Fundamentals of Transport Processes II
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Lecture - 9
Mass Conservation Equation

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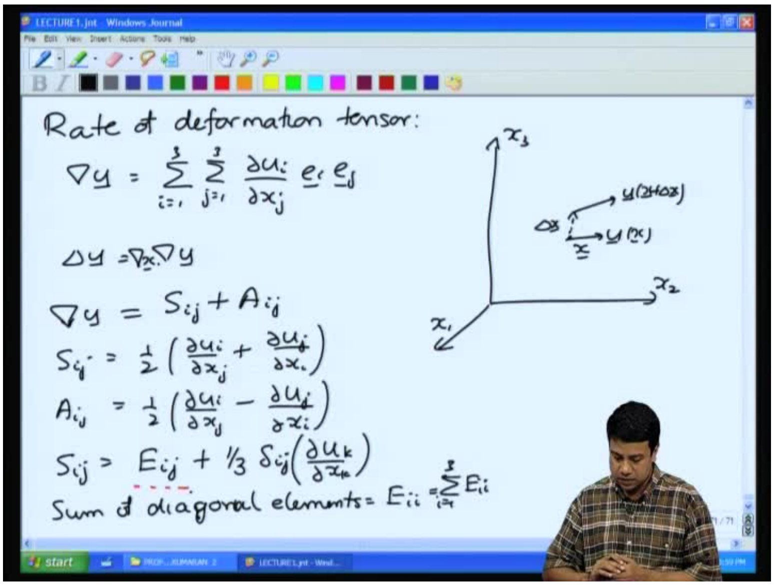

Welcome to this lecture number nine, of our course on fundamentals of transport processes. so far we have been discussing, initially we discussed the vectors, and tensors, and their integral and differential theorems, and then we looked at some results that are obtained, when these vectors operate on the velocity field, a fluid velocity field. In the last class we had discussed the, rate of deformation tensor. It is a rather important subject so we will briefly review it again, before we proceed. So this is the rate of deformation tensor. Second order tensor, it is the gradient of the velocity, it is also written as partially u i by partially x j. And if I want to write it in expanded form, then I would write it as; sigma i is equal to 1 to 3, sigma j is equal to 1 to 3, of partial u i by partial x j e i e j. So at a given point in the fluid, if I mark a location x and I go a small distance delta x, to some new location.

You have a velocity at x, and this is the velocity at x plus delta x. The difference in velocity delta u, is equal to grad u, you can there have a delta x dotting that. So basically the displacement dotting with grad u. The dot product is, between the displacement vector, and the gradient, in the rate of deformation tensor. So this basically gives you the rate at which, positions are moving related to each other, as the fluid flows. It is a second order tensor, it has nine components; however as we discussed in the last class, this can be decomposed into fundamental modes of deformation. So the gradient, which is the second order tensor, can always be written as the sum of two parts; one is the symmetric part, and the other is the anti-symmetric part, where the symmetric part is equal to half of the gradient of the velocity, plus x transpose.

The transpose is obtained just by interchanging rows and columns, or by interchanging the two indices of the tensor, because one index represents the row, the other index represents the column. The anti-symmetric part, is equal to half of the matrix minus, its transpose, half the matrix minus its transpose. This symmetric part can, once again be written as the sum of two tensors; one is the symmetric traceless tensor, and the other is an isotropic tensor. The isotropic part, is just equal is just propositional to the identity tensor. It has equal diagonal elements, and zero half diagonal elements. The symmetric traceless tensor, is symmetric, and the sum of its diagonal elements is equal to zero. So this can be written as, this symmetric traceless part plus one-third delta i j times partial. In the previous lecture, I have written this as the divergence of u.

Partial u k b partial x k is also the divergence, because there is a repeated index, so there is a dot product. So this thing is a symmetric traceless tensor, what that means, is that the sum of the diagonal part, diagonal elements of this tensor, is equal to zero. And I showed you in the last lecture that the sum of the diagonal elements, of this tensor, is just equal to, is e i i, because if I have e i i and I expand it out using my indicial notation, there is one repeated index; that means there is a dot product. So there is one summation, and no unit vector, so this is just equal to, summation i is equal to 1 to 3 of e i i, which is basically the sum of the diagonal elements of separate matrix.

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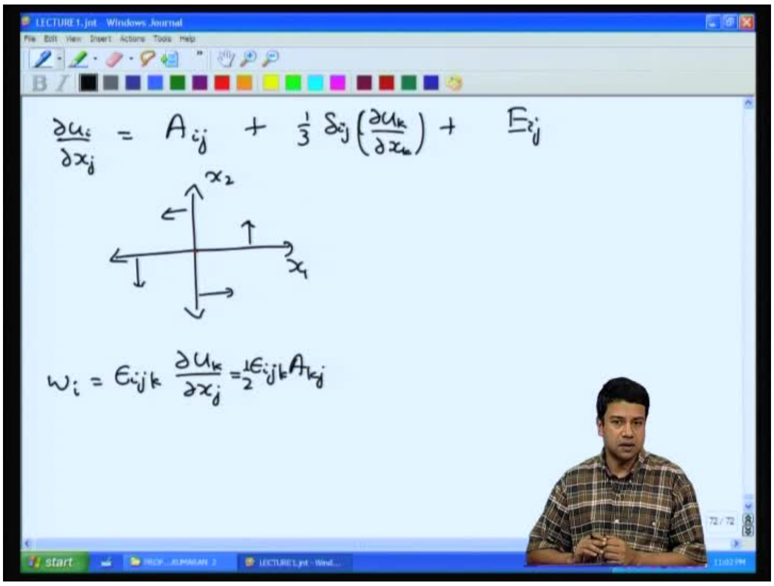

So this rate of definition tensor can be separated into three parts, is equal to the anti-symmetric part plus the isotropic part which was plus e i j. The three parts, and I had shown you, that there are specific forms of definition, associated with each of these three components. This tensor I have separated into anti symmetric isotropic, and symmetric traceless, and if you look at the local deformation has point to each individual part, we find for that anti symmetric part. The deformation is basically a solid body rotation; that is if I am sitting at some point, within the fluid at that center, and I look at the velocity with which nearby points are moving relative to the point at which I am sitting. You find that the nearby points are moving, in a rotational form; that is, they are there is a solid body rotation, around this central point.

So that is what, is captured by the anti-symmetric part, to the rate of deformation tensor. And we saw in the last class that, the rate of deformation tensor, is related to the verticity. So the verticity, which is the curl of the velocity, partial u k by partial x j; that is the verticity vector, can also be written as epsilon i j k times a k j. So the curl of the velocity vector, is also equal to epsilon, double dotted with the anti-symmetric part of the rate of deformation tensor. So if I take epsilon, and double dot it with a, anti-symmetric part of the rate of deformation tensor, i get a vector, whose direction is perpendicular to the plane of rotation, so that is what you get from the anti-symmetric part of the rate of deformation tensor. In fact you can actually, in fact there is a half sitting in front, please note that.

