereas, plain strain problem and this (( )) clause of problem is more prevalent in case of soil and rock mechanics. Any problem which has got one of the dimensions very long in comparison with other dimension like for example, you are trying to compute the stresses around a tunnel. For instance, in this case, this dimension is very long in comparison with other dimensions than this type of problem is called a plain strain problem.

So, here what is happening really is in the long direction, you do not develop any strain or you can actually neglect the strain in the long direction. So, what you end up with are these components. So, you have got any strain component, if you call this long dimension as z direction, then in this problem you can neglect all those strains which has got z in it. For instance, all these components, they are going to be non-existent. So, this clause of problem is called plane strain problem and this clause of problem is called plane stress problem. You could have a general three-dimensional problem as well you need to consider all the six components of stresses and strain of stress and strain.

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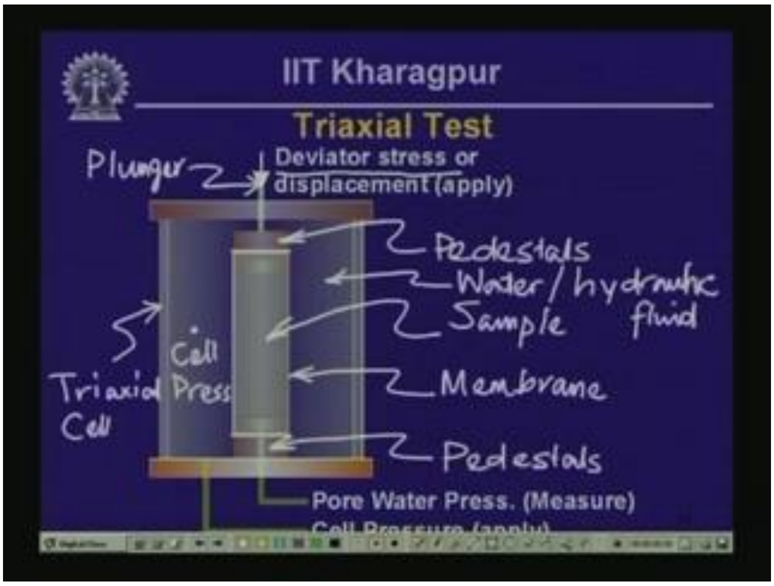

Then, the question comes how do we define, how do we derive the stress strain behavior of a soil or rock or a small piece of soil or rock? So, what we do typically is to conduct a triaxial test in order to find the stress strain behavior of soil or rock. So, here we take a cylindrical sample of soil or rock, and in this particular sketch, the sample of soil or rock is here and that sample is wrapped around within a water tight membrane and then, we place it in between two pedestals. So, those are the pedestals and then, what we have is, we enclose this element within a bath of water or hydraulic fluid, and this particular fluid is pressurized, and then through a plunger near the top apply a deviated stress or apply a stress in addition to the pressure to the cell pressure.

So, the pressure here is called the cell pressure. So, this pressure is called the cell pressure, and this is the triaxial cell. So, what we do? Actually we apply a pressure in addition to cell pressure through a plunger near the top, and we increase that stress that is called deviated stress until the sample fails, or it cannot support anymore any higher value of the deviated stress. Sometimes these tests are carried out in a displacement control manner in which a displacement constant rate of displacement is applied through the plunger on the top pedestal. So, that is how a triaxial test is carried out.

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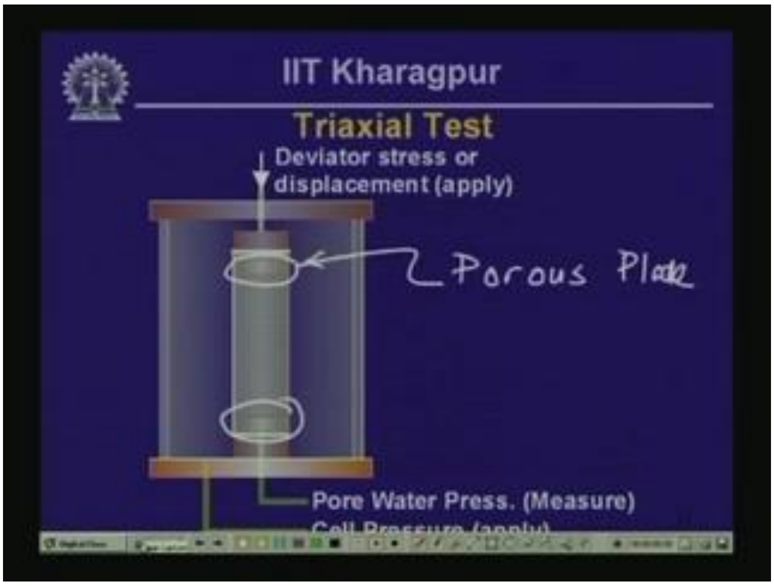

What you also should notice is that within the top portion, within the top pedestal, there is a porous plate and the use of that will be illustrated later and similarly, there is another porous plate at the bottom and that allows us to measure the pore water pressure within the element, and we are going to see, we are going to explain what is meant by that when we go further on soil and rock testing in one of the later presentations.

So, this is how we actually conduct a triaxial test. So, what we do is the procedure to apply certain amount of cell pressure and take the sample to failure, and then you take another sample of the same rock mass or soil volume or the type of soil, and conduct another test at different value of cell pressure, and note down the new deviated stress and conduct a series of such tests and from that, we can estimate shear strength parameters of soil and rock mass. So, let us see how these tests are conducted.

