Fluid Mechanics

Prof. T.I. Eldho

Department of Civil Engineering

Indian Institute of Technology, Bombay

Lecture - 9

Kinematics of Fluid Flow

Welcome back to the video course on fluid mechanics. In fluid kinematics, in the last lecture, we were discussing about the potential flows. We can define the velocity potential as: u is equal to del phi by del x, v is equal to del phi by del y and w is equal to del phi by del z. We have defined the consequence rotationality of flow field for 3 dimensional flows and the potential flow where we have defined with respect to the rotational flow fields. Further, we have discussed about the potential flow, stream function and then we have derived the Laplace equation which governs the inviscid incompressible irrigational flow fields.

In potential flows, we have seen how we can define a problem and how the boundary conditions are defined. As we discussed potential flows which is the theories applicable for inviscid incompressible irrotational flow fields given by the Laplace equation and lines of constant potential is equipotential; the stream function is also defined and then the lines of constant function is called stream line. We have also seen some examples related to the potential flows. Today, we will discuss about the potential flow; we will see the basic potential flow; then, super position of potential flow.

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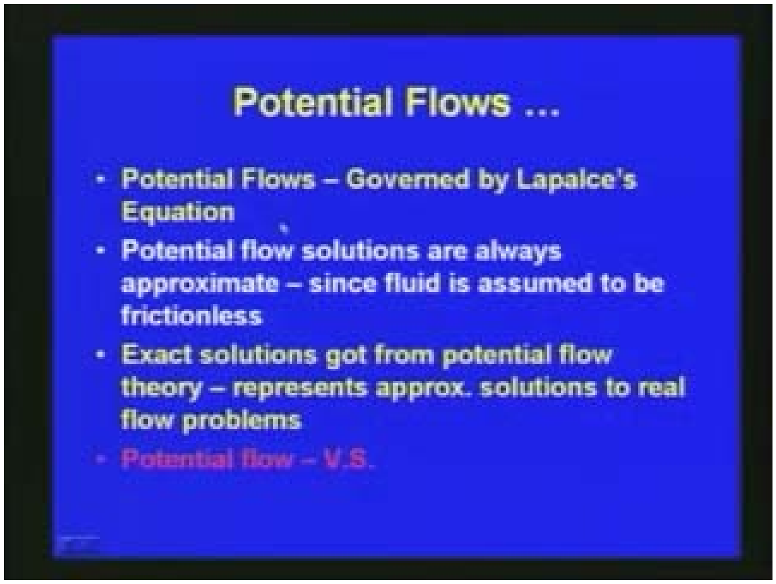

Potential flow is as we have seen it is governed by the Laplace equation. Potential flow solutions are always approximate since most of the fluid, the assumption in potential flow is irrotational, the viscosity is neglected and hence we are assuming as frictionless. Potential flow solutions are always approximates since fluid is assumed to be frictionless. Exact solutions are got from the potential flow theory and represents approximate solutions to real flow problems. Even we can derive the exact solutions for potential flows but as far as real fluid flow is concerned this is an approximation for the reality or the real fluid problem. The exact solution obtained in by using potential flow theory gives only or represents approximate solutions to the real flow problem. So, the potential flow which we have seen here is that we are assuming the flow as potential but the reality is different; the solutions which are derived for the potential flow are just approximation for the real fluid flow.

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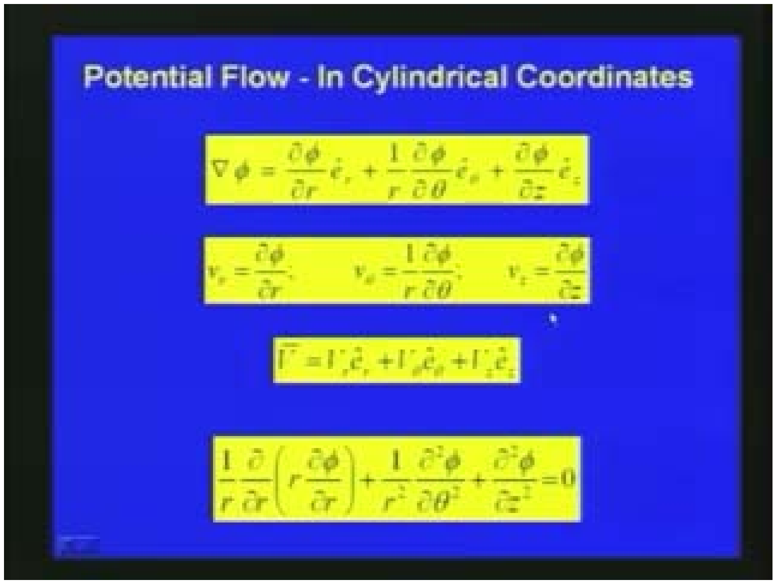

We have already seen the Laplace equation in the Cartesian coordinate system and now for potential flow in the cylindrical coordinate system, we can describe this del phi as shown in this slide here: del phi is equal to del phi by del r e_r plus 1 by r del phi by del theta e_{theta} plus del phi by del z e_z. In the cylindrical coordinate system, we are defining in terms of r, theta and z. We can use the unit vector e_r e_{theta} and e_z and then we can represent the radial velocity as del phi by del r and then the tangential velocity v_{theta} we can represent as 1 by r del phi by del theta and v_z is represented as del phi by del z.

In the cylindrical coordinate system, the potential flow is represented with respect to r, theta and z. This can be represented in terms, here the radial velocity. So, in a cylindrical coordinate system the parameters are described as r, theta and z. The velocity or the parameters can be described in terms, as we have seen for the Cartesian coordinate system, here represents the velocity in xyz as uvw.

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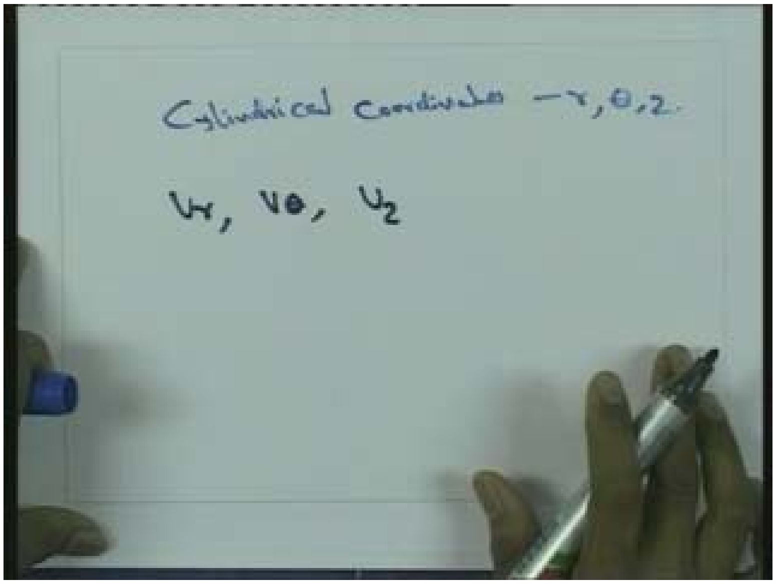

Correspondingly, in cylindrical coordinate system we can represent the radial velocity v_r, the tangential velocity v_{theta} and the velocity v_z which are defined here as v_r is equal to del phi by del r with respect to the potential function; then, the tangential velocity is represent as 1 by r del phi by del theta and the v_z is represented as del phi by del z. Finally, the velocity can be represented as v_r e_r; the unit vectors plus v_{theta} e_{theta} plus v_z e_z and the Laplace equation in the cylindrical coordinate system can be represented as 1 by r del r of r del phi by del r plus 1 by r square del square phi by del theta square plus del square phi by del z square is equal to 0. So this equation represents the Laplace equation cylindrical coordinate system. Some type of problems where we will be using theta in z coordinate system or the cylindrical coordinate system we can use the equation in this particular form.

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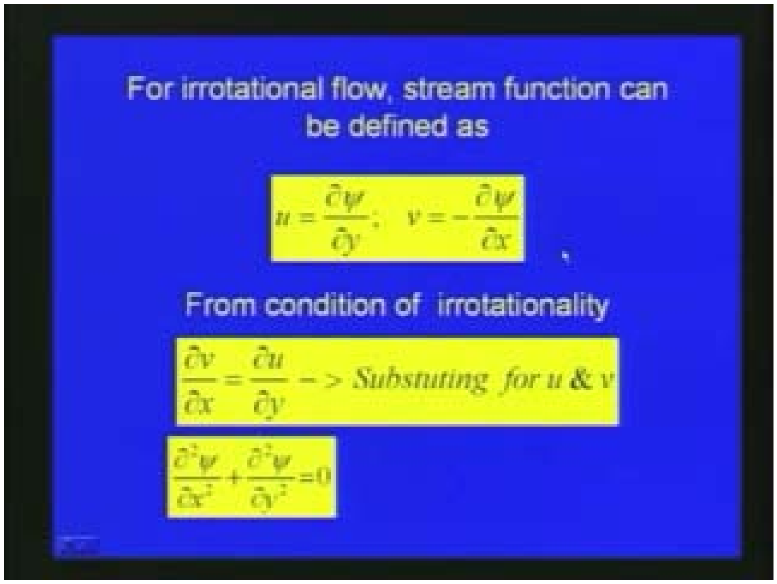

Now, we have defined the potential function; we have defined the stream function here.