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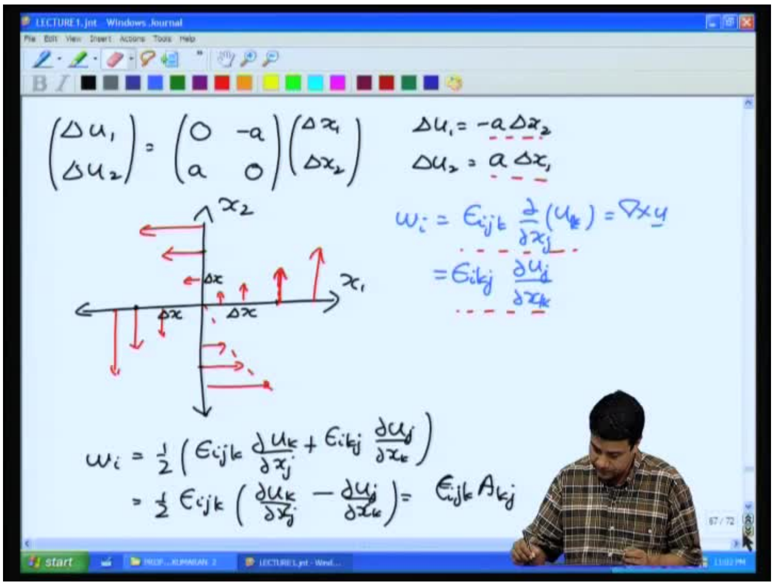

Now, that is because the anti-symmetric part has two partial u i by partial x j minus partial u j by partial x i, and once, let me just go back and check. You can see that there is. I should make it correction here; half of partial u k by partial x j minus partial u j by partial x k, this itself is the anti-symmetric part, so there should be no half in front here, because half of the difference between these two, is just the anti-symmetric part. So please correct that in the previous lecture.

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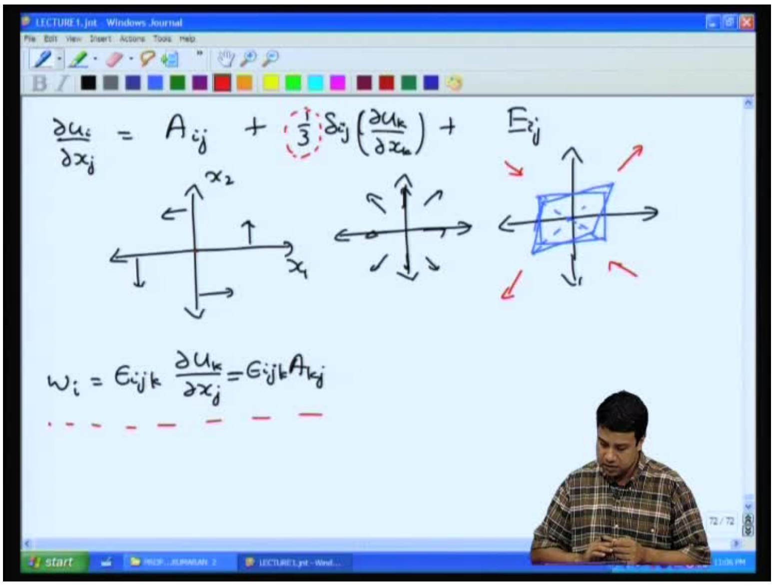

And I had written a half here, so that half should not be there. So that is the anti-symmetric part of the rate of deformation tensor. The second part was the symmetric traceless part, and that. I am sorry. The isotropic par, tand that as I showed you corresponds to readily outward, or inward flow. If the divergence of the velocity is positive; that means that the flow is readily outward. If the divergence of velocity is negative, the flow is readily inward. And this divergence of velocity is non-zero, only if you have a source of fluid, as I said the volume has to increase for the divergence to be non-zero, because the divergence of velocity integrated over a volume, is equal to integral of u dot n, over the surface.

So that is non-zero that means there is net fluid, coming out of that surface, so there has to be a source, or the density has to decrease. So unless, if the density is remaining a constant, and there is no source of fluid, then this part will be identically equal to zero. And the third part, was pure extensional string. We had seen in the last class that this, resulted in a deformation in which you have expansion, along two axes, and compression along the perpendicular two axes. This is expansion along two axes, compression along the perpendicular two axes.

So there is one extensional direction, and one compressional direction in two dimensions. So that if I have some differential volume here, and if I have look at what this extensional motion, how this that will deform, this differential volume. After some time I will have the volume looking something like this. It is going to deform in such a way that it extends, along one axis, compresses along the other axis, in such a way that the area, is preserved in two dimensions or the volume, is persevered in three dimensions. And these two extensional compression axes, also do not rotate, because rotational motion is associated with the anti-symmetric part. So the symmetric part has no rotational motion, so the axis remains the same.

There is no increase in volume, because the trace was zero, because of that you cannot have any increase in volume. So deformation is such a way that volume is persevered, as well as there is no rotation. So that is what is represented by the symmetric traceless part of the rate of deformation tensor. This is simplified picture, in two dimensions, if you go to three dimensions, you can have you have a symmetric traceless tensor, which is a 3 by 3 matrix. Similarly, the anti-symmetric part is also a 3 by 3 matrix. However the anti-symmetric part, this still holds. Note that what I have written here is for three dimensions, this one third. In the last class I showed you that for two dimensions, is equal to 1 by 2. So in general is equal to 1 by d, where d is the dimensionality of the system.

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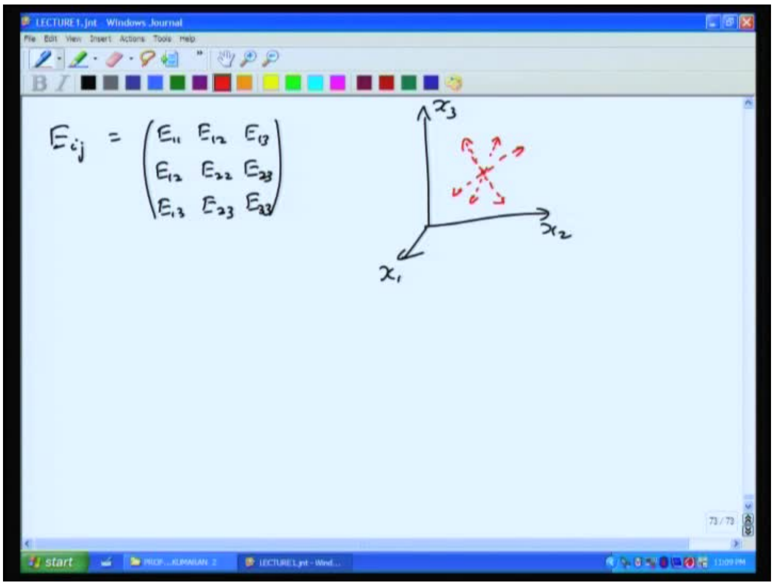

So in three dimensions these deformations can be of different forms, so e i j is a symmetric traceless tensor, so it is symmetric e 1 1, e 1 2, e 1 3, e 1 2. So it is a symmetric tensor, and the sum of the diagonals are all zero; e 1 1 plus e 2 2 plus e 3 3 is equal to zero. Symmetric tensor, it is self ad joint, these transpose is equal to itself, means it is a self ad joint matrix; that means that it has eigen values, are real, and eigen vectors are orthogonal to each other, and the eigen vectors form a complete basis at origin.

