Mass Transfer Operations-I
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Module - 1
Diffusion Mass Transfer
Lecture - 3
Fick's law of Diffusion

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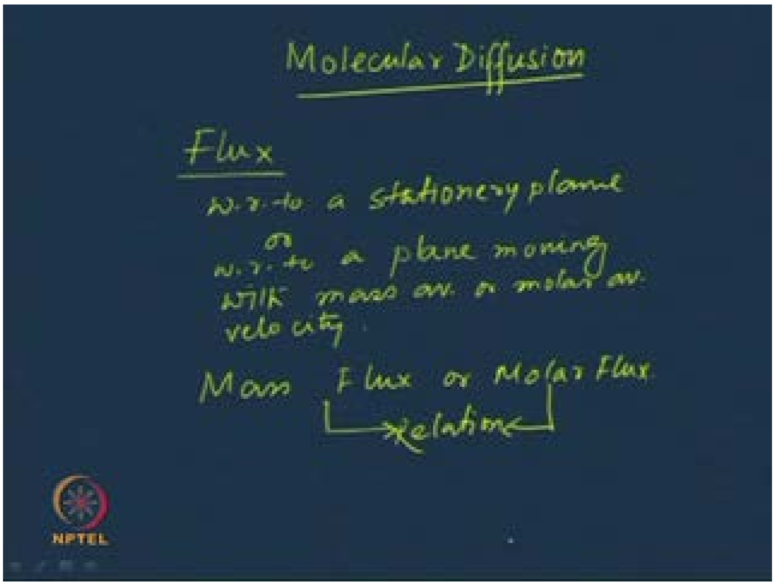

Welcome to the third lecture on mass transfer operation; the third lecture will be on the Fick's law of diffusion under steady state and unsteady state conditions. So before going to the third lecture, let us have a small recap on the last lecture we have; the last lecture was on molecular diffusion. Here, we have seen that the molecular diffusion is much slower compare to the eddy diffusion or turbulent diffusion, and we have seen that the effect of barrier on the molecular diffusion is very important, if we put some stagnant layer on the water surface, it reduces the diffusion by around 600 times. Then we have seen the concentration gradient is very important for the mass transfer for a single phase, and in that case the concentrations are defined in various ways. So, we have seen how the concentration is defined in terms of the mass concentration and in terms of the molar concentration. And we have also seen how to calculate these concentrations for a particular system; we have solved few examples on it.

Then we have seen the molecular velocities or the diffusion velocities is important for the mass transfer or diffusion, we have seen there are two types of velocities - one is mass average velocities, another one is molar average velocities; and how to define that and how to calculate for a particular systems that we have discussed. We have also discussed the fluxes; flux and we have seen that the flux maybe defined with respect to a stationary plane with respect to a stationary plane or with respect to a plane moving with mass average or molar average velocities.

So, depending on that it can be defined as the mass flux or molar flux, mass flux or molar flux, then we have seen the relations between, between the mass average and molar average velocity. And then we had questions and discussion sections at the end.

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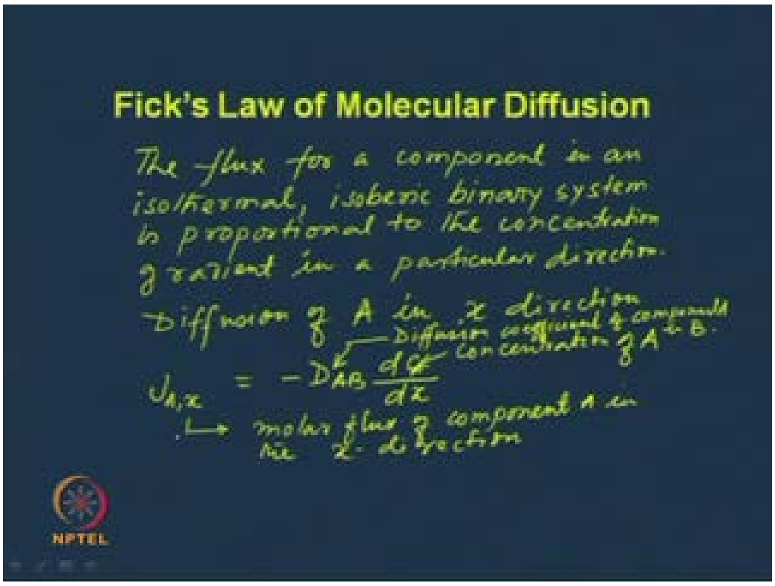