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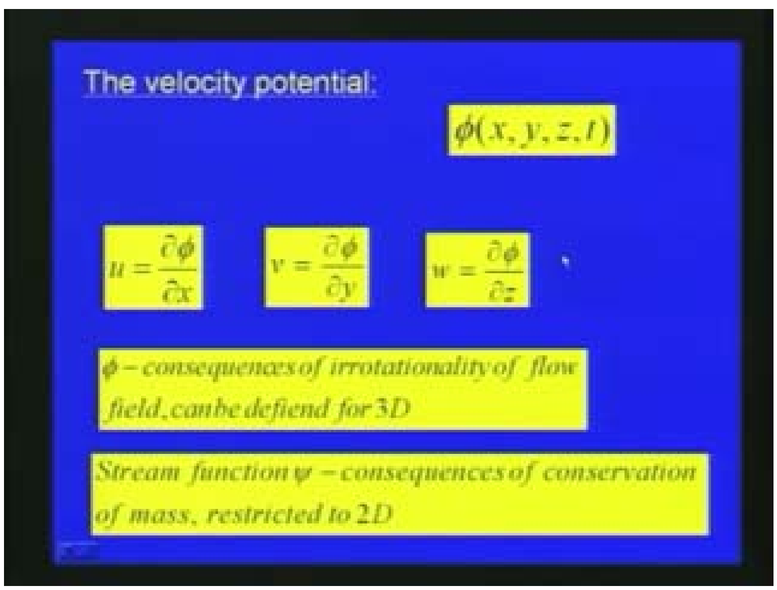

So here with respect to the stream function.

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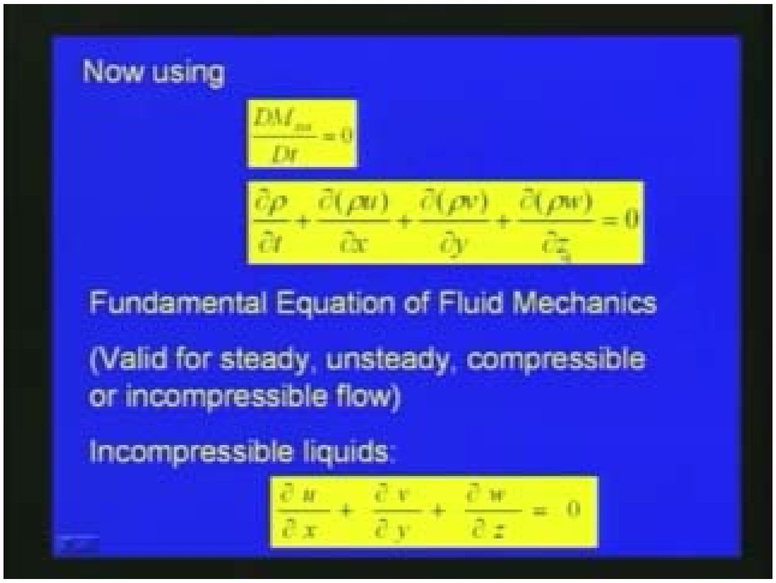

We can represent the velocity for irrotational function flow. We have already seen the stream function can be defined as u is equal to del psi by del y and the velocity in y direction is minus del psi by del x, where phi is the the stream function and then the velocities here are represented in terms of x and y direction.

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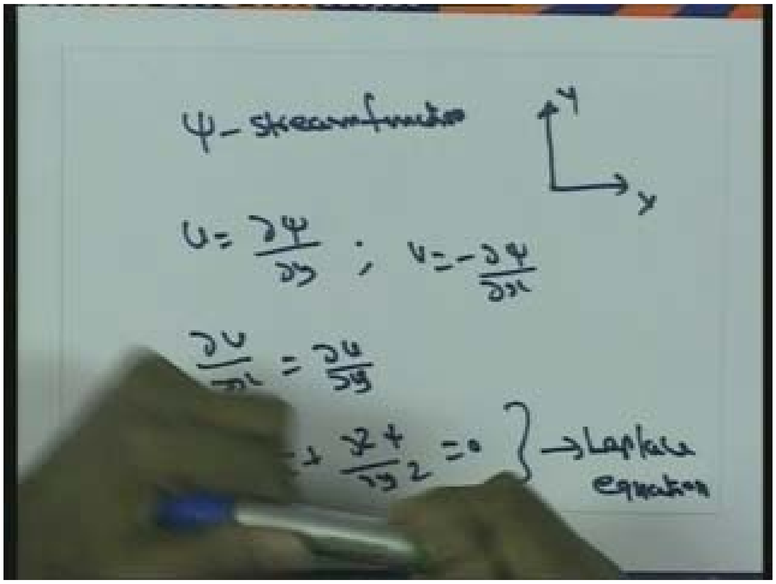

This is x this is y and psi is the stream function. When we come as 2 dimensional flow with respect to stream function we can write u is equal to del psi by del y and v is equal to minus del psi by del x which we have derived earlier. From the condition of irrotationality we can write del v by del x is equal to del v by del y. From the condition of irrotationality which we have seen earlier by substituting by $omega_z$ is equal to 0, we have show that del v by del x is equal to del u by del y. If you substitute for u and v from these equations here as shown then we will get del square psi by del x square plus del square psi by del y square is equal to 0s. This is another form of the Laplace equation derived from the condition of irrotationality and the definition of the stream function. Finally, we get a del square psi by del x square plus del square psi by del y square is equal to 0. This is again Laplace equation in terms of stream function.

Depending up on the problem if we can represent the flow field in terms of potential function or if we can represent the flow field in terms of stream function we can write either of these two equations: if you represent phi then we can write del square phi is equal to the 0 for the potential flow problems; if we can represent the flow field as a stream function then we can use del square psi is equal to 0. Both equations are valid for theorems depending up on potential flow problems depending up on whether you are using the potential function or stream function. The stream function similar to what we have seen in the case of a potential function, we can represent the stream function in terms of cylindrical coordinate system, r_{theta} and z.

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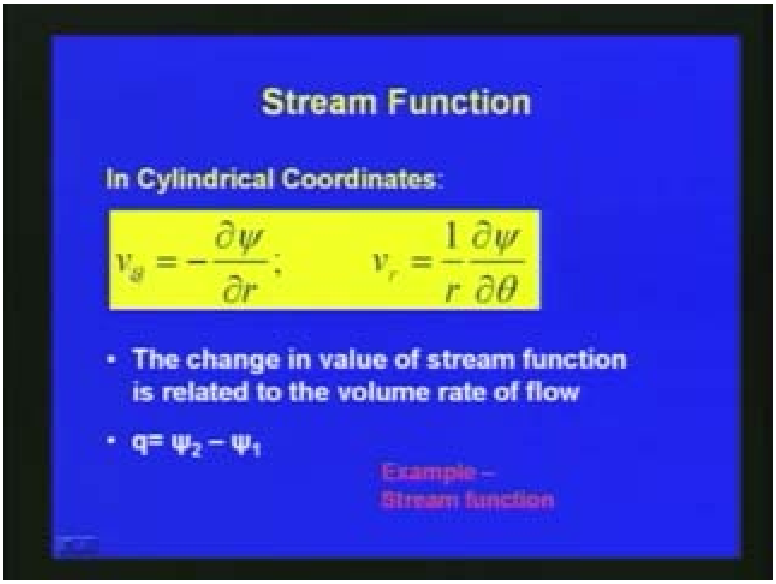

Since z coordinate system is not coming in the case of stream function or since we are considering two dimensions so v_{theta} is equal to minus del psi by del r; this is the definition of the tangential velocity with respect to the stream functions psi. So v_{theta} is equal to minus del psi by del r and v_r is equal to 1 by r del psi by del theta. In cylindrical coordinate system r_{theta} with respect to stream function we can write the tangential velocity as minus del psi by del r and the radial velocity v_r is equal to 1 by r del psi by del theta and change in value of the stream function is related to the volume rate of flow.

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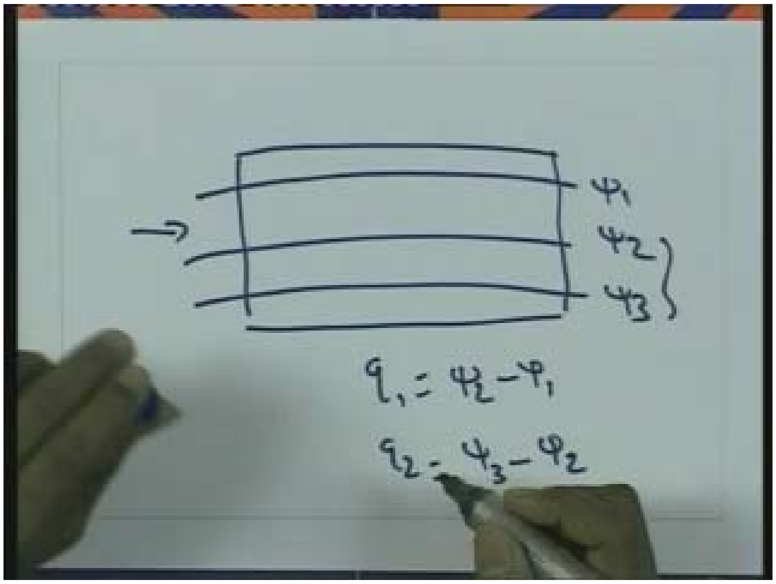

That means if you draw the stream function with respect to stream line, if we consider a flow field like this and then if we can draw the stream lines like this psi_1, psi_2 and psi_3 extra, the stream in change in value thing function from one position to another that represent actually, the volume of …(11:38). This is psi_1 and psi_2, then the volume rate of flow q is equal to between psi_2 and psi_1. One we can write the volume rate of flow is equal to psi_2 minus psi_1, that is, q_1. Similarly, between this psi_2 and psi_3 we can write q_2 is equal to psi_3 minus the stream functions psi_3 minus psi_2. Stream function q is the volume rate of flow can be represented as the difference between this stream function. This potential flow with respect to the stream function we can use to calculate the volume rate of flow between the fluid flows which we will be considering. Now, with respect to this stream function let us consider a small example.

