

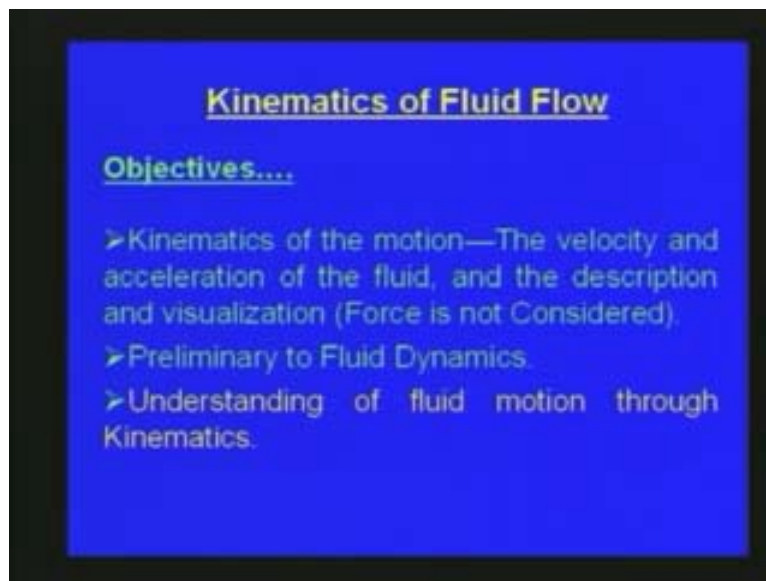
Fluid Mechanics
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Lecture 6
Kinematics of Fluid Flow

Welcome back to the video course on fluid mechanics. In the last few lectures we are discussing about the introduction theories of fluid mechanics introduction to various fluid properties, we discussed about fluid statics its related theories, we have discussed about buoyancy pressure fluid pressure center of pressure met center extras. Today we will discussed the another topics fluid kinematics.

The main object is of the topic on kinematics of fluid flows to introduce various aspects of fluid motion without being concerned with the actual forces necessary to produce the motion. The other object is of this topic are to introduce the kinematics of the motion the velocity and acceleration of the fluid the description and visualization without considering the force.

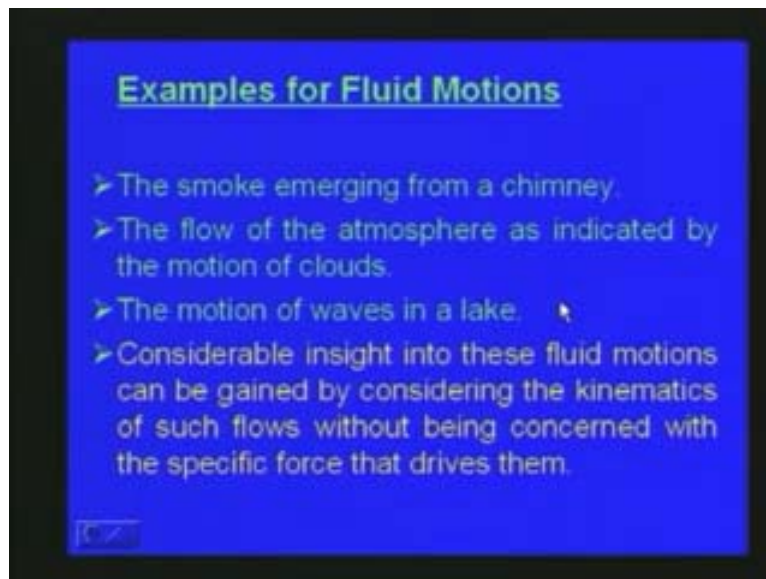
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we will introduce the fluid dynamics and finally to understand the fluid motion through the kinematics fluid flow.

That is the important object that is of the section on kinematics of fluid flow. As we have already discussed the kinematics of fluid flow means the fluid flow is taking place due to various forces. But when we consider the fluid motion here in this kinematics of fluid flow consider the forces as such but without considering the force what happens to a fluid or how the fluid is moving how the velocity can be calculated how the acceleration can be calculated, we will go to the various the principles of fluid mechanics like consideration of mass consideration of momentum based up on the fluid kinematics. If you consider the various examples of fluid motions,

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Like smoke emerging from a chimney or the flow of the atmosphere as indicated by the motion of clouds or the motion of waves in a lake. All this fluid motions here generally when we try to analyze all this fluid motion there can be consider by inside to this fluid motion by considering the kinematics or such flows without the concern with the specific force that drives them. The smoke emerging from the chimney there is force which is taking place which is driving this smoke or the wind on the atmospheric motion or the motion of clouds and or the waves in the lake all there are same forces but we the

analyses be much simpler when we consider this fluid phenomena or fluid motion without much concern to the specific force that derive them.

That is the importance of this kinematics of fluid flow. we will be considering the section the kinematics of fluid flow with respect to this various fluid motions without giving much attention to the force driving this the driving force which we are which due to this the fluid is moving. We considering the various aspects of kinematics of fluid flow starting from the velocity field we discuss the Lagrangian and in concepts we will discuss about the acceleration fields etcetera. The fluid parameters,

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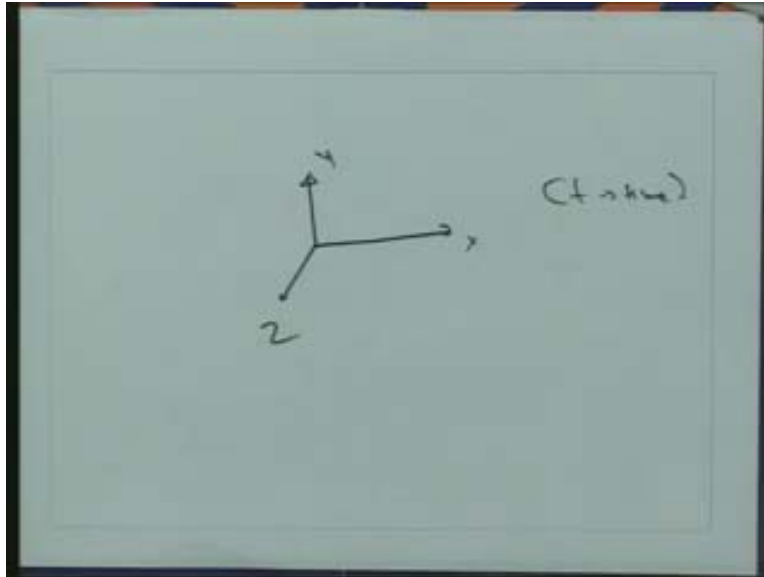
Fluid parameters by field representation } → Spatial Coordinates and time.

Eg. Velocity Field

$$\vec{V} = u(x, y, z, t)\hat{i} + v(x, y, z, t)\hat{j} + w(x, y, z, t)\hat{k}$$
$$V = |\vec{V}| = (u^2 + v^2 + w^2)^{1/2} = \text{Speed of flow}$$

By field representation as I mentioned earlier the fluid moment is then you consider the fluid moment it the moment is in with respect to space as well as time. If you consider the space we consider the 3 dimensional motion of a fluid with respect to xyz axis time t will be al will be there.