Today we will discuss on the Fick's first law, and Fick's law of molecular diffusion says that flux for a component in an isothermal isobaric binary system, binary system is proportional proportional to the concentration gradient in a particular direction. So, if we consider diffusion of A in x direction we can write the flux of A J A X is equal to minus D A B d C A dx, where J is the molar flux of component A in the x direction, C A is the concentration of A, and D A B is the diffusion coefficient diffusion coefficient or diffusivity of component A in B.

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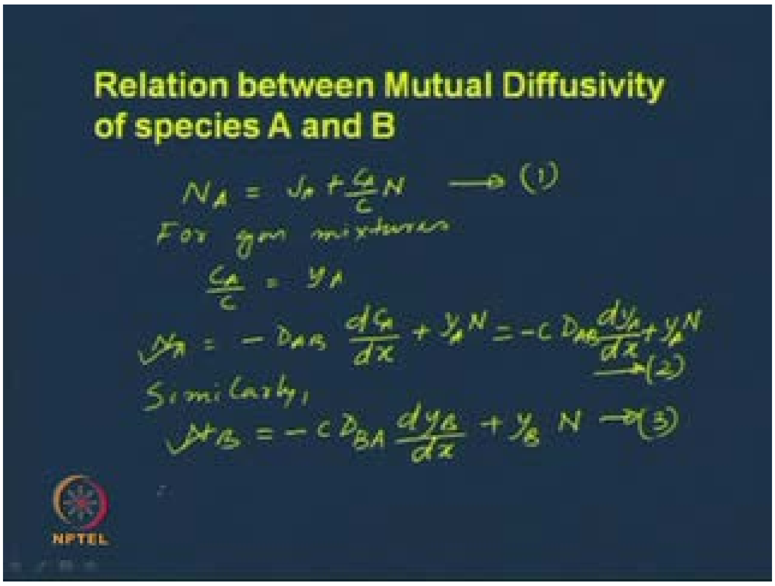

So, we can relate between flux as per the Fick's law of molecular diffusion, if there are two species A and B; and they are diffusing both the ways in that case we can derive a relation between their mutual diffusivities of component A and B. So, for a binary mixture if we consider the flux N A we can write is J plus C A by C into N, and for gas mixtures we can write C A by C is equal to y A. So, this is equation one and we know that from this we can write N A is equal to J we know, D A B d C A dx plus y A N. So, this we can again write is equal to minus C D A B d y A dx plus y A N, this is equation 2. Similarly, we can write N B is equal to minus C D B A d y B dx plus y B into N; N is the total flux. So, if we sum between N A and N B, so N A is this and N B is this, if we sum between N A and N B.

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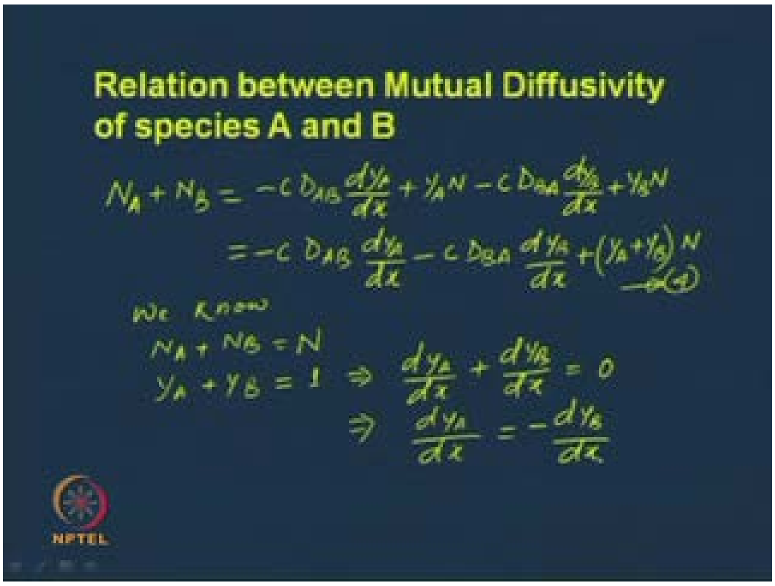

So, we can write N A plus N B is equal to minus C D A B d y A dx plus y A N minus C D B A d y B dx plus y B N. So, we can write minus C D A B d y A dx minus C D B A d y B dx plus y A plus y B into N. So, this is equation number 4.