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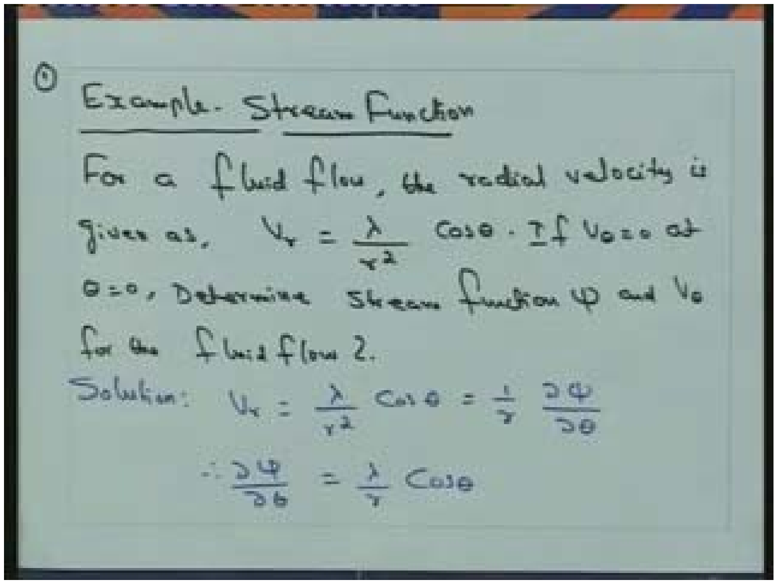

We can see an example for a fluid flow. With fluid flow, radial velocity is given as v_r is equal to lambda by r square cos theta and if v_{theta} the tangential velocity is equal to 0, theta is equal to 0. Determine this two stream functions psi and v_{theta} for the fluid flow?

The problem is the data given are in terms of the cylindrical coordinate system r and theta; the radial velocity is already given v_r is equal to lambda by r square cos theta where lambda is constant and then a condition is given v_{theta} is equal to 0 and theta is equal to 0 we have determine the stream functions psi and then the tangential velocity v_{theta} for the fluid flow.

Already given v_r is the radial velocity equal to lambda by r square cos theta. This with respect to over definition here v_r is equal to 1 by r del psi by del theta, this lambda by r square cos theta is equal to v_r that is equal to 1 by r del psi by del theta or we can write del theta is equal to lambda by r cos theta. Now, we got an expression for psi with respect to theta as del psi by del theta equal to lambda by r cos theta.

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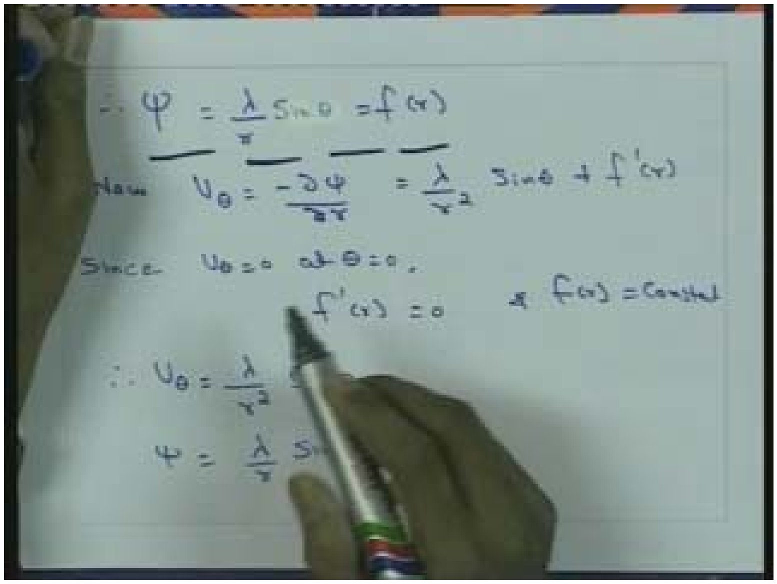

To get the stream function we can just integrate this with respect to theta. If we integrate theta del psi by del theta we get psi equal to lambda by r sin theta f r. So, this stream function is obtained as psi is equal to lambda by r sin theta that is equal to fr. If you want to determine the tangential velocity v_{theta} definition is minus del psi by del r. Here, we have already derived the stream function so we can just differentiate with respect to r to get the tangential velocity. We differentiate the expression with respect to r we get v_{theta} is equal to lambda by r square sin theta plus f dash r which is constant.

One condition is given that the tangential velocity v_{theta} is equal to 0 at theta is equal to 0. We can apply this condition here. So, v_{theta} is equal to 0 so we get fr, f dash r is equal to 0. Finally, we can obtain v_{theta} is equal to lambda by r square sin theta which is the expression asked in the question.

So have found the stream function as lambda by r sin theta and we got the tangential velocity v component v_{theta} is equal to lambda by r square sin theta. As shown in this problem by using these kinds of the polar coordinate system or cylindrical coordinate system we can solve this kinds of problem in terms of the theta, the radial velocity or in times of v_r- the radial velocity or the tangential velocity v_{theta} and with respect to r psi we can determine various functions like v_{theta} v_r are distinct function. This is about the representation of the potential flow with respect to the stream function.

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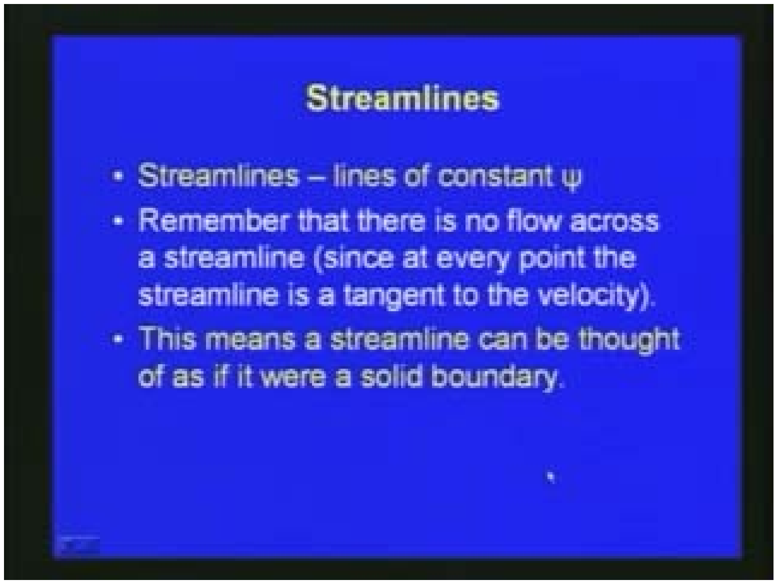

As I mentioned we can represent the streamlines are the lines of constant psi. We can draw like this in the figure shown here (Refer slide Time: 16:30), Psi_1 psi_2 psi_3, these lines are constant and hence these lines are called stream lines. We can see that there is no flow across a stream line, flow is in the direction of the stream line once the stream line with respect to stream lines concept there cannot be a flow across a stream line. So, flow across this stream line is impossible. Since at every point stream line is tangent to the velocity we can say that there is no flow across a stream line. This means that a streamline, we can consider as a solid boundary so that flow cannot cross across a streamline. The streamline concept is very useful to solve many of the fluid mechanics problems especially when we can approximate it is next and as potential flows. The stream lines there cannot be of any flow across a streamline and then the streamline can be considered as a solid boundary where we can consider as a boundary between two various flow problems. So, streamline is defined here.

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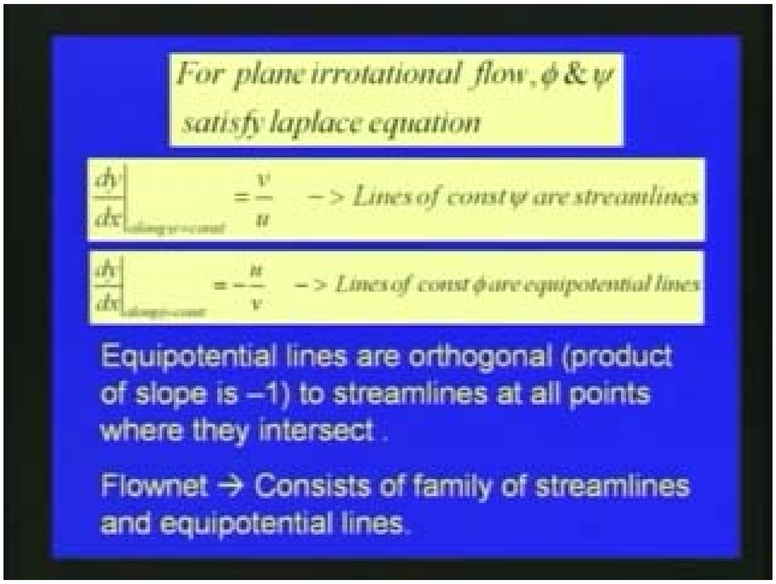

With respect to this streamline and also for the plane of irrotational flow we have defined the potential function and stream function. We have already seen that both satisfy the Laplace equation. So, in Laplace equation which we have derived del square phi is equal to 0, del square psi is equal to 0. So, both del square phi is equal to 0 function and both potential function satisfies the for plane irrotational flow satisfy the Laplace equation.

Now as per our definition if we use the definition of the stream function and potential function we can write its dy by dx along psi is equal to constant and is equal to v by u, that is, the lines of constant psi or which we have discussed the streamlines and then dy by dx along the potential psi is constant that means equal to, as per definition, that is equal to minus u by v, where u is the velocity in the x direction and u the velocity in the y direction. These lines are constant where the potential psi is constant where these lines are called a equipotential lines. With respect to the concept of the potential velocity potential and the stream function we can draw the stream lines where dy by dx along psi is equal to constant which is equal to v by u which are called the stream lines. Then we can draw lines where the potentials are constant or dy by dx along phi is equal to constant as minus v by u. These are lines of constant potential which are called a equipotential lines.