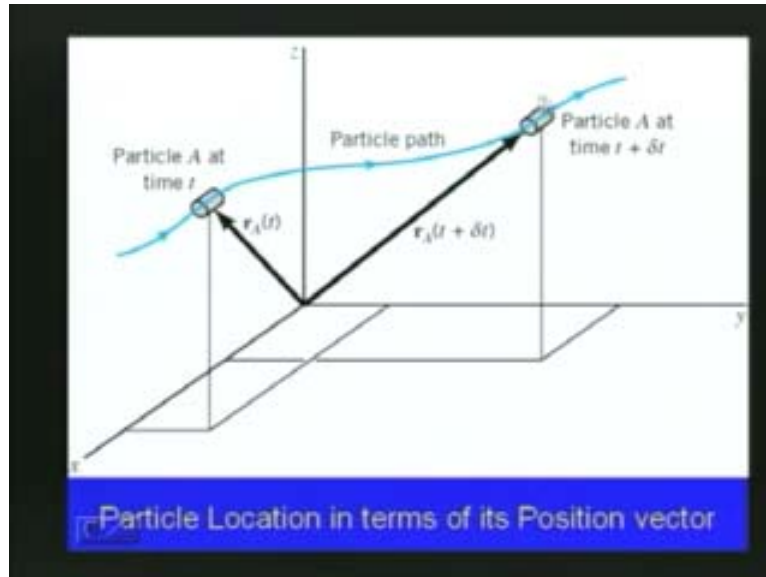
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The fluid motion is with respect to space with respect to time the fluid parameters are generally represented by the space the space coordinate xyz time t . for example the as shown in this figure here the fluid is moving in a pipe or in an open channel when the fluid is moving. The motion is with respect to space with respect to time that we can represent that show in this figure the velocity fluid can be represented as v is equal to U xyz of i the unit vector plus V xyz of the unit vector j plus w xyz the unit vector k .

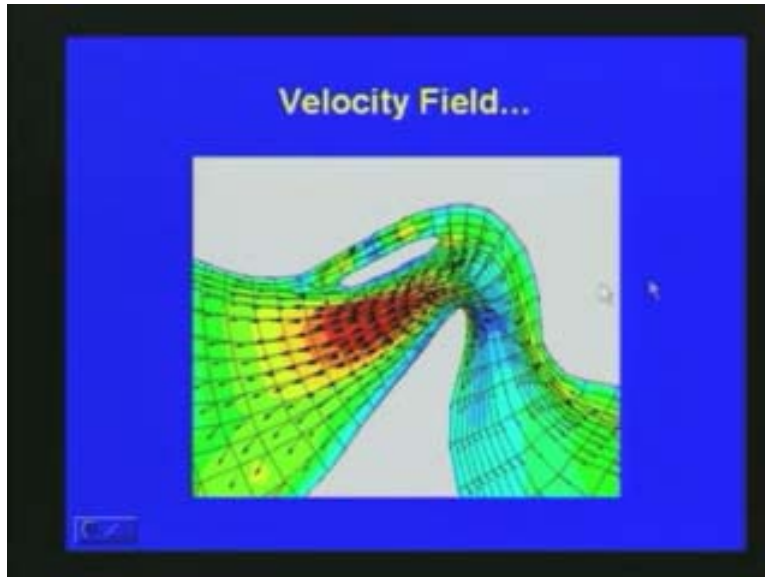
Since the velocity is it has got a direct direction as well as magnitude that we represent a early with respect to i , j and k which are the unit vectors U here uvw are the magnitude of the velocity moment and finally the velocity field can be represented as v is equal to ui plus vj plus wk where, uvw are depend up on the spatial coordinate xyz and time t . Finally, the speed of flow we can represent when we are considering a pipe flow or when you are considering an open channel flow or any kind of flow we can represent the speed of flow as it will be the absolute value of this v the it can be represent as square root of u square plus v square plus w square as shown in this slide. In the speed of flow is square root of u square, plus v square plus w square that use the speed of flows. The velocity field is the generally represented as the function of spatial coordinate xyz and time; here you can see that a particle is moving.

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From a position A to another position at time t position is here and the time t plus delta t the particle is moving from this position to the new position. The position vector if you draw the position vector. This $r_A(t)$ is the initial position like vector $r_A(t + \Delta t)$ is the present position on the fluid particle. With respect to this the particle is located in terms of the position vector with respect to this only we want to describe the moment of a fluid particle in any fluid media. When we represent the velocity here this position will be with respect to xyz coordinate and time the current time here, the new position will be all represented in terms of the new position xyz time t, the particle location in terms of the position vector.

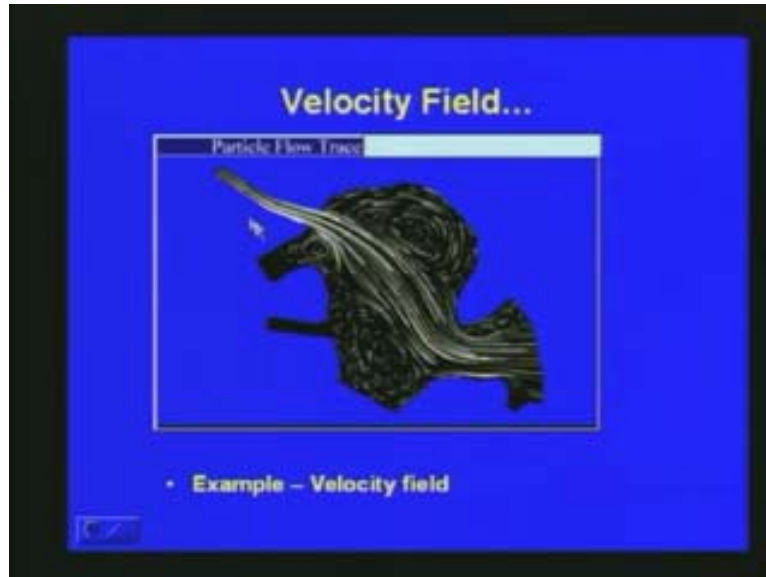
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Here in the slide we can see that velocity field is for example, in a river there is a flow type space here you can see that the flow is with respect to various flow conditions the flow can be this direction of the other direction. The flow velocity vectors the velocity vector is we can represent with respect to spatial coordinate xyz and also with respect to time. Here, this figure shows the actually this figure here theme contamination or theme glom is introduced in this river system how it is moving with respect to that generally, we will be solving the for the this kind of problem solving first the hydro dynamics the velocity we will be determining then with respect to the that the velocity. We are describing the contaminant moment or the glow moment will describe with respect to the velocity vector.

It is very important that we should note the velocity field that means with respect to space and with respect to time how the velocity is changing that is very important that is actually the major problem. As for as a fluid dynamics or the fluid flow is concerned, the velocity field of the velocity determination with space and time is very important. As you can see in this slide also the velocity field we can with respect to article float race.

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We can trace as shown in this figure we can trace the particle how it is moving with respect to the path line or with respect to either stream line we can represent the velocity the flow how it takes place we can represent as shown here this is actually a river system joining an [10:46]. How it is behaving with respect to the velocity field we can represent with respect to the path lines then stream lines and as shown in the previous slide we can also use the velocity vectors to represent the velocity field. As I mentioned here we will just discussing brief example problem how to determine the velocity field, as I mentioned since the velocity varying with respect to xyz direction. That we will determine the velocity field with respect to the xyz coordinate as well as the time the problem here is the example problem here is.

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①
Example: Velocity Field
The velocity in a flow field is expressed as:
 $\vec{v} = (7t + 2xy)\hat{i} + (-2yz - 4t)\hat{j} + (-yz + \frac{z^2}{8})\hat{k}$
Determine the magnitude of velocity at a point Q
(x=3, y=-1, z=2, t=3).