So the eigen values of this tensor, basically represents the rate at which, there is stretching or compression, along the principle directions. The eigen vectors represent, the three principle directions, along which there is deformation. So for example, in a flow field. You know that the eigen vectors of the symmetric matrix at each and every point in space are all orthogonal to each other; that means that if I have one particular point in space. I have three, in the most general case I will have three perpendicular directions, represented by the eigen vectors at this point in space. So there are three perpendicular directions, at one point in space, so there are two in the plane and one that is coming out of the plane, and along each of these three perpendicular directions, I have one eigen value, along each of these three perpendicular directions I have one eigen value, which basically it gives you the rate at which there is extension or compression, along that particular direction. Note that this is a traceless matrix; that means that the sum of the diagonal elements is equal to zero. And we know from linear algebra, that the sum of the diagonal elements, is also equal to, the sum of the eigen values. Therefore, the sum of the three eigen values has to be equal to zero. There is the reflection of the fact that volume is preserved, so either you have to have stretching in one direction and compression in the other directions, in such a way that volumes do not change. Therefore, you can have either two positive eigen values, and negative. You could have two negative and one positive, or you could have, one zero, one positive, and one negative.

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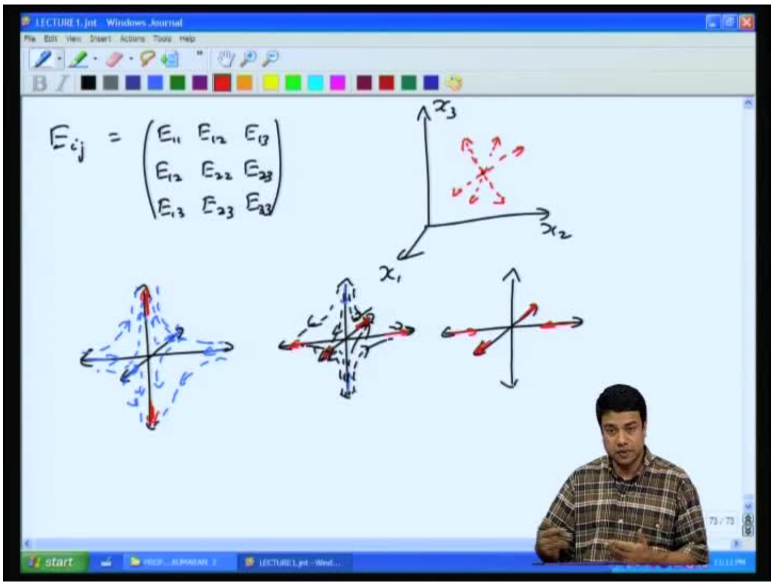

If I have one positive eigen value, and two negative eigen values; that means that at this point, along the three principle directions, I am having extension in one direction, and compression in the other two directions, because there are two negative eigen values, and two, and one positive eigen values. So, basically flow was coming in, from two directions I was getting, is going out along the third direction. So this is called uni-axial extension, there is extension along one axis, and there is compression along two other axis. If two eigen values are negative, and one is positive. The other option is for two eigen values to be positive, and one to be negative.

So in that case you have, stretching along two directions, and compression along the third direction, has stretching along two directions, along the positive eigen values, and compression along the third direction in such a way that the divergences the velocity is equal to zero. So in that case, you will have, flow that looks like this, it will come in along one direction, and it will flow out along two directions. The only other option is to have, one zero, one positive, and one negative. If all three are zero, of course there is no deformation, so you can have, so that is a trivial case.

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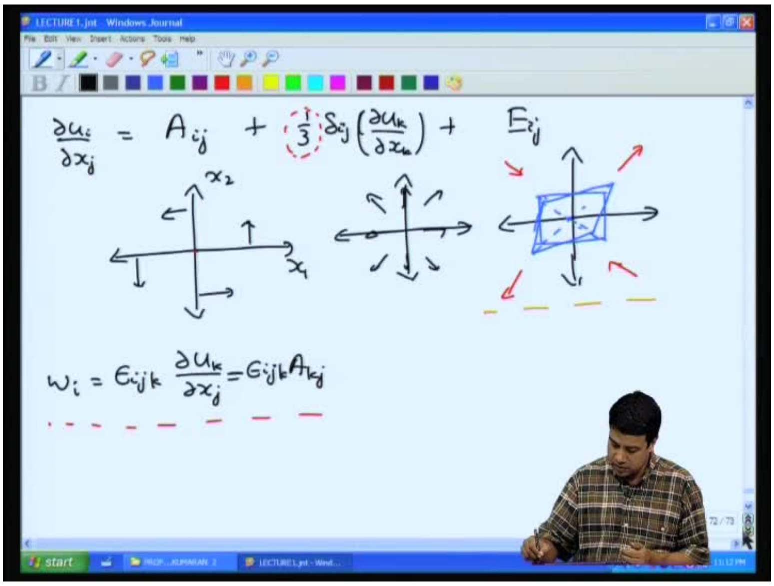

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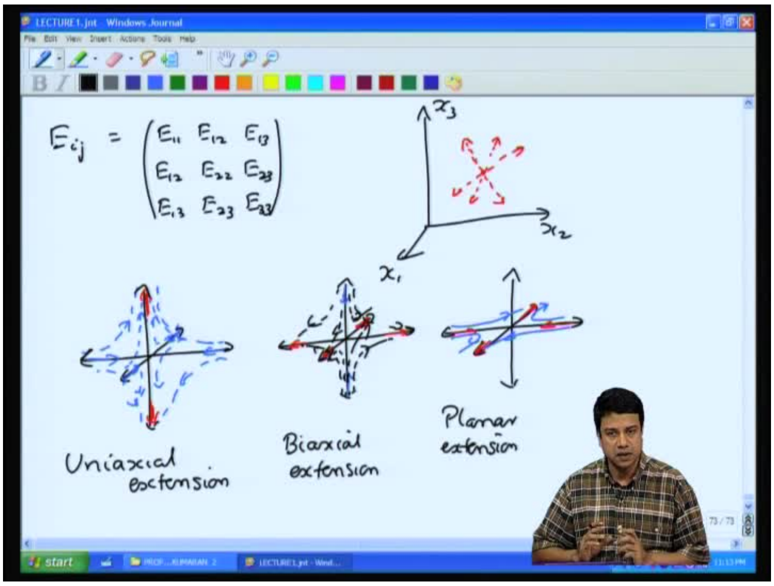

But you could have one positive, one negative, and one zero. In that case, you have deformation that is coming in, along one direction, it is going out, along one direction, and there is no deformation in the third direction. Therefore, you just have fluid that is in the plane, the same figure that I showed you in the previous lecture this one, deformation in the plane; that is when the third eigen value is equal to zero, and the other two have to be opposite in sign, and equal in magnitude. So you just have fluid that comes in this way, deformation in the plane. So this is called uni-axial extension, extension along one axis, this is by-axial extension, and this is what is called planar. So these are different components of the rate of deformation tensor, and as we shown c a little later.