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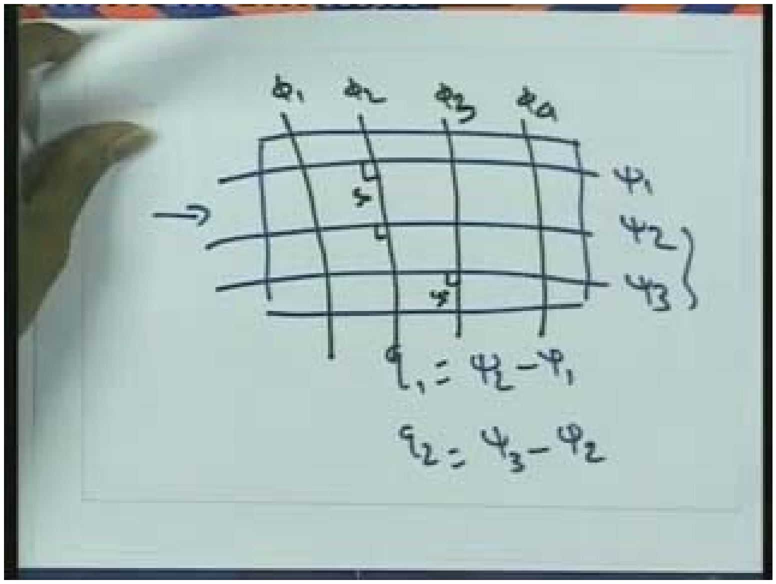

This equipotential lines are when we draw with respect to fluid flow we can see that these equipotential lines are orthogonal to the streamline. So if I draw here you can see that these are the streamlines here. If you plot the potential lines we can see that these are orthogonal to the 90 degree angle. So this is phi_1, phi_2 and phi_3, like that phi_4. We can draw the potential equipotential lines and then the streamlines already drawn here. We can see equipotential lines are orthogonal to the streamlines at all point where they intersect or equipotential lines intersect with respect to the streamlines at 90 degree or they are orthogonal since the product of slope is minus 1. When we draw the streamlines and then when we draw the equipotential lines these lines together consists a family of streamlines and equipotential lines called flow net. This flow net concept is very useful in many of the fluid flow problems. So, flow net consists of family of streamlines and equipotential lines.

We have seen how to draw streamlines with respect to the direction of the fluid flow and we have also draw the equipotential lines; together they are orthogonal or the product of slope is minus 1 and then the family occurs with respect to streamlines and equipotential lines from the flow net.

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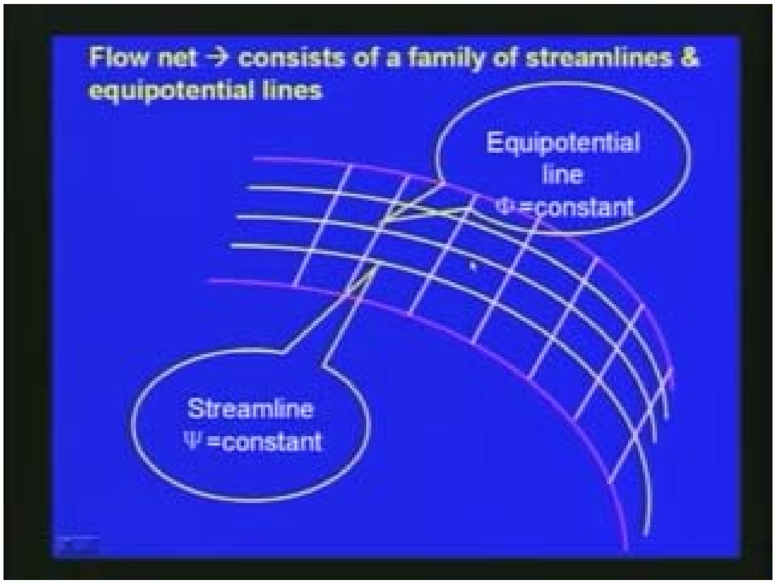

So, here you can see the flow net. Flow net consists of streamlines and equipotential lines; the streamlines are drawn here just like this red color and yellow color and equipotential lines phi is constant is also drawn. So they form the flow net. This flow net concept is very useful in many of the flow net.

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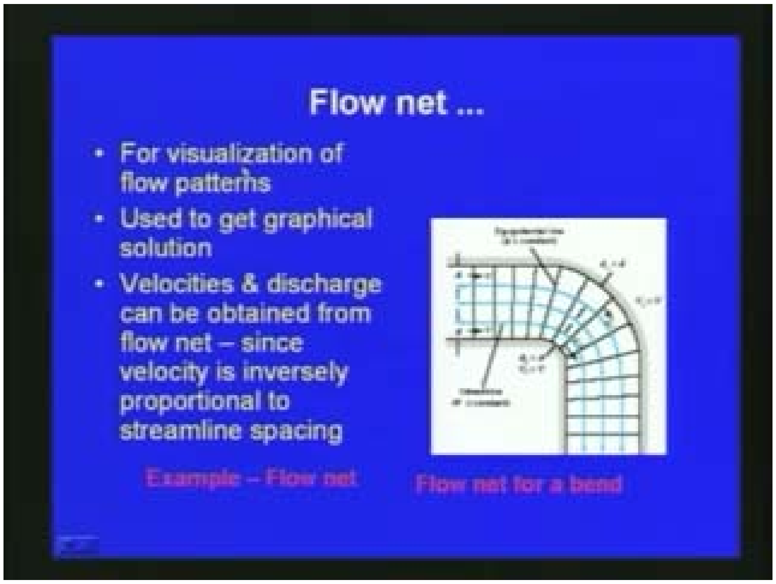

This flow net we can use for visualization of flow patterns as shown here. It can be also used to get graphical solution for fluid flow problems. They can also determine the velocity and discharge can be obtained from the flow net since velocity is inversely proportional to streamline spacing. Here this figure shows a flow net for a fluid flow in a bend so there is a flow through a bend.

The streamlines are plotted here like this and then equipotential lines are also plotted. If we can draw the streamlines and then equipotential lines finally get the flow net we can get a visualization of the pattern as per the fluid flow is concerned especially for potential flows or the flows which we can the approximate as potential flow. Many of the problems like flow through an earth dam we can use this concept.

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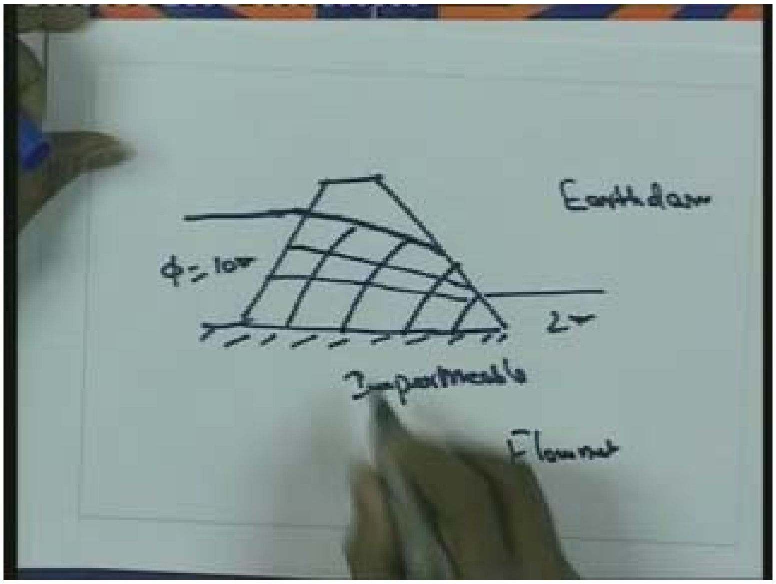

For example, you consider an earth dam like this. In this earth dam, let us assume that it is impermeable. Here, if the head or the potential is 10 meter and here is 2 meter you can see that there is fluid surface and then with respect to this for the earth dam problem. We can draw the streamlines for this and then correspondingly we can plot the equipotential lines which give the flow net. The flow net for an earth dam is drawn here. With respect to this, once the flow net is drawn we can use this flow net pattern to calculate the velocities or discharge since the velocity is inversely proportional to this streamline spacing so this concept of the streamlines or the flow net with respect to streamline and equipotential lines. There are large numbers of applications like in earth dam so flow through a bend, wherever the flow problems can be approximated as a potential flow we can use the concept of the flow net.

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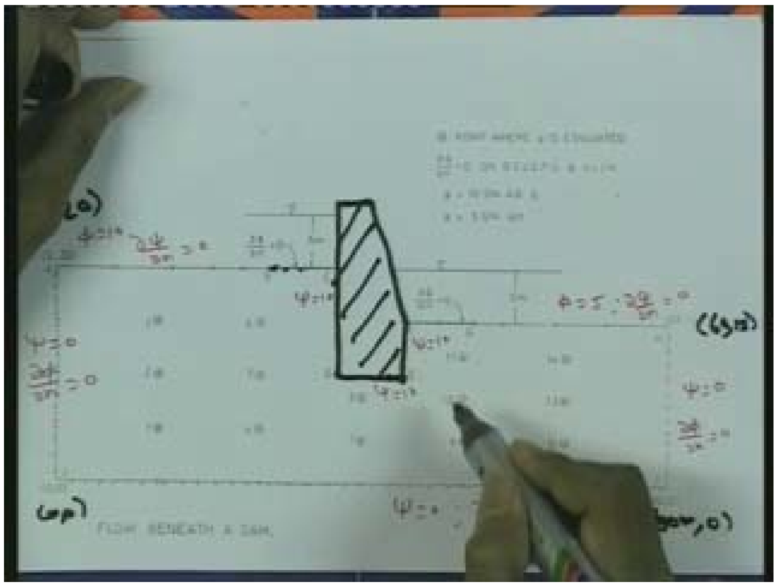