Solution: $\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$
 $\therefore u = 7t + 2xy$; $v = -2yz - 4t$; $w = -yz + \frac{z^2}{8}$
At Q (x=3, y=-1, z=2, t=3)
 $u_Q = 7(3) + 2(3)(-1) = 15$; $v_Q = -(2)(-1)(2) - 4(3) = -8$
 $w_Q = -(-1)(2) + \frac{2^2}{8} = 2.5$; $\therefore V_Q = \sqrt{u^2 + v^2 + w^2} = \sqrt{15^2 + 8^2 + 2.5^2}$
 $= \sqrt{225 + 64 + 6.25} = \sqrt{295.25} = 17.18$

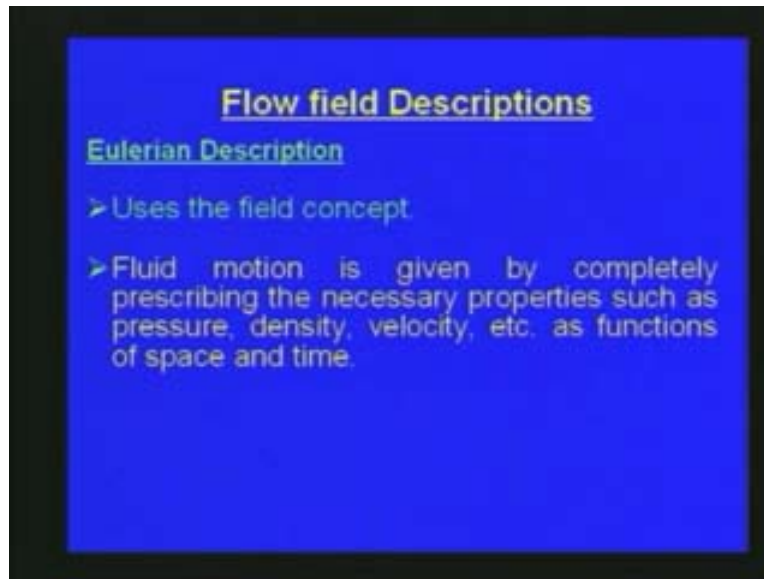
The velocity field the velocity in a flow field is expressed as \vec{v} is equal to the velocity vector is equal to 7 plus 2 xy I the unit vector I plus minus 2 yz minus 4 t unit vector j plus minus yz plus z square by 8 unit vector k. This is the velocity field is represented by this function given with respect to I j and k. The magnitude is magnitude xyz directions are represented by this the quantities the x direction that means velocity component u if is given by 7 t plus 2 xy and the velocity component is implied and direction v is given by minus 2 yz minus 4 t and the velocity component in z direction w is represented here as minus yz plus z square by 8. We want to determine the magnitude of velocity at a point q the position of q is x equal to 3 y is equal to minus 1 z is equal to 2 and at time t is equal to 3. The coordinate system s xyz is given here and time al given we want to determine the magnitude of the velocity at a point x equal to 3 y equal to minus 1 z is equal to 2 a time t is equal to 3. Here this function the velocity field is represented as already given by this v is equal to this function.

Do this we can represent as a vector we have discussed the velocity can represented as the velocity vector uvw and with respect to the ijk as ui plus vj plus wk where, ijk are the unit vectors in a xyz direction and uvw are the velocity in xyz direction. Here v is represented as I mentioned u is here with respect to the given equation for velocity for u is 7 t plus 2 xy and v is minus 2 yz minus 4 t and w the velocity z direction is minus yz plus z square by 8.

The position of the point is given x equal to 3 y is equal to minus 1 and z is equal to 2 and time t is equal to 3 that we can determine the magnitude of velocity in x direction u_q we will substitute values here in for u . We can write u_q is equal to that means the velocity component u at point q is 7 into time 3 t is equal to $3 \cdot 7$ into 3 plus 2 into 3 into minus 1 . that will give 15 as the result here u_q is equal to 15 v_q the velocity in y direction at point q is given as v_q is equal to minus here the values minus 2 y minus 4 t^2 into minus 1 since y is minus 1 in time is t z is 2 minus 2 into minus 1 2 plus time is 3 , 4 , 3 that will give the q as minus 8 similarly we can determine w_q . w_q is given as minus yz plus z square by 8 that is if you substitute for y and z y is minus 1 minus of minus 1 into 2 , z is 2 plus 2 square by 8 that will give the value of 2.5 .

Finally, as we have seen the velocity can be the magnitude of the velocity at the point q can be determined by taking square root of u_q square plus v_q square plus w_q square. Here the u_q the magnitude of the velocity equal to square root of 15 square plus minus 8 square plus 2.5 square that is equal to square root of 295.25 and that is equal to 17.18 . Its corresponding units can be put here the meter per second or a numeral second depending up on the problem. like this the velocity can be represented in xy with respect to the space xyz time, if you want to determine the new position of the velocity if the function the velocity variation as represented in this figure is known then we can determine the magnitude of the velocity. This is the velocity field here the velocity can be represented with respect to space and time. Like this the velocity field is determined.

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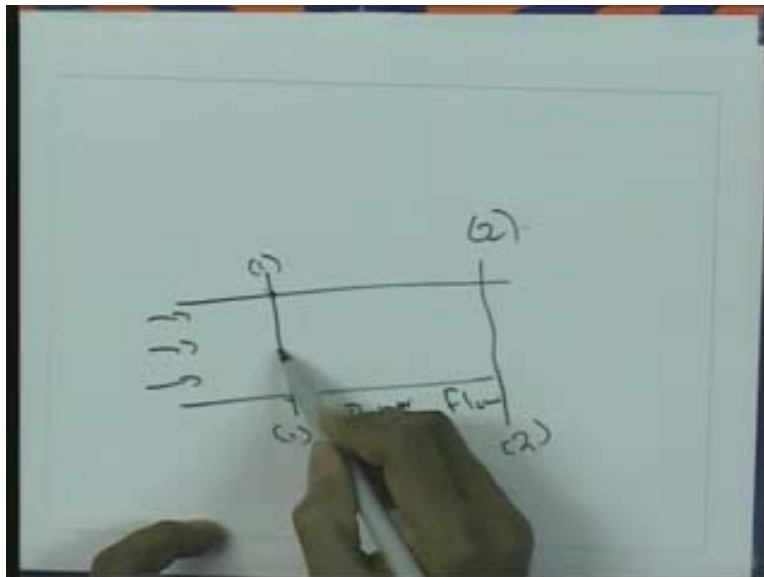


Now we will discuss about the flow field description. In the introductory lecture we have seen the various aspects of how a flow field can be represented as there we discussed about two types of fluid flow field description. First one is eulerian description second one was legrangian description. Here further we will discuss since we are discussing the kinematics of fluid flow further you will discuss the two description fluid flow field description the first one is the eulerian description. As we have seen earlier the eulerian description uses the field concept as we have already seen the velocity field or the acceleration field or the pressure field whatever it is. Here the eulerian description uses the field concept and fluid motion is given by completely prescribing the necessary properties such as the fluid property such as pressure density velocity extra as functions of space and time.

As we have already seen the velocity here is represented as a function of xyz coordinates and time t . In the eulerian description the fluid motion is given by prescribing the necessary properties which can be velocity or pressure or density whatever it is as a function of this and time. if you use the eulerian description from this method you obtain the information about the flow in terms of what happens at fixed points in space at the fluid flows pass those points. What will be doing same if you consider the fluid motion in an open channel or pipe flow for example let us consider river flow here.