Rotation, solid body rotation, does not change the distance between nearby points in the flow. So there cannot be a stress, due to the anti-symmetric part of the rate of deformation tensor. If the fluid is compressible, the isotropic part of the rate of deformation tensor is identically equal to zero; so all of the stresses have to be only due to the symmetric, traceless part of the rate of deformation tensor. One more topic in kinematics, before we proceed to deriving equations of motion; and that is the substantial derivative.

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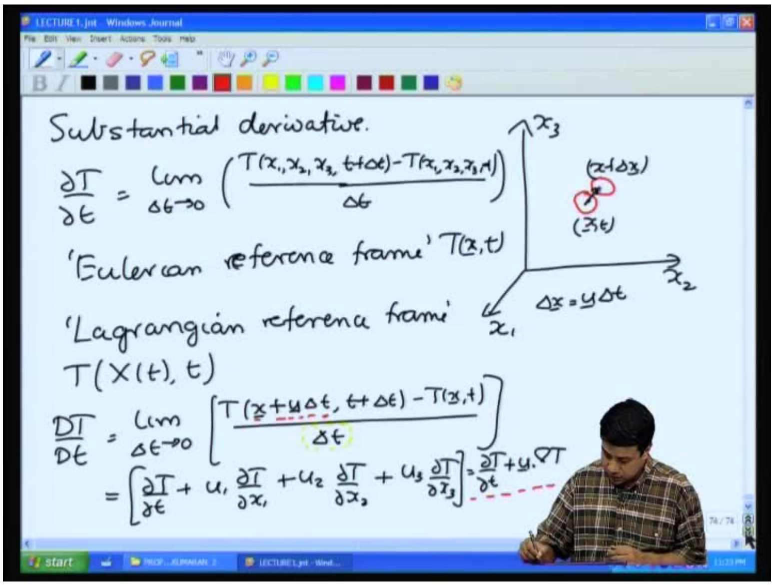

Now when you take the partial time derivative, implicitly you are keeping the position the same, and finding out the change in property, when between two subsequent instants of time. So the partial so for example, you have a temperature field in this three dimensional system partial t with respect to temperature, is equal to limit, as delta t equals to zero, of t at x 1 x 2 x 3 t plus delta t minus t i divided by delta t. So that is the partial derivative, the difference in temperature between two time instants, divided by the time interval in the limit as that interval goes to zero. However if you have some kind of a fluid flow, if you have some kind of a fluid flow, that is taking place. If I have some differential volume that is here, at time t, after a small interval of time, it would have travelled to some other location. It would travelled, and it would have deformed, so it would have travelled to some other location; x plus delta x. That is because the mean fluid is carrying the properties, along with a, the temperature is travelling along with the fluid itself, we have a hot parcel of fluid, and there is a mean flow, that parcel is actually being convicted by that mean flow.

So in order to find out what is the temperature difference along the moving parcel of fluid. This partial time derivative, is not the appropriate derivative, because this is in a fixed reference frame. The fixed reference frame in is this what is called an eulerian, reference frame, where as it is fixed in space. On the other hand, the reference that is moving, with the mean fluid velocity, is what is called a lagrangian reference frame. So in the eulerian reference frame, you would write the temperature as some function of position in time, absolute position in the absolute time at each instant in time, is you just find out what is the temperature, for the given position. In the lagrangean reference frame, you write it down, as a function of the moving parcel of fluid, that is carrying its temperature along with it.

So in this case you write it as t as function of, because the location of that parcel itself, is now a function of time. It has some one value at this instant, one value at that instant, and it is moving in space, and therefore, it is carrying the temperature or the concentration along with that, so that is the lagrangian reference frame. Now in order to implement a lagrangian scheme, one has to know if a parcel of fluid is at this location, at one instant of time ,where it was in the past, what was the past location, of that parcel of fluid, which is at one particular instant at this present location. That is usually a difficult task, because you have to trace out the entire history of the fluid. However one can define a derivative, which is called the substantial derivative, which contains in it the elements of the lagrangian moving reference frame. And that is if a fluid element was at some location x at time t, after the time t plus delta t it has moved to a new location.

So a difference in temperature, between the location at t plus delta t minus, at instant t plus delta t minus the temperature at location x at time t, so that is the, how the substantial derivative is defined. So, let us look at that it is usually written as d t by d t is equal to limit as delta t equals to 0 of t at x plus u delta t t plus delta t minus t at x t. Note that I am taking the position x plus u delta t at time t plus delta t. Why is that? Because if the fluid element was at location x at time t at time plus delta t it has moved a small distance, time t plus delta t it has moved a small distance to some new location x plus delta x, what is that new location x plus delta x? The new location, the displacement, is going to be equal to, the velocity times, the time interval, the distance moved is going to be, the displacement vector is going to be equal to the velocity vector, times that time interval.

So it is the difference in the temperature between x plus u delta t and t plus delta t minus t at x t divided by delta t. So that is the substantial derivative, a derivative in a reference frame that is moving with the parcel of fluid. And you can easily evaluate this in terms of the partial derivatives by just using the chain rule for differentiation, because x x 1 plus u 1 delta t at time t plus delta t minus x 1 and t at time t. So I just evaluated by taking partial derivatives, and what you will get is, this is partial t by partial t plus u 1 partial t by partial x 1 plus u 2, because the distance travelled was u 1 delta t. Therefore, the difference in temperature is equal to u 1 delta t times partial t by partial x 1, and then i divide by this delta t in the denominator, divide by this delta t in the denominator, and therefore, i get u 1 times delta t partial t by partial x 1. So this is the substantial derivative, which recognizes the fact that fluid elements are moving along with the mean velocity of the fluid.