Now, we also consider here another problem with respect to a flow beneath a concrete dam. Here, in this slide we can see there is a concrete dam; this is the dam position and then we are considering a domain 60 meter length, this is 0,0 then 60,0; the depth is 60 by 15 and here 0,20. We are now considering there is a concrete dam on a permeable foundation like this. You can see that here the flow condition, the boundary condition here it is 5 meter depth and here it is the downstream end; it is 5 meter. There is a level difference, here the boundary conditions and here the potential if we consider with respect to this line as datum, you can see that there is a potential meter of 10 meter. So on the upstream side, the flow comes and the potential function phi is equal to 10 meter and downstream side this is the water level. The downstream side phi is equal to 5 meter and since we are considering this boundary at the bottom as impermeable we can say that del phi by del n that means no flux can cross this boundary. So, del phi by del n is equal to 0; here also del phi by del n is equal to 0 and this side also del phi by del n is equal to 0. Also we can assume that stream functions psi is equal to 0, psi is equal to 0 here and here also psi is equal to 0. Then, similarly, here there is a concrete bed and the stream function here, let us assume psi is equal to 10 and on this also del phi by del n is concrete dam; then in upstream there is a concrete bed; in downstream also there is a concrete bed. So del phi by del n is equal to 0 on this phase also. With respect to this problem we can solve now. The equations of the del square phi is equal to 0 we can solve and del square psi is equal to 0. The Laplace equation in terms of phi the potential function and then the Laplace equation terms of stream function also we can solve. In this particular domain, the boundary conditions are given here and then we can determine the phi and psi at various points like this, at various points we can find the potential function and stream functions. Then we can interpolate between two to get the stream lines and the potential lines like in the figure.

So here we have drawn with respect to the problem given here and the boundary conditions and then the use the Laplace equation in terms of phi and then use the Laplace equation in terms of psi del square phi is equal to 0 and del square psi is equal to 0.With respect to this boundary conditions we can get the potential function and the stream function in various locations and finally we can call this stream lines like this.

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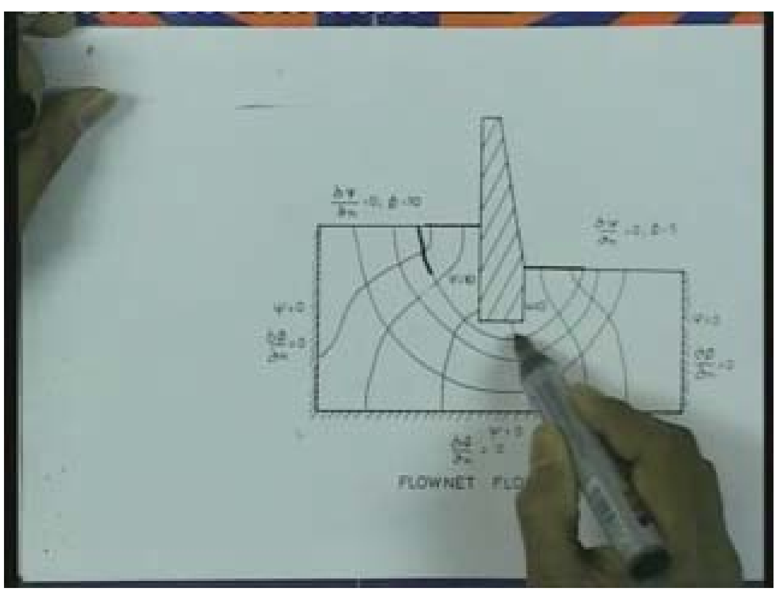

You can see here these stream lines are plotted like this and then the potential functions are also plotted. So, a flow net is formed here for the flow beneath the concrete dam and this can be used to calculate how much will be the discharge. These are the equipotential lines and these are stream lines. So, finally, we get a flow net for this particular problem. This can be used to find the discharge or the flow which can go through the pores media from this place to this place and then finally it will exist at this place. This can be used to find the velocity; these are discharge between with respect to the potential flow equations and potential flow theories.

Potential flow theory has got lag in applications as described in the earth dam here or the flow beneath concrete dam as described here. Further this will be this aspect will be discussed later. Now, after the flow net we will see some of the basic potential flows. The potential flow where the applications directly we can apply Laplace equation and where the simple flow surface and we can call some basic potential flows like uniform flow, like a source since and double x. Since the potential flows are governed by the Laplace equation which is a linear equation we can have number of particular solution and with this particular solution can be added to yield solution for complex problem.

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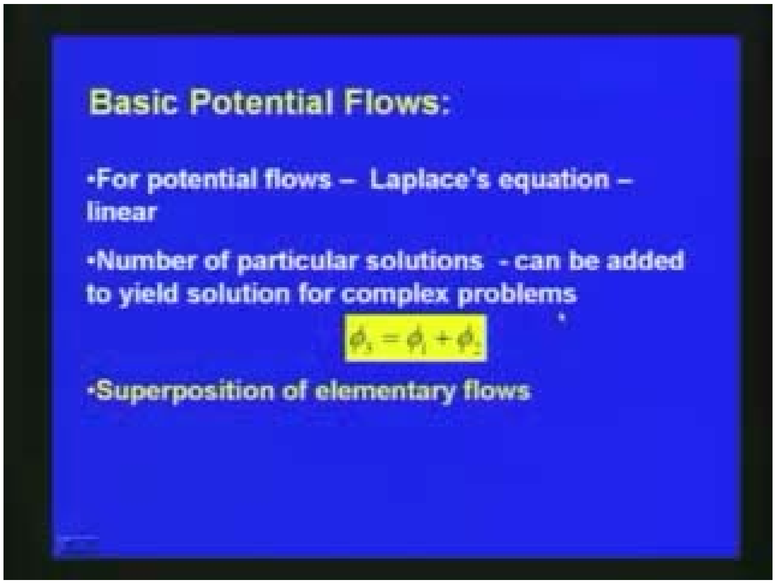

If we have a problem like where the potential functions for example potential function phi_1 phi_2 phi_3 extra are known then since the Laplace equation which we are considered del square phi is equal to 0 or del square psi is equal to 0; this is the linear form of the linear equation. We can superpose or we can add to yield the solution of complex problem. If phi_1 phi_2 phi_3 extra the solution obtained then for various problem we can superpose the problems to the elementary flows like uniform flow, like a flow with source and sink and then we can superpose whether to get complex flow problem. This concept we can use to solve many of the complex problem which can be kindly approximated with respect to the potential theories. This we will discuss before going to the complex problems. You will see the basic potential flows.

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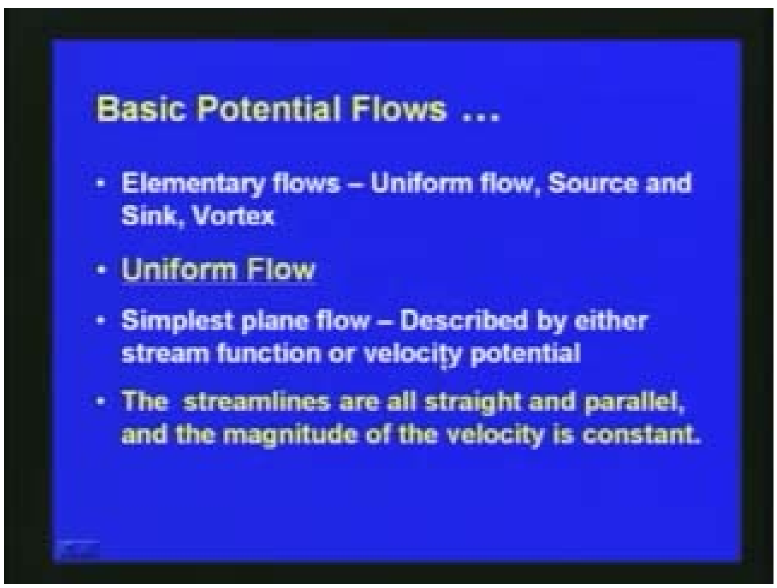

As mentioned elementary flows are uniform flow, source, sink and the vortex. These are the elementary flows which we consider in the potential flows theory, uniform flow source and sink and vortex so each one of this discuss in detail. Now we will discuss the basic potential flows.

First, we will discuss the uniform flow. This is the simplest plane flow described by either stream function or velocity potential. Some of the flows like the ground water flow without the pumping or velocities very low then it can be approximated as uniform flow sometimes depending up on the flow conditions. The uniform flow concept becomes most simple or simplest plane flow where there is no complexity; there are no sources or nothing.

In the uniform flow we can approximate using potential flow theory. This we can either represent using the stream function equation or the potential equation and the stream lines are straight and parallel as far as the potential flow is concerned and the magnitude of the velocities is constant.

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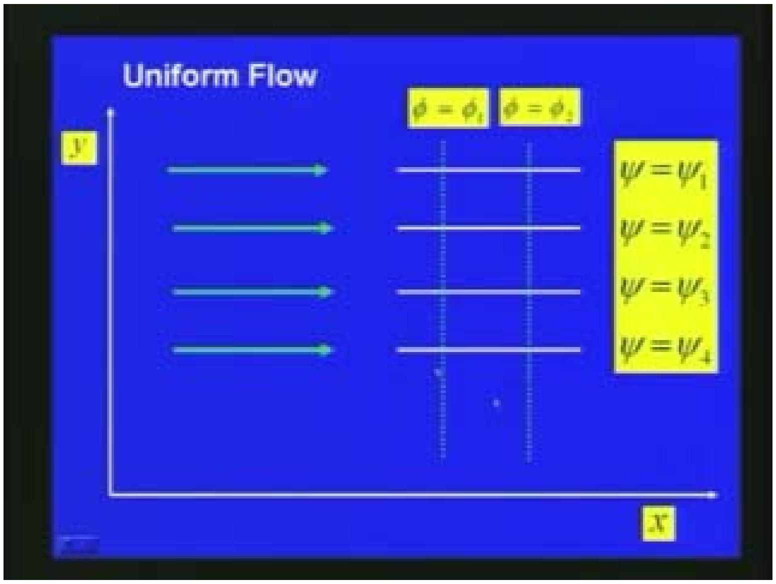

Here, we can see that it can be horizontal or implant but as you can see here the stream lines are straight and parallel so that is the peculiarity of uniform flow and magnitude of velocity is constant. Here you can see the psi is equal to psi_1 or psi is equal to psi_2 and then the streamlines are psi is equal to psi_1 or psi is equal to psi_2 or psi is equal to psi_3 or psi is equal to psi_4 like that. We can represent the uniform flow with equipotential lines and streamlines are drawn as shown in this uniform flow or it can be implanted like with respect to an angle alpha.