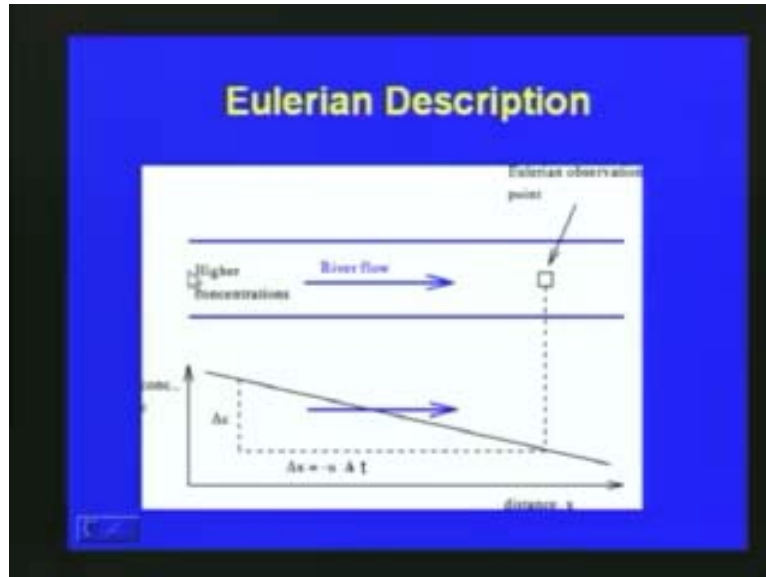
This is a river flow what we are doing here is with respect to flow is taking place all the times. We will be considering particular sections here one or section two and between the sections one and two to what happen to this various fluid flow properties like velocity how it is changing or how the pressure is changing or the various other fluid flow a parameter are changing. That is what we are generally describing in eulerian description generally using most of the fluid flow problems as we are discussed earlier we will be generally using eulerian description. From this we are getting the information as the flow progresses in terms of what happens fixed point here the fixed points which are considering this section one.

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Or even it can be a fixed point or it can be a section like this with respect to this happens for the fluid properties when the fluid flow is moving from one section to another or at that particular section with respect to time what happens to the fluid flow. That is the way which we will be describing the fluid flow in the eulerian and eulerian description. Here you can see in this figure here a river flow takes place and.

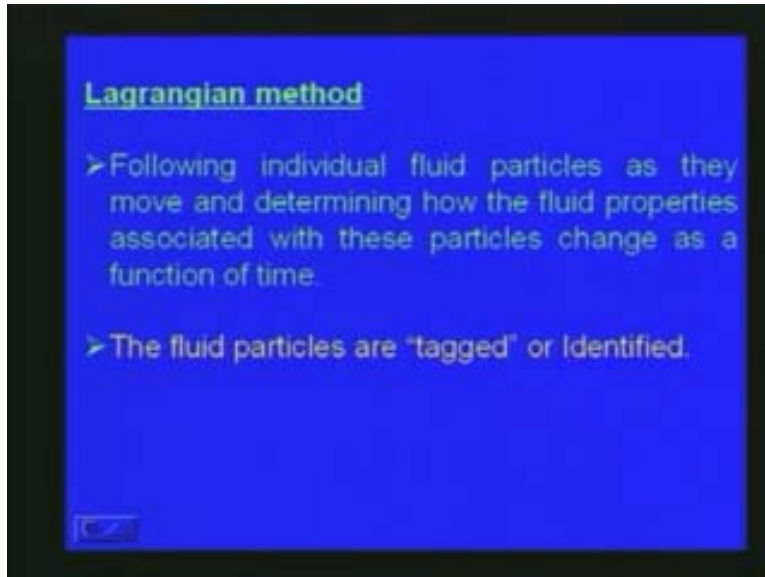
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The concentration higher concentration or the contaminant glom is used at his position we want to see in the eulerian description what we can d is there can be a acceleration point as shown in this time there can be observation point like this. Then what will be describing is when the river flow takes place with respect to time will be taking what happens whether the contaminant the concentration how it is vary with respect this particular section with respect to time how it is varying. If you brought this for you have particular position then you have see the contamination the concentration will be reducing as the flow proceeds like this graph shows how it will be behaving.

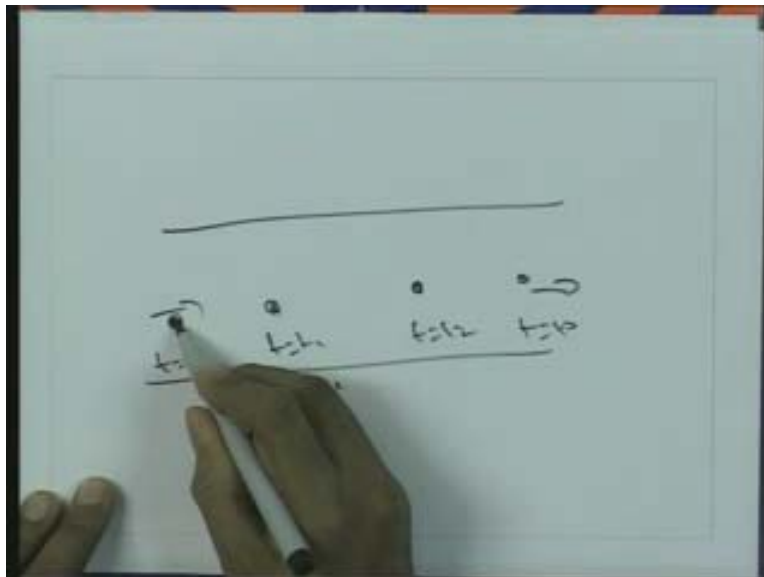
This shows the eulerian description in a fluid flow as we have discussed earlier. Generally the eulerian description is very much useful in fluid mechanics and most of our fluid flow analysis will be based up on the eulerian description.

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The second method which we have discussed earlier is the lagrangian method. The lagrangian method as I have mentioned in the lagrangian method we will be flow into the fluid particle what will if fluid flow in a open channel or in a river flow or in a pipe flow then that is what we are discussing is same if this is the flow river flow then we will be chasing a particular particle. them with respect to this t is equal to zero then t is equal to t_1 or t is equal to t_2 or then t is equal to t_3 .

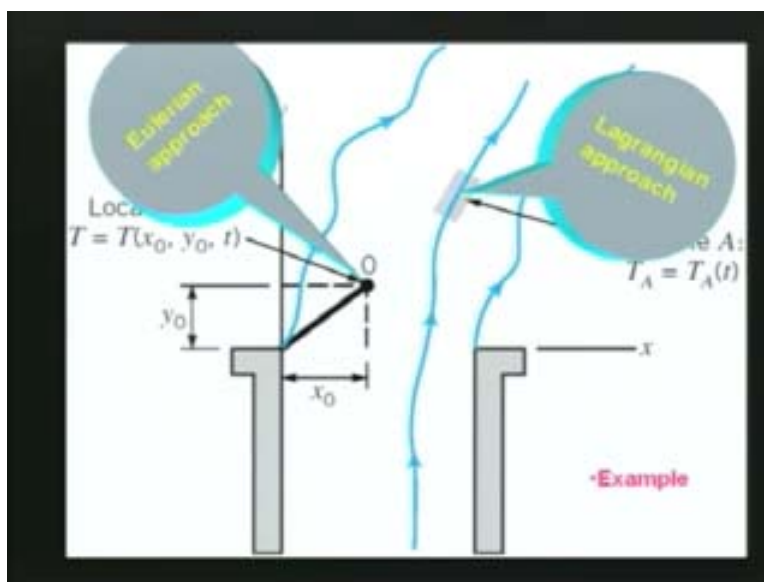
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Like that what happens to this particular fluid particle that is the way which we are chasing out in the lagrangian method? The lagrangian method flow the individual fluid particles as they move under the determined how the fluid properties associated with this particles change as a function of time. This is mainly with respect to time what happens and position is anywhere with respect to the fluid particle moment it is always changing the particular fluid particles are traced what happens those particles. It is what we are doing the lagrangian method here the fluid particles are tagged or identified ask time progresses happens to those particles. That e are what we are describing and since as you can see that fluid motion or fluid flow is concern it is very difficult to track the individual particles like this see what happens to those particles.