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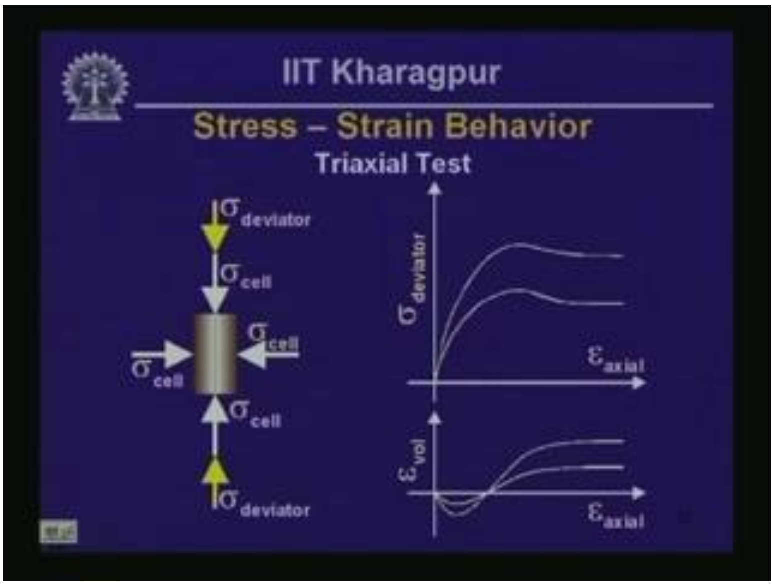

So, on the left is shown the schematic of a triaxial specimen and that is subjected to all round cell pressure and an axial deviated stress. So, let us begin the first test. So, what happens is, we increase the deviated stress mark with a yellow color there and we gradually increase that deviated stress. We take the sample to failure and what we get are those responses. So, we have got deviatoric response in which we plot the deviated stress versus the axial strength and then, we have got a volumetric response in which we plot the volumetric strength versus axial strength, and if you recall volumetric strength is the summation of the axial strength on all three coordinate directions on the three orthogonal coordinate directions is going to give you the volumetric strength, and this is the response for the first test.

Then, what we do? We increase the deviated stress. We do the same thing as we did in case of the previous test and what we get is this. Here you noticed that we increased the cell pressure. The cell pressure in this case is larger than the previous test, and we increased the cell pressure even further and we do the same thing. We take the sample to failure by increasing the deviated stress. This one here is the second sample that we tested and incase of the third sample, we are going to increase the cell pressure even further, and what we get for the third sample is a situation like this. Then, what we do? We will use these responses to plot Mohr circle for the three stress conditions in order to find out shear strength parameters, and that process I am going to illustrate in the next little bit, but before I do that, what I want you to notice here is that as the cell pressure increases, you are going to be needing a larger value of deviated stress for in order to fail the sample, and also you notice that the amount of volume change in this case by increasing cell pressure. What we are having? We are triggering more of a contractive volumetric behavior of the soil and rock specimen.

So, these are typical responses of soil and rock samples. So, here what we are doing is that on the positive side, we are plotting volumetric expansion. So, epsilon volume positive in this case indicate volumetric expansion or dilatation as it is called negative contraction.

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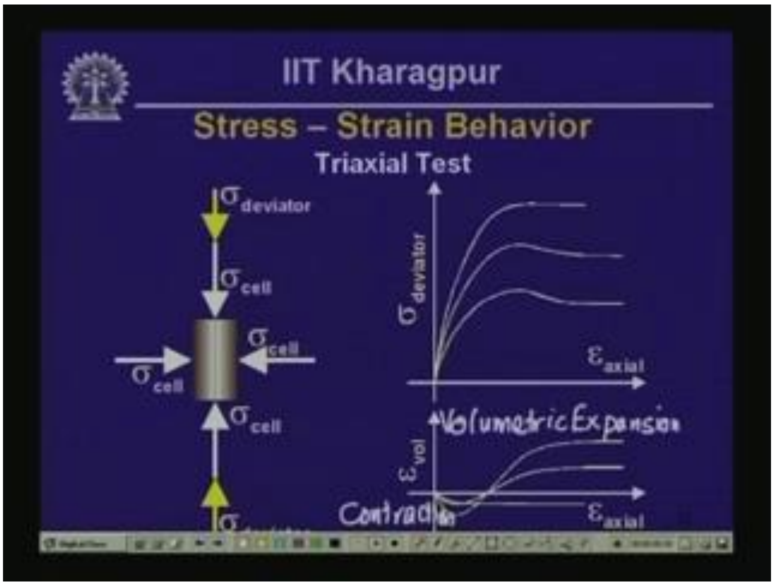

So, that actually gives you the response in a triaxial test and based on this, what we are going to do is, we are going to plot Mohr circle concerning the stress condition in these triaxial tests, and we will be trying to identify the strength parameters from this series of testing of three triaxial, three cylindrical samples of soil or rock mass or intact rock, and what we are going to do is, to estimate the sigma one value or the major principle stress value in each one of these tests, and the minor principle stress value in each one of these tests noticing that the horizontal and the vertical surfaces in this case are free from any shear stress.

So, they are really the principle planes or the planes on which principle stresses are acting and plot Mohr circle, but we are going to continue this discussion in the next lesson, and we are going to see how we construct Mohr circle and estimate the strength parameters from the series of test in the next lesson. So, until we meet for the next time, bye for now.