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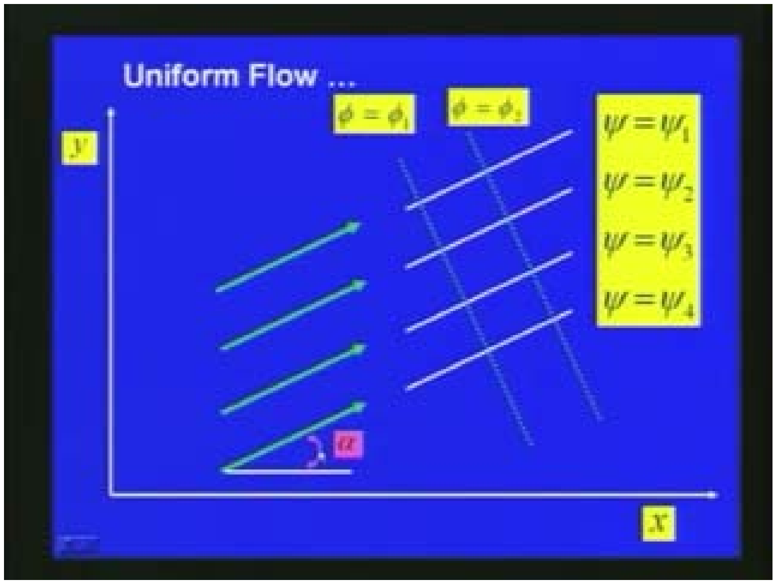

Here also phi_1 and phi_2 represent the equipotential lines and psi_1 psi_2 psi_3 psi_4 represent the streamline.

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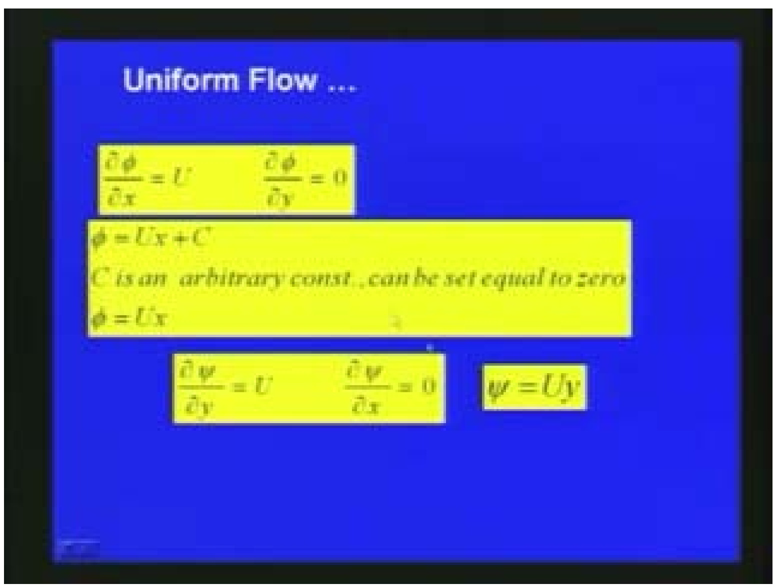

With respect to this now we will use the potential flow theory. As we have represented the velocity in the x direction it can be represented as u is equal to del phi by del x and the for this particular problem del phi by del y is equal to 0 for the case of the uniform flow. So, this is the flow in one direction; we can represent del phi by del x is equal to u and then we can write phi is equal to ux plus c, where c is an arbitrary constant and this can be said to 0. So that for the uniform flow we can write phi is equal to Ux, where U is the velocity the x direction, phi is equal to Ux plus c and c is an arbitrary constant which we said to 0, phi is equal to Ux. So that we can write now del psi by del y is equal to u as per the definition of the stream function; del psi by del y is equal to 0 and also the uniform flow del psi by del x is equal to 0. Since we assume that v is equal to as per the definition of the uniform flow, the stream lines are straight and parallel and the magnitude of the velocity is constant. So, with respect to this we can write that phi is equal to Ux and then psi is equal to Uy. So that del phi by del x is equal to U and del psi by del y is equal to U. Finally, we get the expression for the potential as phi is equal to Ux plus c and the expression for Ux plus c, we said c is equal to 0, we get phi is equal to U into x and psi is equal to U into y. This represents the uniform flow.

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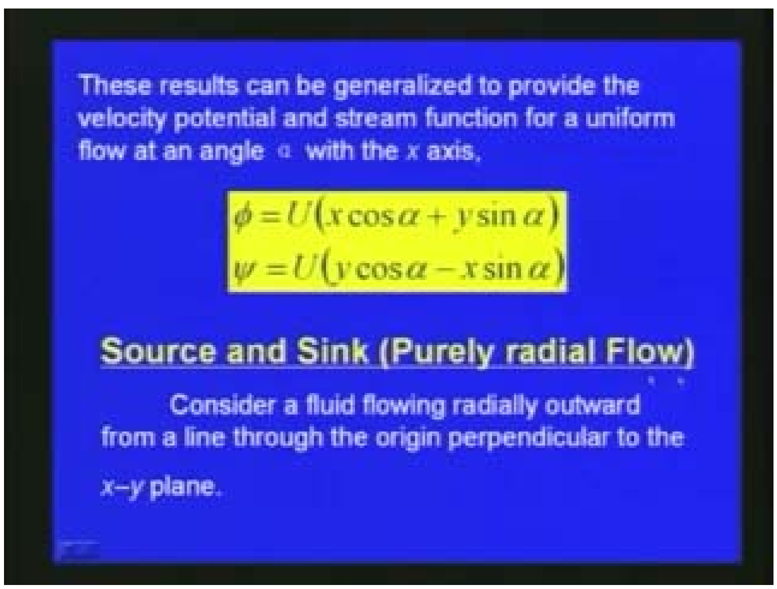

These results can be generalized to provide the velocity potential and stream function for a uniform flow at an angle alpha with the x axis. As shown in the previous slide, these lines are present: phi is equal to Ux and psi is equal to Uy and if you consider certain angle as shown here then we can represent the psi and phi like this. For phi is equal to U into x cos alpha plus y sin alpha and psi is equal to U into y cos alpha minus y x sin alpha

So the results are not generalized, provided the velocity potential and stream function are uniform flow pattern angle alpha. This is called uniform flow and the uniform flow represented with respect to the potential function and then the stream function as phi is equal to Ux and psi is equal to Uy as shown here.(Refer Slide Time: 37:36). phi is equal to Ux and psi is equal to Uy for the horizontal type of flow like this and for inclined type flow it is phi is equal to U into x cos alpha where alpha is the angle plus y sin alpha and psi is equal to Y into y cos alpha minus x sin alpha, this is about the uniform potential flow with respect to the uniform flow which we have discussed. This is the simplest plane flow. As discussed here uniform flow is the simplest flow described by either stream function and we have seen the stream function and velocity potential.

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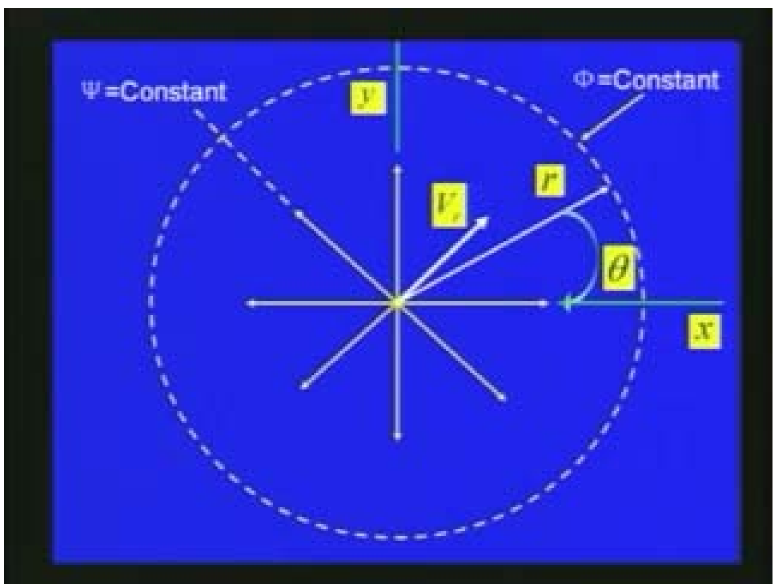

Now, the second kind of the basic or elementary potential flow which we represent is called the source and sinks; the source and sink is purely radial flow type. So the fluid flow is radialy outward from the origin perpendicular to the xy plane only we have to consider the v_r- the radial velocity here and psi is equal to constant. This line represents psi is equal to constant and this dash line represents the phi is equal to constant.