Generally, this lagrangian method is very rarely use in fluid flow problem due to it's the difficulty to track the particular particle to track particular particle what happens to that particular particle it is very difficult to do this kind of as analysis. Generally, for most of the fluid mechanics problem eulerian method is used since it is simpler we get the information what we are looking for especially at particular section with respect to time we are getting. That is the most of the time we will be using in eulerian method in the fluid mechanics of fluid flow analysis. Finally, in this slide here this slide show how the eulerian approach and lagrangian approach is used as for as fluid flow analysis is concern.

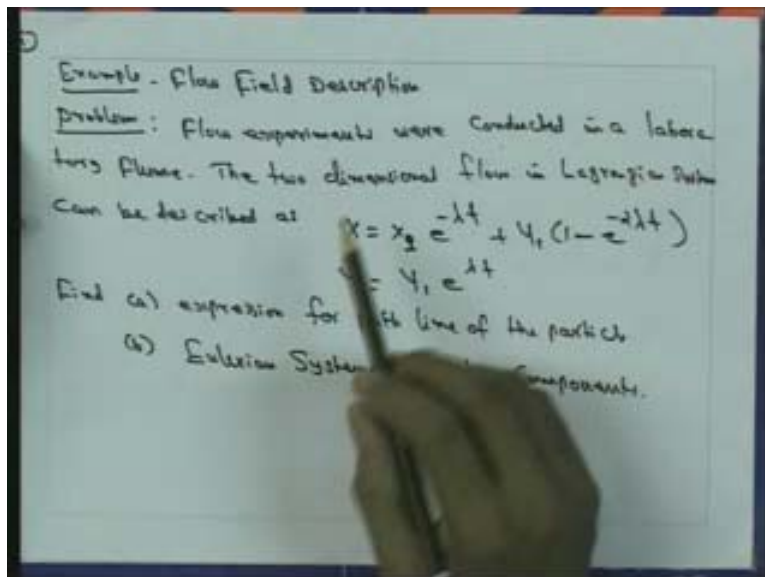
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Here you can see that the fluid is moving from a section a channel like this. here in the eulerian approach we are taking consider a particular point at a particular section here like this the point is that is from this x at the distance of x_0 and y_0 we are considering a particular point the location is t is equal to t $x_0 y_0 t$. What happens to fluid flow as for as the particular point is consider that is we will the studying in the eulerian approach this will be generally with aspect to time how the fluid is behaving but as far as lagrangian approach is concerned here you can it is mainly with respect to this time.

We are we have already tracked on particular particle and this position to this position how it was work like that we are chasing on the particular particle, we are describing the flow property with respect to the properties for this particular particle is concerned and that is the lagrangian approach. Even though we rarely used lagrangian approach but once the fluid flow parameters are known we can convert to eulerian description or from the eulerian description to the lagrangian description al the conversion is possible. Just let us discuss a small example how we can utilize this eulerian approach lagrangian approach as for as fluid flow field description is concerned here the problem which we are discussing s here the flow field description.

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In an experiment of flow description were conducted in laboratory flow. the 2 dimensional flow in a lagrangian system is already obtained as x is equal to x_1 into e to

the power λt plus y_1 into $1 - e^{-\lambda t}$. In the λ is a constant and t is time x is the x axis here with respect to x with respect to y time this is a two-dimensional problem the fluid flow properties are changing with respect to the space xy and xy direction and time t . Here already the two-dimensional flow in the Lagrangian system is given as x is equal to $x_1 e^{-\lambda t} + y_1 (1 - e^{-\lambda t})$ and also for y direction is concern is given as y is equal to $y_1 e^{-\lambda t}$. This y_1 and x_1 gets what is the initial positions this is $x_1 y_1$ the variation with respect to x and y are given with respect to time t by these equation number 1 equation number two. We want to find an expression for path line of the particle with respect to this Lagrangian system given we want to get the corresponding equation Eulerian system velocity components we want to determine.

Problem is we want an expression for path line of the particle Eulerian system velocity components these we can solve first the first problem is we want to find path line for the particles. Path line means it should be an expression without any times we will use this equation number one and two.

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Solution: a) Eliminate t to get path line

From $e^{-\lambda t} = \frac{y}{y_1}$

Put in eqn. for x

$$x = x_1 \left(\frac{y}{y_1} \right) + y_1 \left(1 - \left(\frac{y}{y_1} \right)^2 \right)$$

b) Eulerian System:

Vel. component $u_x = \frac{dx}{dt}$

Diff. $x = x_1 e^{-\lambda t} + y_1 (1 - e^{-2\lambda t})$

$$u_x = \frac{dx}{dt} = \frac{d}{dt} [x_1 e^{-\lambda t} + y_1 (1 - e^{-2\lambda t})]$$

We will eliminate t the time component to get path line from the second equation from second equation you will get $e^{-\lambda t} = \frac{y}{y_1}$ from y_1 equation is given y is equal to $y_1 e^{-\lambda t}$. The from that we can write $e^{-\lambda t} = \frac{y}{y_1}$

λt is equal to y by y_1 this e to the power λt is already there in equation number one here. you will substitute that for this e to the power λt in equation number one here for the expression for x we will get x is equal to x_1 into y_1 by y since into the power λt already obtain as y by y_1 e to the power minus λt y_1 by y . x is equal to x_1 into y_1 by y plus y_1 into $1 - y_1$ by y whole square. This gives the expression for the path line description of the particle in the lagrangian system.

here there is no time component we want all gives the expression for path line of the particle the second part of the problem is u_1 to get eulerian system velocity component as for as this with respect to the lagrangian system given. The eulerian system the velocity component we can write as since the eulerian system which we are discuss the as here you can see that at a particular position what happen that gives the velocity component. Here with respect to this slide here we will get the velocity component U_x equal to dx by dt and u_y will be dy by dt that means velocity y direction will be dy by dt and velocity next direction will be dx by dt . We will just differentiate this equation number one here give here that you differentiate that will be velocity component the x direction. we will differentiate the equation x equal to x_1 into the power minus λt plus y_1 into $1 - y_1$ into e to the power $2\lambda t$ if you differentiate u_x equal to dx by dt that is equal to d by dt of x_1 into e to the power minus λt plus y_1 into $1 - y_1$ into e to the power $2\lambda t$. If you simplify if you differentiate and simplify you will get an expression for velocity that is the eulerian systems. u_x is equal to minus λx plus λy into e to the power minus λt plus e to the power minus $3\lambda t$.

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The image shows a whiteboard with handwritten mathematical work. At the top, the expression for the x-component of velocity is given as $u_x = -\lambda x + \lambda y (e^{-\lambda t} + e^{3\lambda t})$. Below this, it says "vel. y component" and shows the differentiation of the y-component: $u_y = \frac{dy}{dt} = \frac{d}{dt} (y_1 e^{\lambda t})$. The final simplified result is $u_y = y_1 \lambda e^{\lambda t} = \lambda y$.