I can also write this in vector notation, as partial t by partial t plus u dot grad t partial t by partial t u dot grad t is u 1 partial t by partial x 1 plus u 2 partial t by partial x 2 plus u 3 partial t by partial x 3. So this is the difference in temperature between the location x plus delta x at time t plus delta t minus the temperature at location x at time t; that is this substantial derivative, in a moving fluid, if for example, the temperature does not change, with time for a moving parcel of fluid. In other words, if I have to neglect completely the diffusion, or the conduction of temperature if the temperature is a constant, on a moving volume element of fluid, then this derivative is 0. If the temperature is a constant in a moving element of fluid; this is the substantial derivative at a zero, not the partial derivatives. That is because as the fluid moves, this volume element that was at location x goes to x plus delta x, some other fluid occupies the location x and there may be spatial variations between nearby volume elements of fluid. If the temperature is not diffusing, then the temperature on a moving element of fluid, is constant it may be specially varying.

So in that case it is a substantial derivative that will be zero, the partial derivative in general will not be zero. So that is the substantial derivative, it is in a lagrangian reference frame, and it recognizes the fact that, as the fluid moves, the fluid element occupies a new location, in after an instant of time in comparison to the original location, and you are taking the derivative on that moving volume element of fluid. So next we go down to start deriving the conversion equations. In the previous lecture course on fundamentals of transport processes one, we did this by actually writing out a differential volume in different coordinate systems, looking at what comes in, and what goes out, and writing the balance law

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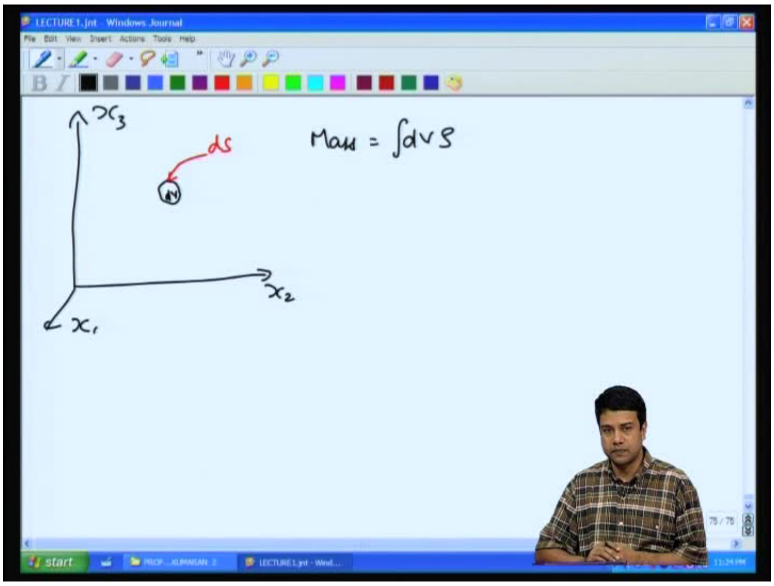

The net change in the mass, the heat in a differential volume, is equal to what comes in minus what goes out, plus source minus sign, rather than do it that way, what we will do, is to look at the rate of change of mass, on a moving differential volume, without reference to a specific coordinate system.

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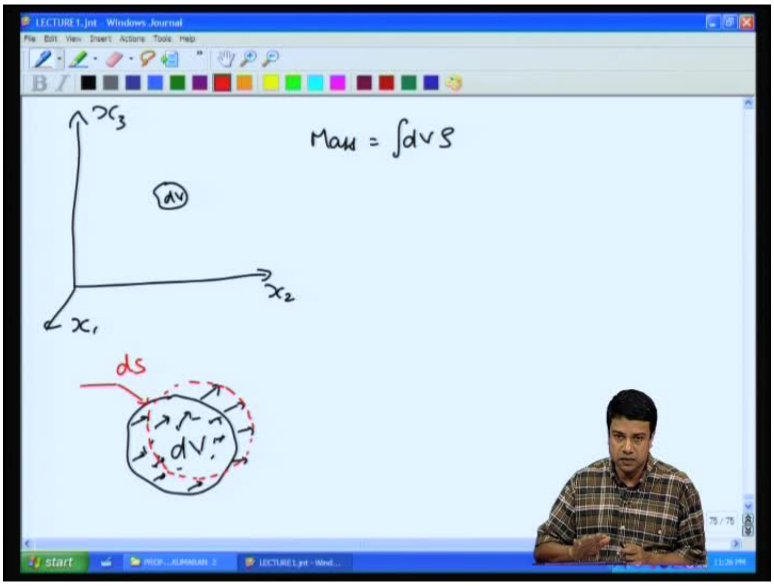

So for that, we have to do some back ground work. If I have a volume d v with the surface t s, this volume d v with the surface d s, on the surface. The net mass with in this volume, is going to be equal to integral over the volume d v of the density within that volume, there is a net mass within the volume. The mass conversion equation, basically states that, the mass within this volume has to be conversed. The volume itself is moving as a function of time, so let us take this volume d v, I will draw it bigger over here. This is the volume d v, with a surface d s. Now this volume itself is a function of time, it is a fluid material volume, and has time progresses, the fluid particles within this volume will move, and those on the surface will also move. The fluid elements on within this volume, we will move as a function of time, those on the surface will also move.

So at some later time, at some time t plus delta t, this volume would have gone to some other location. If the surface is that, the volume itself is defined by the motion of the surface points; the points on the surface, how they move? The points on the surface, move with the same velocity as the fluid velocity on that surface, the points within move with the same velocity, as the fluid velocity of within this volume. Note that the velocity is a vector that is defined, at each particular, at each point in space, it is a continuous fluid. Now as this volume moves, the points that are on the surface will continue to be on the surface, provided they are moving with the same velocity, as the fluid velocity at that point. The points that are within will continue to be within, once again provided the they are moving with the same velocity, as the fluid velocity at that point. What that means is that points on the surface, continue to be on the surface, the points within, continue to be within; that is because the fluid velocity as I said, is a single valued function of position, at each point the fluid velocity has only one value.

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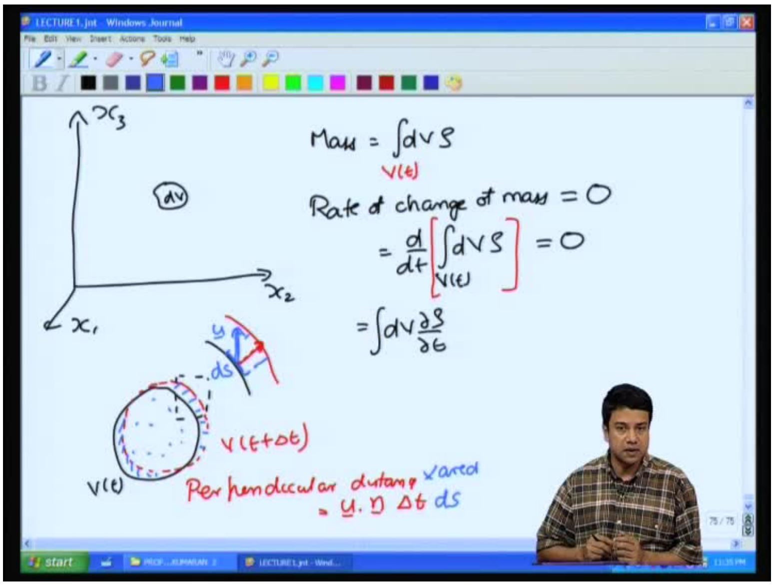

If a point within has to cross and go outside the surface, then obviously the velocity, it has to intersect with the trajectory of a point on the surface. If a point within, has to travel and go outside the surface, it has to intersect with the trajectory of some other surface point; that is travelling on the surface, if it has to go from outside to inside. Similarly, for point that is outside has to come inside, it has to cross trajectories. This crossing of trajectories is not permitted, because at that particular location, it implies that the velocity has two values. Whereas, we have defined the velocity vector a single valued function in space. Therefore, fluid points cannot cross from inside to outside the surface, because the surface is moving at the mean velocity at that location at the surface, inside is moving at the same mean velocity as the inside points. And if it has to cross, that will a double valued function, which it is not, it is a single valued function at each point in space.