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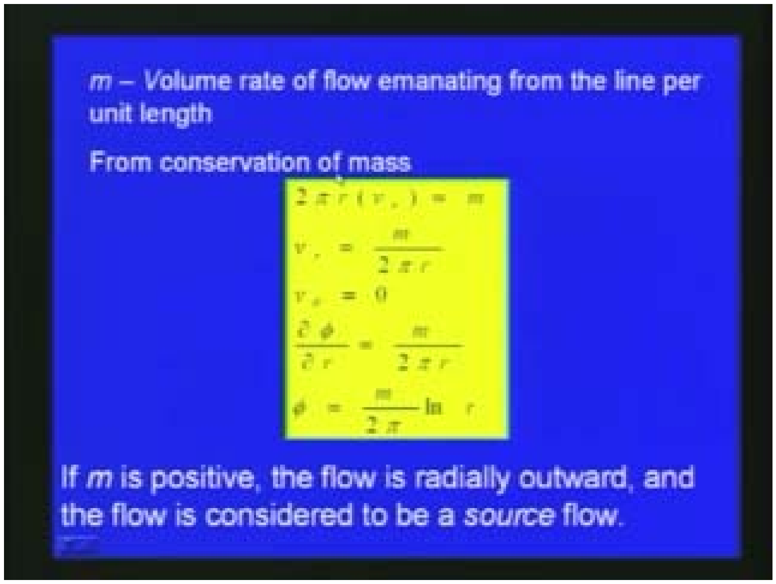

If we consider the flow with respect to an angle theta, we can derive various values of the potential function and the stream function. Let us consider m as the volume rate of flow emanating from the line per unit length as shown in this figure. If m is the volume rate of flow emanating from the line per unit length, if we use the conservation of mass we can write this as 2phi r into v_r is equal to m.

With respect to this figure, we can write m is equal 2phi r into v_r, r is shown here. If we consider the radial distance which we consider here 2phi r into v_r is equal to m where v_r is the radial velocity. Finally, for the considered source sink we can write the volume rate of flow emanating from the line per unit length v_r is equal to m divided by 2phi r. So with respect to this conservation of mass we can write v_r- radio velocity is equal to m by 2phi r and for the potential flow, the tangential velocity is equal to 0. As defined here it is purely radial flow; there is no tangential component for the velocity. So, the radial velocity is equal to m divided by 2phi r and V_{theta} is equal to 0. So that we can write del phi by del r which is the radial velocity component del phi by del r is equal to m by 2phi r. Finally, we can get an expression for phi. The potential function, phi is defined as phi is equal to m divided by 2phi by r. The integration of expression del phi by del r is equal to m by 2phi by r. We get an expression for the velocity potential phi as 2phi by r. So this represent as far the source flow is concerned as shown in this figure the potential function phi is represented as m is equal to m divided by 2phi natural log r, where m is the volume rate of flow emanating from the line per unit length as shown in this figure. So, if volume rate of flow, m is positive the flow is radially outward and the flow is considered to be a source flow.

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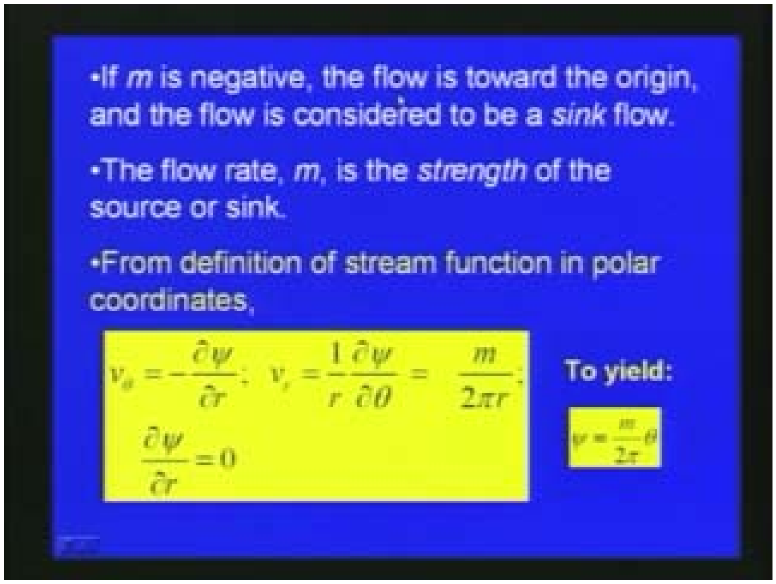

If m is negative the flow is toward the origin and the flow is considered to be a sink flow. This is either with respect to the domain which we are considering what is coming and what is going out; with respect to that particular point we can define if m is positive then it can be the source flow and if m is negative it can be the sink flow. So the flow rate, m is called the strength of the source or sinks. We have considered the source or sink. As shown here whether it can be coming to the domain or going out of the domain. (Refer Slide Time: 42:35). It can be a sink or a source depending up on the case whether m is negative or positive, where m is defined as the volume rate of flow emanating from the line per unit length. This m is called with respect to the potential function phi is equal to m by 2phi natural of r. Here, this m is called the strength of the source or sink. With respect to stream function, we can define this in polar coordinate system in cylindrical coordinate system define theta is equal to the tangential velocity. v_{theta} is equal to minus del psi by del r and the radial velocity v_r is equal to 1 by r del psi by del theta.

With respect to this for the source or sink it is purely radial flow we can that v_{theta} is equal to 0 so that here del psi by del r is equal to 0 or now find the V r is equal to one by r del psi by del theta so this is the radial velocity which we have already seen so this is equal to m by 2phi r so which will give the stream function is equal to m by 2 phi theta.

So finally, we can derive here del psi by del theta. This we can integrate that we will get psi is equal to m by 2 phi theta, this gives the stream function. For a source or sink type flow which is described here, source or sink which are purely radial flow we have derived the velocity potential as m by 2phi natural log r and then the stream function psi is equal to m divided by 2phi into theta, where m is the strength of the source or sink which we considered. Now we got the stream function and the potential function.

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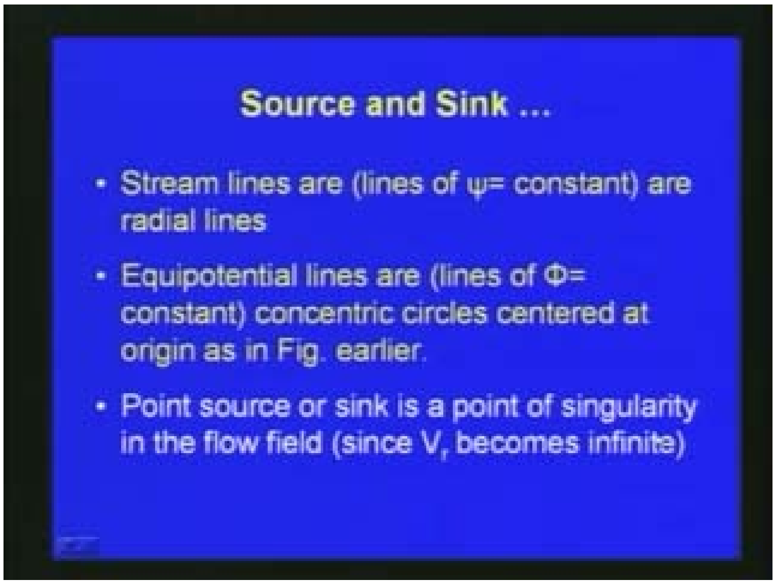

As far as source and sink is concerned which we represent, the stream lines are just radial lines.

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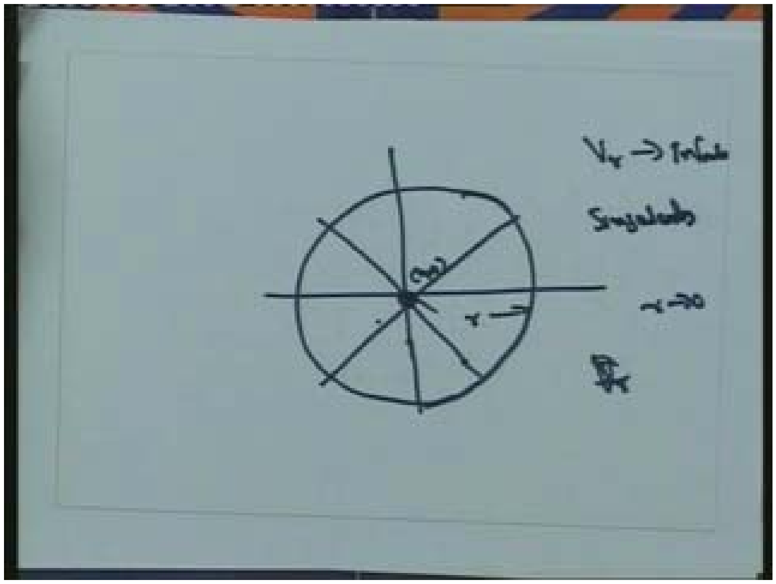

The source the stream lines are radial lines and the equipotential lines are concentric circles centered at the origin as shown in figure. So point source or sink is a point of singularity in the flow field. Since the radial velocity, r is defined like this is the origin so from origin r tends to 0 the sources strength is put on the origin. So, when r tends to 0 we can see that the radial velocity V_r radial velocity v will be tending to become infinity as per the definition. We can see singularity may occur at the point source of singularity so V_r is defined as m by 2phi r, when r tends to 0 as shown here you can see that V_r becomes V_r tending to infinity. Then source or sink point source become a singularity, the point of singularity where r is tending to 0. V_r becomes infinity which is called a singular point or sink is we can represent as a singularity. So this concept of source and sink on applications especially we consider the point media flow so sometimes there are not much complexity. For the flow in media is homogeneous, isotopic cases we can consider. If there is a recharge well or there is a plumbing well application of the potential theory we can approximate and then we will try to solve the problem with respect to the pores media flow. These are some of the applications which will be discussing later. With respect to these now point sources is singularity as discussed and now the third one is so called the vortex flow.