Here this after differentiation we will simplify. Finally, you will get the expression for the velocity as u_x is equal to minus lambda x plus lambda y into e to the power minus lambda t plus e to the power minus 3 lambda t . Similarly, the velocity component in y direction you will get u_y is dy by dt that we can just differentiate since the initial equation the expression for y is given in the lagrangian system y is equal to y_1 in e to the power lambda t . if you differentiate you will get u_y is equal to dy by dt that is equal to d by dt of y_1 into e to the power lambda t . that is u_y is equal to lambda 1 into y_1 into lambda into e to the power lambda t . That will give this can be written as since y is y_1 into e to the power lambda t u_y can be written as lambda y that gives the expression for the velocity component in y direction for the eulerian system.

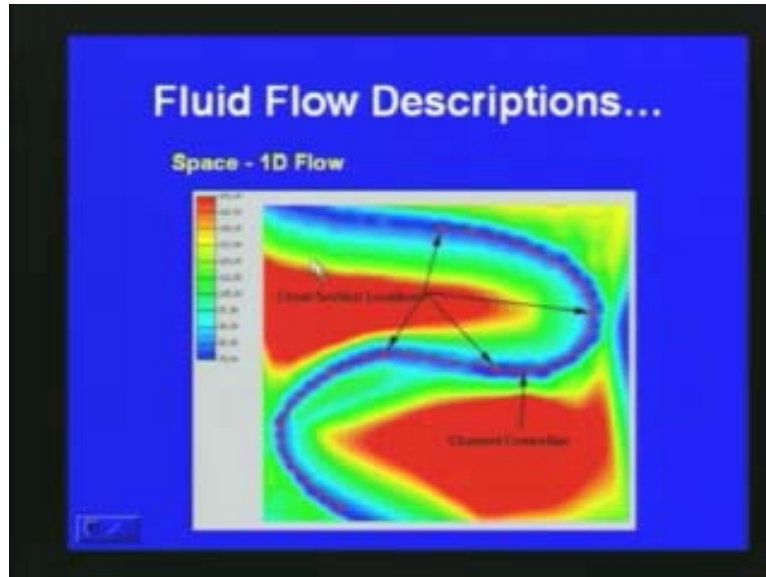
This is the velocity field or in flow particular fluid flow parameters is known in one system either lagrangian system or in eulerian system convert to other system; with respect to the various mathematical relationship available with respect to both the system. We have seen how we describe the various fluid flow properties with respect to discussion the eulerian description and the lagrangian description as I mentioned. Generally we will be using the eulerian description using the eulerian description since it is much easier and it is easily the analysis must simpler and we can get results especially since fluid flow much bother about what happens at particular section than just tracing some fluid particle.

But some cases also some problems will be taken in the lagrangian approach by tracking particular particle. That also we will discuss later with respect to this the field description the fluid flow as we have discussed earlier can be described briefly again review this fluid flow descriptions., space wise as I mentioned the flow can be either one-dimensional two-dimensional or three-dimensional flows as I mentioned. Generally the all the flows are three dimensions in nature it is varying with respect to xy and z and al time. But many of the problems for example as I mentioned when we discussing a river flow if you want to know a particular with respect to the longitudinal direction l here or here this x direction. If you want to know with respect to this longitudinal direction what happens to the flow properties like velocities or the head or the pressure or the parameters then it is better that we consider the fluid flow as one-dimension or when we need more details the same problem we will be describing as two dimensions that this x and y both component we will be describing.

That the velocity in the lateral direction this in u and v will be al considered and al correspondingly the time the other parameters will be constant two-dimensions and otherwise if you are looking for accurate the analysis if you are looking for a problem in three dimensions. That we have to consider for example the flow in a river is concern here we are to consider the depth wise also, what happens with respect to z direction not only the longitudinal the lateral we have concerned the depth.

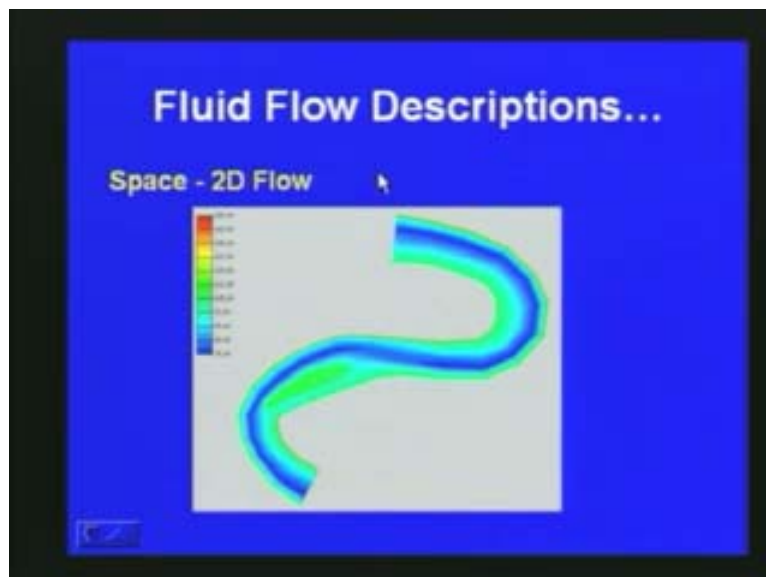
Also depending up on the problem most of the fluid flow is concerned this three dimensional nature but we will be simplify the problem into one-dimensional two-dimensional three-dimensional problem. As you can see here it is a river flow.

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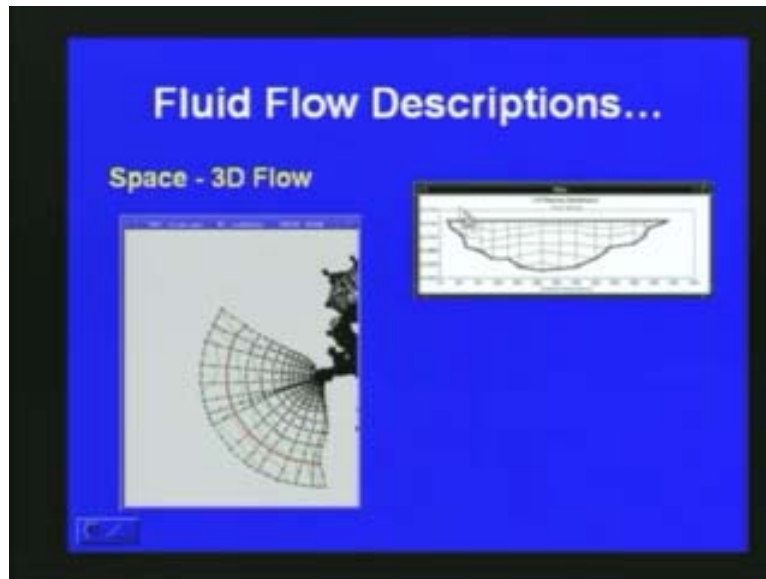
Here this slide, shows what happens with respect to the flow here the analysis is various section here considering analysis with respect to this one dimension what happens. Space is one dimension with respect to time the variation is with respect to this the length of the river. Here with respect to x time we will be analyzing this is special in one-dimensional flow is concerned analyzing with respect to what is happening in one d. this is fluid flow in 1 dimension when we analysis as 1 dimension the next here.