Therefore, one cannot have the crossing of trajectories, what is outside is outside, what is inside is inside, and what is on the surface, continuous to be on the surface, provided it is a material point which is moving with the same velocity as the fluid velocity on that surface. And within this moving surface, the mass is defined as the integral over this volume, which is a function of time of integral d v times rho. Now the rate of change of mass, is equal to the time derivative of the integral over this volume of d v times rho. The rate of change of mass is the time derivative of the integral over this volume of d v times rho. Now the volume itself is a function of time, the density also present general the function of time and space. So you going to get contributions to this derivative, both from the volume, the dependence of density on time, as well as from the fact that the volume itself is changing, as time progresses. Now, how do we combined these two values, so there are two contributions to this rate of change of mass.

We know that from mass conversion equation, the mass within that volume has to be conversed, because points that are inside continue to be inside, points that are outside continue to be outside. And therefore, I require that for mass conversion condition, this has to be equal to zero, the rate of change of mass, because mass cannot be created or destroyed, unless you take into account nuclear reactions for example, so we will work within the classical regime, where this has to be equal to zero. And this relation that I have got, is for a moving element of fluid, and I need to convert that into a relation for a fixed volume element. So let us do that, so we go back to what we had earlier.

We have this volume, this is the volume at time t, and after a little bit of time t plus delta t, this goes to some other volume. This is v at t plus delta t, what is the change in this integral d v times rho between t and t plus delta t. There are two components to this change in the volume, change in the mass; one is because the density within the volume itself is changing at various points within the volume we have a density that is defined. The density within the volume itself is changing, and the other is because the surface itself has moved, so their certain parts of the surface, that have come in to the volume, which were not previously in the volume.

That has to be incorporated into the mass within this volume, the certain parts of the surface that were previously within the volume, but have now left, so that is to be subtracted from the mass. So there are two contributions; one is t to the change in density within this volume itself, the other is because certain regions are come in, and some other regions are left. So the rate, the change in mass due to the volume, change in density itself, is just integral d v of partial rho by partial t, so that is the change in mass, because the density is changing, within this differential volume. In addition there is certain parts that have come in, and certain parts that have left; so for example, if I expand out a small region over here. Surface, which was initially at this location, has now moved to a new location, because of the fluid velocity, the fluid velocity could in general be in some direction at this location. This is the fluid velocity u, in some general direction, with respect to the surface, because the fluid is moving, the surface is also moved. So you should take this little patches surface, I will call it as d s, because of this little patches surface, how much of surface, how much of volume has come in, to this differential volume, because the surface is moved in this way.

The volume that has come in, is basically equal to this distance, which is the perpendicular distance of the surface, the perpendicular distance that the surface has travelled; that is this perpendicular distance, that the surface is travelled. What is the perpendicular distance that the surface is travelled in a time delta t, perpendicular distance travelled is equal to the velocity, along the perpendicular times time. So perpendicular distance travelled, is equal to the velocity times time; the velocity along the normal direction. so the velocity along the normal direction is u dotted with n times delta t, so that is the distance that is travelled. Note that u dot n is a velocity, is normal velocity.

The component of the velocity u, along the unit normal to the surface n. Note that this is the outward unit normal to the surface, so that is the distance travelled. The total volume that has come in is equal to the distance travelled, times that surface patch. So the total volume that has come in, into area, so volume that is come in, is this times d s. So there is a total volume that has come in, because the surface is travelled. Now the mass that has come in is just equal to these times, the density itself, density at that particular location; that is the mass that is come in.

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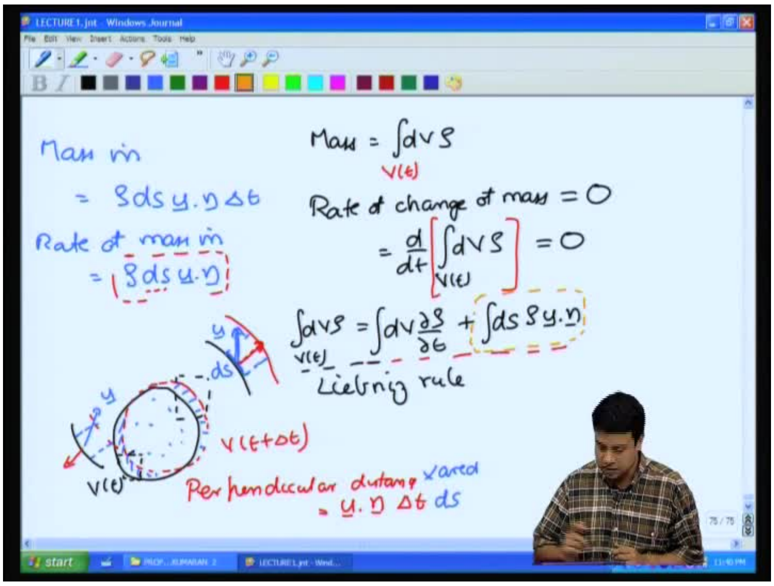

So therefore, the mass in, let me just get rid of this, so that we can write it all the same, is equal to rho d s times u dot m delta t; that is the mass that has come in for this little patch of surface, it has travelled, the distance u dot n delta t. So that is the mass m unit time delta t. Therefore, the rate of mass m, is going to be equal to this mass m divided by the time, this going to be equal to rho t s u dot n, this is the rate at which mass is coming in, for this little patch of surface, because the surface is moving. Now if you take a patch of surface on the other side. If you take a patch of surface on the other side, this surface initially it was something like this, so if I take this little patch, this surface initially it was here, and after some time it goes to some other location, after some time it goes to some other location.