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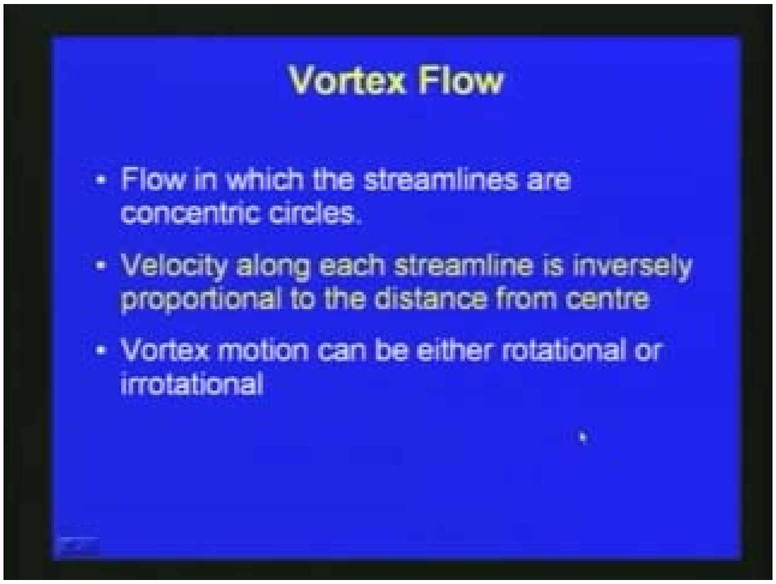

The vortex flow: it is the flow in which the streamlines are concentric circle. First we discussed the uniform flow; second one, we discussed the source and sink and third type of basic or elementary potential flow is called the vortex flow. The vortex flow is the flow in which the streamlines are concentric circles and the velocity along each streamline is inversely proportional to the distance from the center. This kind of flow is called a vortex flow and the vortex motion can be either rotational or irrotational.

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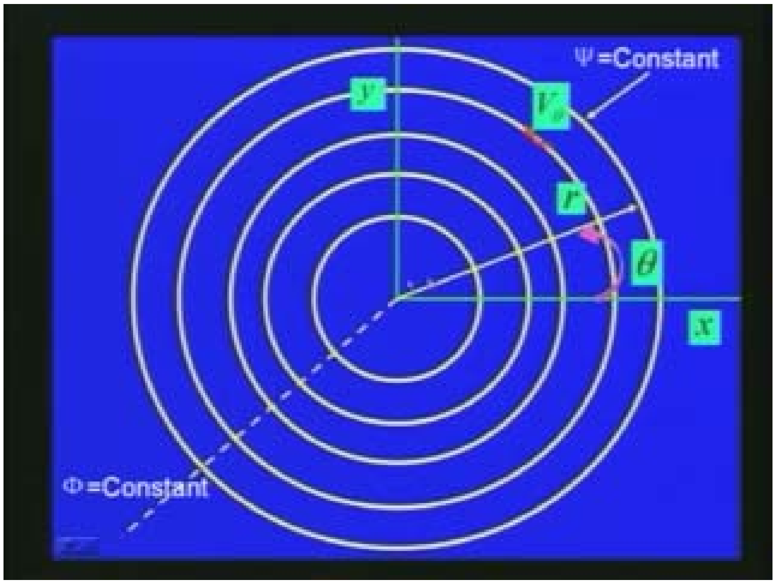

This represents the rotational type which we have discussed here; the velocity, the flow, streamlines are concentric circles and velocity along each streamline is inversely proportional to the distance as shown in this figure. So, phi constant is the dotted lines here and these lines represent the psi constant lines.

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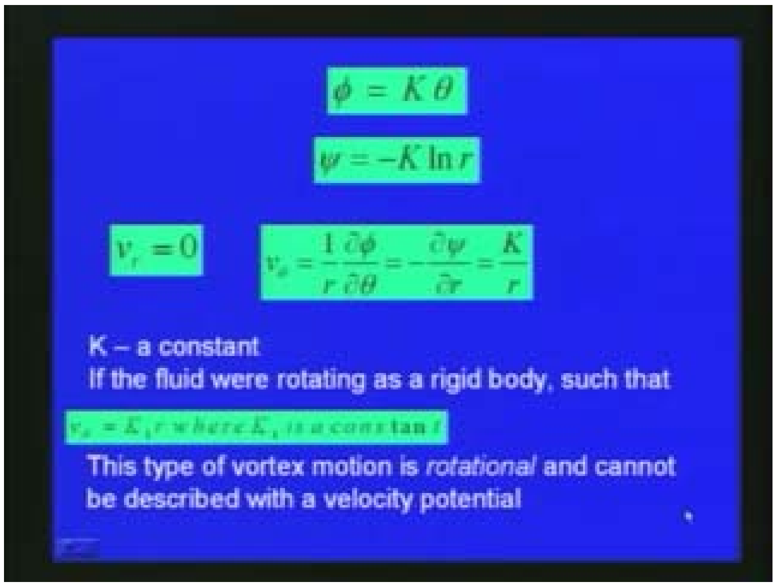

Then we can define the potential function with respect to as phi is equal to K theta, where K is a constant, theta is show in this figure. So phi is equal to K theta and psi can be represented as psi is equal to minus K natural log r. Finally, in this case, vortex flows V_r is equal to the radial velocity component is equal to 0 and v_{theta} is defined as 1 by r del phi by del theta which is equal to minus del psi by del r. This is equal to v_{theta} is equal to minus K by r. For vortex flow we define phi is equal to K theta; the potential function for vortex flow is defined as phi is equal to K theta and psi is equal to minus K natural log r, where K is the constant. If the fluid were rotating as a rigid body has already shown earlier v_{theta} is equal to K_1r, where K_1 is a constant.

This type of vortex motion is rotation and cannot be discussed with respect to velocity potential; the vortex flow can be either rotational or irrotational.

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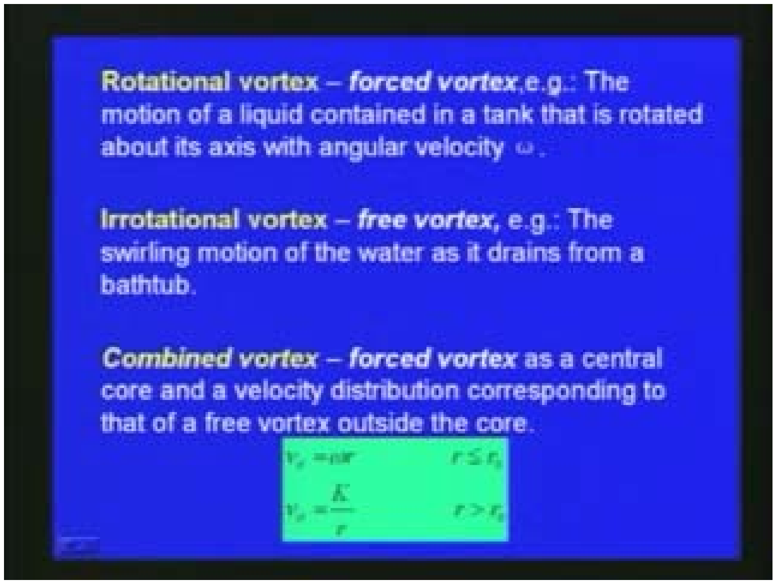

We call the rotational vertex as forced vortex, for example, the motion of a fluid or the motion of a liquid contained in a tank that is rotated about its axis with angular velocity omega. This is called a rotational vortex. Then irrotational vortex, free vortex, for example the swirling motion of the water as it drains from a bathtub. This represents the irrigational vortex.

The irrotational vortex is called the free vortex and for example swirling motion of the water as it drain from a bathtub is called irrotational vortex and rotational vortex is called the forced vortex so the motion of the fluid contained in a tank that is rotated about its axis with angular velocity omega so this is called rotational vortex and also we can have combined vortex. The combined vortex is the forced vortex as a central core and a velocity distribution corresponding to that of a free vortex outside the core. We can write v_{theta} is equal to omega into r, where r is less than are equal to r_0 and v_{theta} is equal to K by r as represented earlier; v_{theta} is the tangential velocity and K is the constant which we considered. The vortex flow can be rotational vortex which is called forced vortex or it can be irrotational vortex called free vortex or we can also have combined vortex which is called forced vortex as defined. So this is about the vortex flows.

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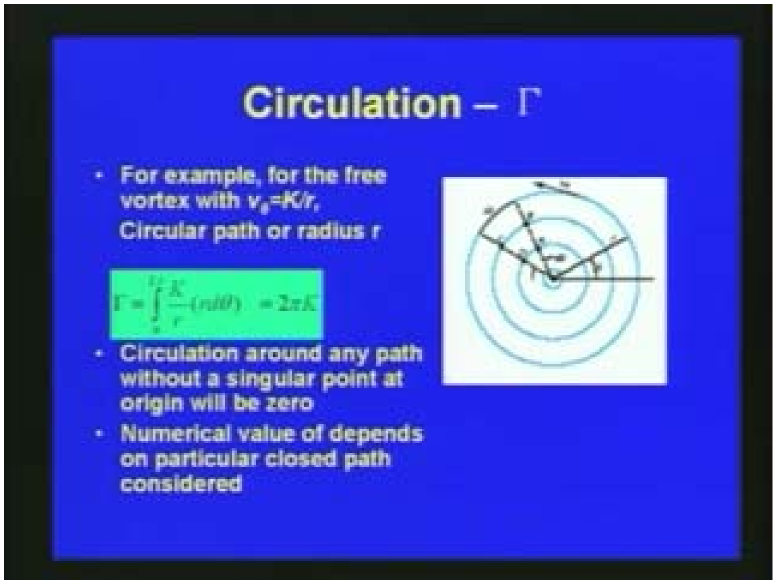

Further, we will be discussing about the circulation with respect to the vortex flows and then we will be discussing the doublet; then the combination of all the elementary basic flows to represent complex flow system which can be in certain places we can apply for the real fluid flow problem. So, this will be discussing in the next lecture.