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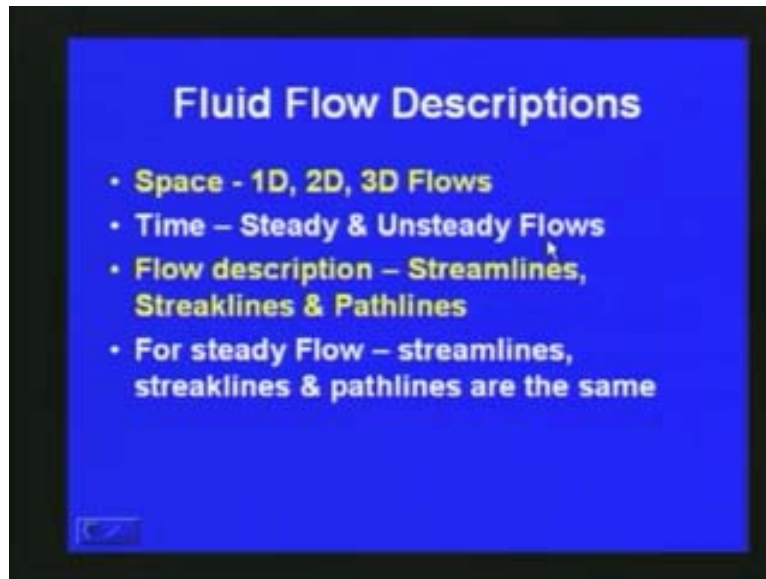
This slide shows this space two d shows this what will this happening with respect to two dimensions. That is why all this here in this blue with respect to what happens in y direction latterly also we have considered that is why here the space wise this is two-dimension flow which we are concerned the flow is the definitely three-dimension that we are simplifying flow to two-dimension we are analyzing as a two-dimension flow next here you can see the space three dimension.

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Here the depth is also considered in a flow with respect to the length of the channel later direction. It is totally three-dimension depending upon the necessity depending up on the type of problem which we are dealing the fluid flow analysis can be one-dimension two-dimension or three dimensions. Also as we have discussed earlier the time wise the flow can be most of time the flow will varying with respect to time. But many times we can also that once there is not much variation with respect to time then we can consider there is no variation with respect to time for the fluid properties then you can consider as study state flow with respect to time we can consider the fluid flow steady state flow and unsteady state flows. In unsteady state flow definitely the fluid flow properties are varying with respect to time.

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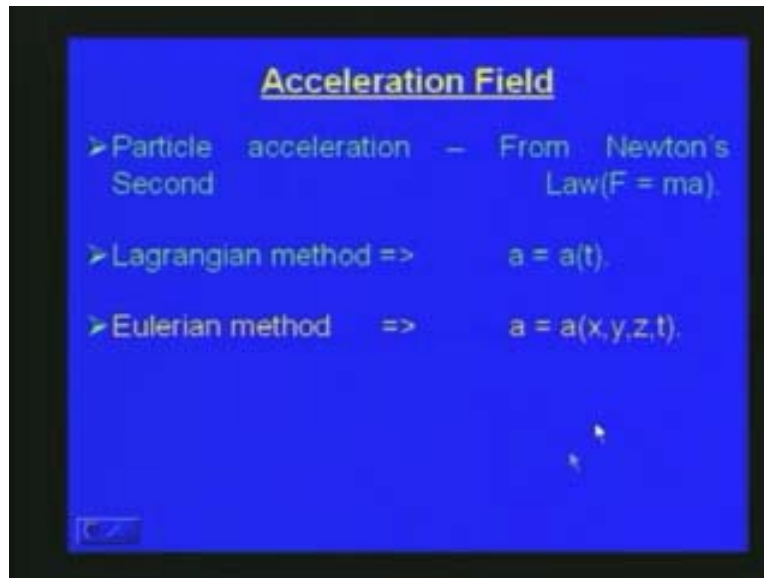


We have to see the spatial variation as well as the time variation but as far as steady state flows as. We will be discussing with respect to spatial direction time wise it is steady state. Also we have discussed generally way to describe the fluid flow we will be using various fluid visualization flow visualization techniques and also certain lines like a stream lines streak lines and a path lines are generally used for to describe the fluid flow. This we have already seen in the previous slide how we are using this kind of analysis as for as the fluid flow descriptions concern. You can see the path lines are used and stream lines are used for the velocity description and here the stream line the velocity vectors are used and with respect to the velocity vector we can find out the stream line. The stream lines path lines steak lines which we have already described discussed in the interacting chapter streak lines or these lines are used for the fluid flow description and with respect to that only will be describing what happens to the fluid flow.

Flow description is we can use this various techniques and in steady state flow the stream lines and path lines are the same. It will be generally used as for as the stream lines will be used for unsteady state flow analysis. We have already seen the velocity field we have seen how we can describe the fluid flow with respect to eulerian description are with respect to legrangian description. The next topic and also we have see how we can describe the fluid flow whether it can be fluid flow can be considering one-dimensional

two-dimensional three-dimensional or steady state or unsteady state like that. We will discuss the acceleration field.

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Acceleration is most of the flow takes place due to acceleration due to gravity of the applied acceleration. Acceleration field is also another important acceleration is al another important property as for as fluid flow is concerned. In the kinematics of fluid flow here you will discuss the acceleration field. The particle acceleration which is generally here you can utilize this Newton second law which is given as force is equal to mass into acceleration. We have seen two methodologies of description of fluid flow one is the lagrangian method.

As for as lagrangian method is concerned, generally the fluid flow is described with respect to time only the lagrangian method the acceleration is described as a varying of with respect to time. That means particle is we are looking in to that and how it is moving or how it is behaving with respect to time. Here the space is not the major issues you are tracking the particle what happens for that particle with respect to time.

As for as acceleration field is al concern here we will be describing the acceleration is with respect to time the variation is with respect to time, but as far as the eulerian method which we have seen earlier is in the eulerian description we are considering a particular sections. We will be discussing what happens to fluid flow properties at the particular

section. Here the acceleration field is concerned in the eulerian description we will be describing a, is equal to a xyz and t time the acceleration field is described with respect to the space xyz and time t and as we know the acceleration time write of change velocity for a given particle the lagrangian method. We can write acceleration a, is equal to as a functional type and the eulerian description you can write as a functional space and time here this.

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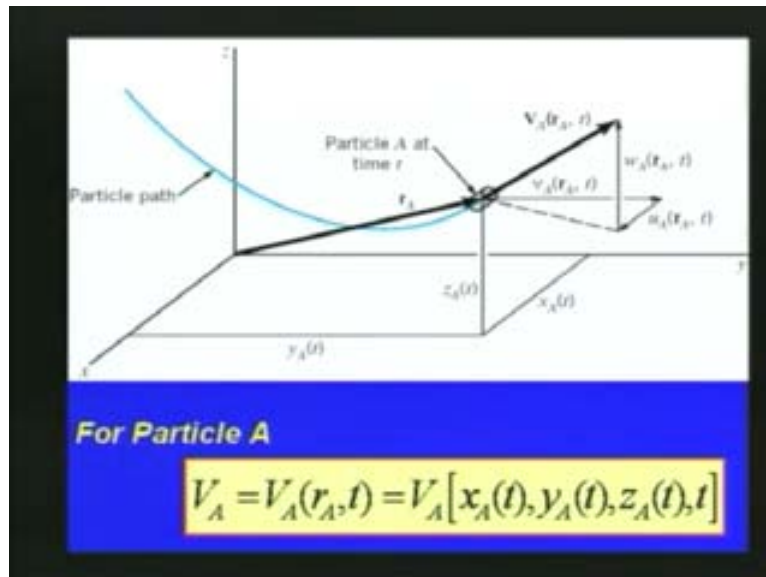


Figure shows if you consider a particular path of a particular fluid particle. For particle a here you can see that the particle path is like this. Particle a, at time t this is here it is described with respect to xyz time t. Its position vector is represented like this with respect to the velocity component t xyz directions. S ua the velocity component x direction the VA the velocity component this VA is the magnitude the final velocity and this shows the velocity on the y direction and this is the velocity component in the z direction.