So obviously some volume elements have left this volume, it has left behind some volume, as it is travelling in this direction, and therefore, the mass has left this volume, mass has been reduced from this volume, what is the mass that is reduced, the argument is exactly the same, I take this little patch of surface. Now the outward unit normal, is in this direction; that is the outward unit normal, at that point on the surface, it is in this direction, may be plotted on the original surface for simplicity. The outward unit normal in this direction, and the velocity vector is in some direction like this u vector is in this direction. The amount that has left is once again u dot n d s, there is a rate at which mass is left. Except that now u dot n is negative, because u and n the angle between them is greater than 90 degrees, if the volume l, if the velocity is going to be opposite, to the unit normal in such a way that, there is some volume that is left behind as the surface moves.

Therefore, for volume elements that are leaving the surface, u dot n is automatically negative, because the velocity is in opposite, the component of the velocity along the unit normal, is along the inward unit normal, it's opposite to the outward unit normal, so u dot n is automatically negative. And therefore, rho s rho d s times u dot n, will automatically be negative, and therefore, this expression gives you the rate at which mass comes in, and mass goes out. If mass is coming in, and if you define n is the outward unit normal, u dot n will be positive that will add mass to the volume. If it is leaving, then u dot n will automatically be negative. So this is the second component, for each patch of surface d s, we get this, therefore, the total, is just going to be equal to, integral over the surface, of rho u dot n. So there are two components; one because the, there is a change in density within this volume itself.

The other is, because as the volume is moving in space, as a function of time. There are elements, volume elements that are coming in, to this differential volume, there is volume elements, which are leaving this differential volume. This thing is what is called the Leibnitz rule t, this identity that integral d v rho over something that is function of time. This is called the Leibnitz rule, and it works not just for density, but for all other things. It works for momentum, for energy, and so on. In all such cases, you have a change in that quantity, because one that quantity is changing within the volume itself.

Second is, because as the volume is moving, there are several regions that are being struck in, and there are certain regions that are being left behind, inside the account for both those regions that has being swept in, as well as the regions that have being left behind. And that those regions that have being swept in, and the regions that are being left behind, are incorporated in this second surface integral term, there are both incorporated and this second surface integral term. So that is the Leibnitz rule, and we know that the rate of change of mass has to be equal to zero, for a mass conversion.

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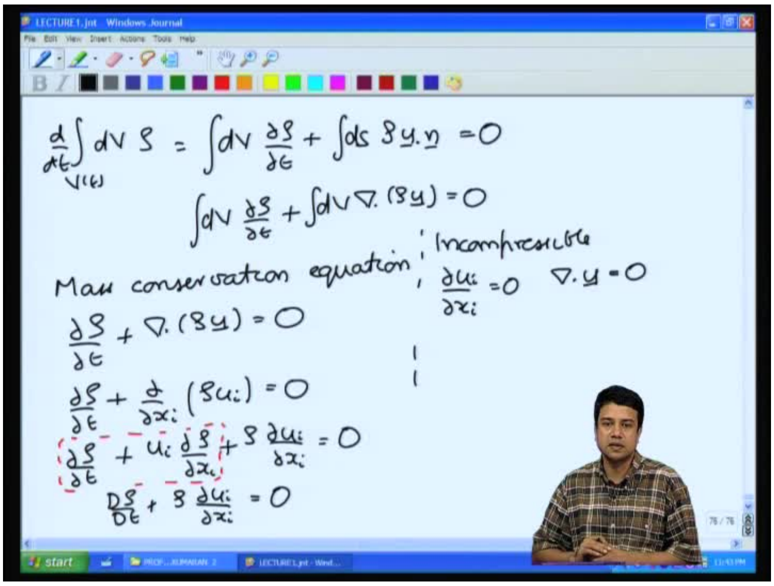

Integral v f t d v times rho, the rate of change of that, which I had derived as integral d v times partial rho by partial t plus integral over the surface d s of rho u dot n, this has to be equal to zero. For the second term here I can use the divergence theorem, vector dot n integrated over a surface, is equal to the divergence of that vector, integrated over the volume. So this integral d v partial rho by partial t plus integral d v of del dot rho u, this is equal to 0. And this has to be true for every volume within the fluid, because it is true at each and every point within the fluid. And therefore, it has to be at every, if it is true for each and every volume that I can consider; this is true at each point within the fluid. And that means that the mass conservation equation; partial rho by partial t plus del dot rho u is equal to zero, that is the equation for the conservation of mass.

In indicial notation, I will write this as, partial rho by partial t plus partial by partial x i of rho u i is equal to zero, dot product repeated index it is a scalar equation for the density. I can also write it as use differentiation by chain rule, to write this as. This differentiation by chain rule, and you can identify this first two terms, as the substantial derivative. These first two terms are identical to the substantial derivative, so I can also write this as d rho by d t plus rho partial u i by partial x i is equal to 0. If the density is a constant, then d rho by d t in a moving reference frame has to be 0, and therefore, I have the mass conservation equation for constant density, incompressible. Density is a constant, I have partial u i by partial x i is equal to zero, over the divergence of velocity is equal to zero.

Divergence of velocity is equal to 0, as I just discuss the isotropic part of rate of deformation, is tensor is zero. There is no radially outward or inward flow.

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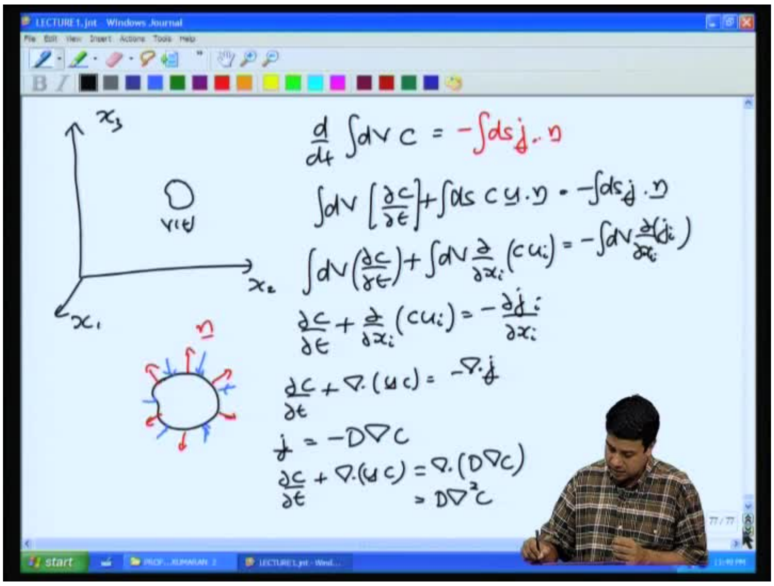

I can derive the other conservation equations, just as easily. If I wanted to derive an equation for concentration field, what I would say is that for this moving volume element for this moving volume element d by d t of the integral d v times the concentration, of the solute. The concentration of the solute, in this moving reference frame is equal to an integral over the surface, is equal to an integral over the surface, of the flux, times a unit normal. So if I have some volume element, which is moving, and I have some flux on the surface. Then d by d t of the integral of the volume times, the concentration is equal to the net flow into this volume, due to the flux is acting on the surface. The flux will increase the concentration within this volume, if it is directed inwards. So the concentration, so the mass within this volume is going to increase, if the flux is directed in the direction of opposite, to the outward unit normal.