Particle A the velocity is described as V_A is a position vector of r and t as described here V_A is the function of r_A and t and that can be put as V_A x_{At} Y_{At} and z_{At} as shown in this slide.

That will be represented like this with respect to xy and z and spatial direction and time t equation of acceleration is concerned.

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The slide has a blue background with white text. At the top, it says "Equation for Acceleration". Below that, three equations are shown in a light blue box: $u = \frac{\partial x_A}{\partial t}$, $v = \frac{\partial y_A}{\partial t}$, and $w = \frac{\partial z_A}{\partial t}$. Below these, a larger equation is shown in a yellow box: $a_A(t) = \frac{dV_A}{dt} = \frac{\partial V_A}{\partial t} + \frac{\partial V_A}{\partial x} \frac{dx_A}{dt} + \frac{\partial V_A}{\partial y} \frac{dy_A}{dt} + \frac{\partial V_A}{\partial z} \frac{dz_A}{dt}$. A small mouse cursor is visible in the center of the slide.

In the legrangian description than we will be writing with respect to u is equal to del X_A by del t the velocity described as here we will consider the equation for acceleration. The velocity component in x direction u is equal to del X_A by del t and v is del Y_A by del t and w is del z_A by del t as shown in this figure.

Acceleration is concerned the variation is with respect to xyz and t that is why the partial is used here. Finally, the acceleration can be written as acceleration is the derivative of the velocity generally the acceleration is represented as a is represented as dv by dt. If we consider the fluid flow at particular position point A, the acceleration can be written a_A t is equal to dv_A by dt which is the total derivative. That with respect to this uvw description we can write del v_A by del t plus del v_A by del x that means the acceleration is not total represent with respect to local acceleration as well as convert the action.

Here the acceleration for this particular fluid particle we will be considering with respect to the time variation with respect to local acceleration conductive acceleration with respect to the fluid flow how it is behaving. That is what we are describing here the acceleration is represent as the total derivative of the velocity vector with respect to time the A is equal to dva by dt. That can be represent as a local acceleration than V_A by del t plus del V_A by del x

dx_A by dt this is with respect to the velocity component in x direction $\frac{dV_A}{dt}$ by $\frac{dy_A}{dt}$ this is with respect to the velocity component y direction plus $\frac{dV_A}{dt}$ by $\frac{dz_A}{dt}$ by dt . This gives the total acceleration as for as the fluid movement is concerned this one part is the local acceleration and other part is the convective acceleration as indicated here in this figure.

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Equation for Acceleration

$$u = \frac{dx_A}{dt} \quad v = \frac{dy_A}{dt} \quad w = \frac{dz_A}{dt}$$

$$a_A(t) = \frac{dV_A}{dt} = \frac{\partial V_A}{\partial t} + \frac{\partial V_A}{\partial x} \frac{dx_A}{dt} + \frac{\partial V_A}{\partial y} \frac{dy_A}{dt} + \frac{\partial V_A}{\partial z} \frac{dz_A}{dt}$$

Here this dx_A by dt can be represent as u and dy_A by dt can be represent as v and dz_A by dt can be represent as w . Finally, if you substitute the total acceleration can be represented as shown in the slide.


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$$\vec{a} = \frac{\partial \vec{v}}{\partial t} + u \frac{\partial \vec{v}}{\partial x} + v \frac{\partial \vec{v}}{\partial y} + w \frac{\partial \vec{v}}{\partial z}$$
$$a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

As \vec{a} is equal to $\frac{\partial \vec{v}}{\partial t}$ with local acceleration plus convective acceleration is given as $u \frac{\partial \vec{v}}{\partial x} + v \frac{\partial \vec{v}}{\partial y} + w \frac{\partial \vec{v}}{\partial z}$.

This is the general the acceleration as a vector representation as acceleration since it varies with respect to space and time the direction and the magnitude is that. Acceleration is can be represented in terms of the x direction y direction and z direction. with respect to general description of the acceleration we can write a_x is equal to $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$ this expression is obtained directly from the general expression for acceleration. This is x direction acceleration a_x is equal to $\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$. Similarly we can write the acceleration in x direction y direction and z direction.

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$$\bar{a} = \frac{\partial \bar{v}}{\partial t} + u \frac{\partial \bar{v}}{\partial x} + v \frac{\partial \bar{v}}{\partial y} + w \frac{\partial \bar{v}}{\partial z}$$
$$a_x = \left\{ \frac{\partial u}{\partial t} \right\} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$


As I mentioned this term here $\frac{\partial u}{\partial t}$ is the local acceleration that means with respect to the x component the acceleration in x direction a axis with respect to the velocity in x direction what happens the local acceleration with respect to time, the terms are called the convective acceleration that means with respect to the velocity uvw how the acceleration takes place. These three terms are called convective acceleration and the first term is called the local acceleration. Similarly, we can write the acceleration in y direction.

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$$a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$
$$a_z = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}$$

a_x, a_y, a_z - Components of Acceleration

As a_y is equal to $\frac{dv_y}{dt} + u \frac{dv_y}{dx} + v \frac{dv_y}{dy} + w \frac{dv_y}{dz}$ acceleration in z direction can be written as $\frac{dw}{dt} + u \frac{dw}{dx} + v \frac{dw}{dy} + w \frac{dw}{dz}$.

The acceleration in xyz direction is represented as local acceleration terms plus convective terms this is the general methodology used for the determination of acceleration. This a_x , a_y and a_z are called the components of acceleration a_x , a_y and a_z are called the components of acceleration. Here you can see that this entire problem which we have seen for with the analysis the fluid flow analysis which we have discussed is mainly.

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Operator

$$\frac{D()}{Dt} \equiv \frac{\partial()}{\partial t} + u \frac{\partial()}{\partial x} + v \frac{\partial()}{\partial y} + w \frac{\partial()}{\partial z}$$

Termed as Material Derivative

Describe time rates of change for given particle

In terms of the without concentric the force which drives the flow we are not concerned with the force. But later this is beginning as for as this topic is concerned later we will discuss that also here the operator which we discussed the total derivative is generally in the previous slide we have seen here the acceleration we have seen in with respect to the xyz components. With respect to this the operator we defined the total derivative as D by dt is equivalent to the local $\frac{d}{dt}$ that means with respect to time u into $\frac{d}{dx}$ plus v into $\frac{d}{dy}$ plus w into $\frac{d}{dz}$. this depends up on the property if it is x direction here will be putting u y direction will be putting v and z direction will be putting w .

This term is called and termed as material derivative. This is termed as material derivative. This describes the time rates of change for a given particle. The time rate of change for a given particle is given by this total derivative termed as material derivative. This will be used in most of our derivations later stages. The rate of change of a given particle is termed as material derivative. The total derivative will be described with respect to the local term plus the convective term here u , v , and w .

Further we will be describing the applications of the kinematics of fluid flow. Kinematics further derivations are derived from various equations as for fluid flow kinematics concerned. Finally, that will be with respect to that proceeding to the fluid flow dynamics.