The mass within this volume is going to increase, if the flux is directed outward, along the inward unit normal, or opposite to the outward unit normal. I have defined my unit normal n as the outward unit normal in this case, and therefore, this is equal to minus integral over the surface of q dot n. When q dot is negative, the mass increases, so when q and n are in opposite directions, the mass increases. So this equal to minus integral over the surface, of the mass flux; that is the amount of material coming in, per unit surface area per unit time. Simplify this using the Leibnitz rule once again, I get a integral d v of partial c by partial t plus integral d s of c u dot n minus integral over the surface of q dot n. And now, we use the divergence theorem, for both the convective part, as well as for the flux.

So then I get integral d v partial c by partial t plus integral over the volume of the divergence of concentration times the velocity is equal to minus integral over the volume of the divergence of the flux itself, of the flux, the divergence of the flux. And this has to be true for each and every differential volume; that means it has to be true at each point in space. And therefore the concentration equation becomes partial c by partial t plus minus partial q i by partial x i or d c by d t plus del dot u c is equal to minus divergence of q. Recall we got exactly the same expression by doing our differential balances of a volumes in the fundamentals of transport processes one. There will be the differentially balances over cubic volumes, spherical volumes, cylindrical volumes. In this case we get the exact same expression without reference to any underline coordinate system.

And then, we have a constitutive relation for the, I should use j for the mass flux. Let me just use j for the mass flux to avoid any confusion, q is better use for the heat flux, so this as j, this is j i this is j, and this is delta j. And then we have the constitutive relation j is equal to minus d times that gradient of c, where d is the diffusion coefficient, and the flux is in the direction of decreasing concentration, so it is opposite to the gradient vector, gradient vector gives you the direction in which there is a maximum increase in concentration. In the constant of proportionalities, the diffusion coefficient, with dimensions of n square by time. And so from this, you will just get the equation, partial c by partial t plus del dot u c is equal to the divergence of d grad c. And is equal to d times the Laplacean, provided if the diffusion coefficient is in a point of position, is equal to d times the Laplacean, if the diffusion coefficient is in a point of position, and this is the exact expression that we got for the concentration equation, in the previous course.

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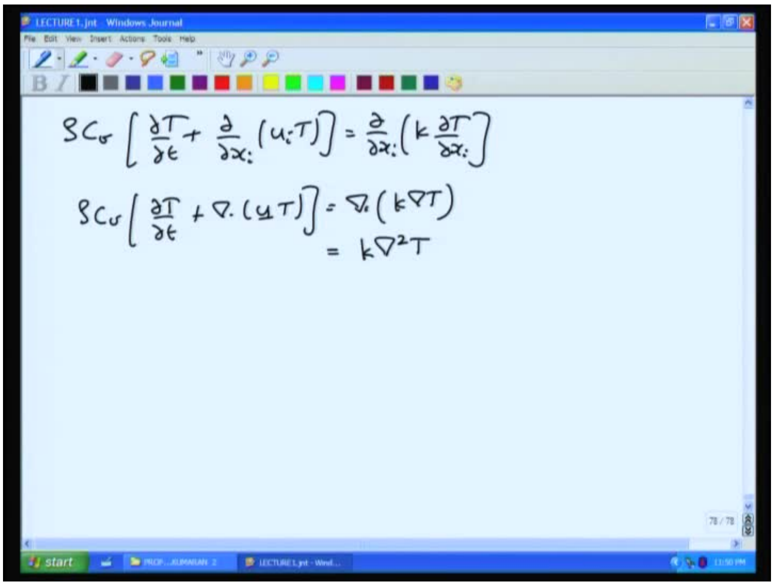

Similarly, one can get an expression for the temperature equation, and if I write it out in in this form we will get rho c v into partial t by partial t plus d by d x i of u times t is equal to of k partial t by partial x i or attentively rho c v into plus del u t is equal to del dot k grad t. And in case, the thermal conductivity is in a point of position, this can also be written as del square, sorry del square is the Laplacian, and I had shown you in the previous lecture, how we derive the Laplacian in the different coordinate systems, spherical, cylindrical, as well as Cartesian. In spherical and cylindrical, you have to account for the fact that, the unit vectors dependent upon position, and therefore, the dependence of unit vectors in position, has to be taken into account, and we derived explicit expressions, for each of these.

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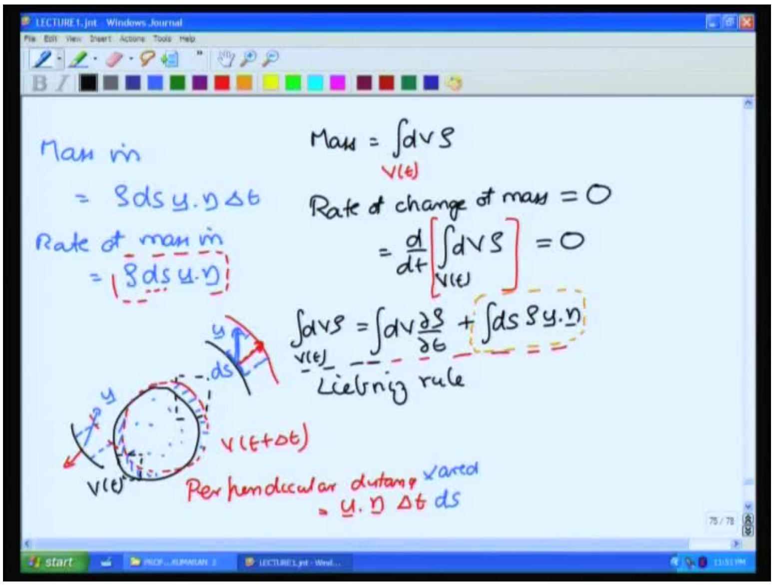

So, these are the mass heat conservation equations. Note that in the previous course, we had actually taken a differential volume, taken it surfaces, six of them, found out what came in, what went out, and on that basis, determined what the conservation equation was; that is in this case we do not have to do any of those. We just use the divergence theorem, and use the fact that, the Leibnitz rule can be applied for a differential volume. In order to relate the change in a moving reference frame, to that in a fixed reference frame, and that is all we require in order to find out the conservation equations. So the next step is to proceed in determine conservation equations for the fluid momentum. This procedure as you can see is much simpler, had I done it the way that I done it previously.

I would have to have three components of momentum, and for each of those, I had to have three components of fluxes. Whereas, here I am just going to treat both momentum and flux, as objects in themselves, independent of the underline coordinate system. So we will proceed with the conservation equations for, momentum for the fluid in the next lecture. So kindly review what we will done here; that is how do you relate the changes in a moving reference frame, to that in a fixed reference frame, and we will proceed in the next lecture, to deal with fluid momentum conservation equation.

So we see you then.