We know that N A plus N B is N, and y A plus y B is equal to 1. So, from this if we differentiate, so it will be d y A dx plus d y B dx is equal to 0. So, it will lead to d y A d x will be equal to minus d y B d x. Now, if we substitute this d N A plus N B is equal to N over here, and then y A plus y B is equal to 1 over here and d y A dx is equal to minus d y B dx.

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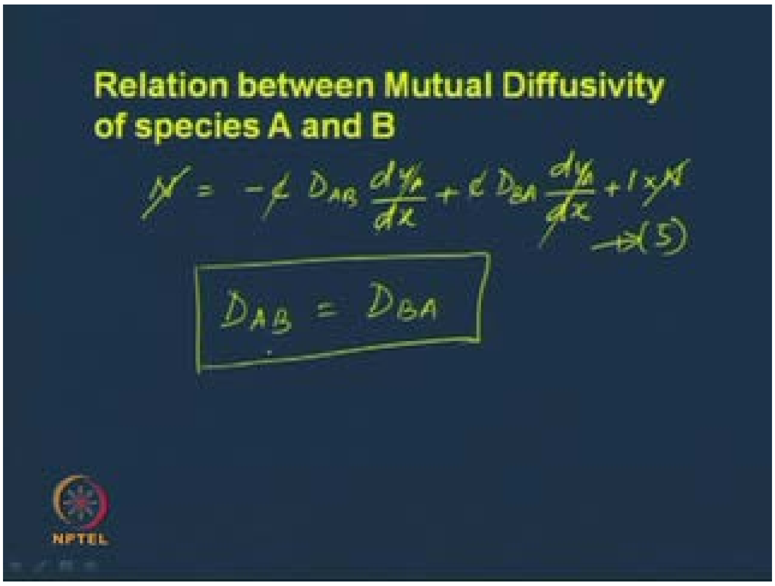

So, it will give you N is equal to minus C D A B d y A dx plus C D B A d y A dx plus 1 into N. So, this is equation number 5, and in this N will cancelled out, hence we can write this also we will cancel out and we will have D A B is equal to D B A. So, we conclude that the mutual diffusivities between A and B; that is D A B and D B A are same.

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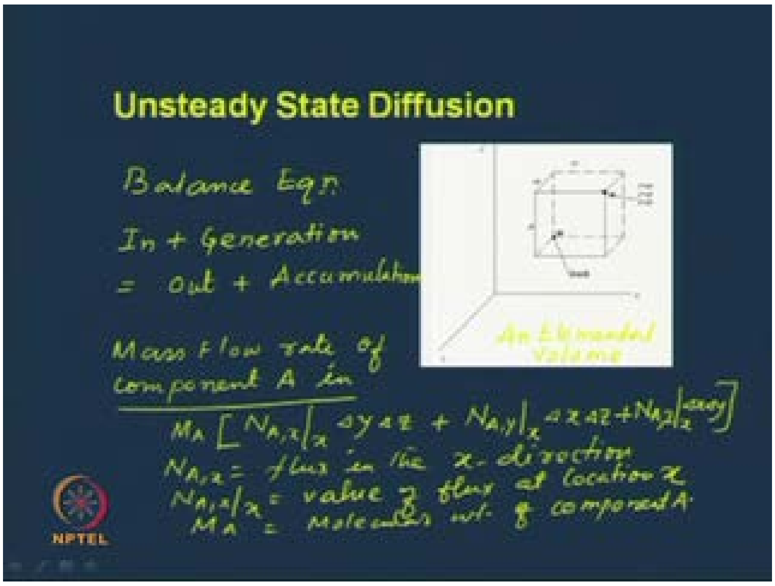

So, now we will move to unsteady state diffusion; the unsteady diffusion it is defined as the change of the concentration of a particular component in the mixture over the time.

So, that is unsteady state diffusion. We will try to derive the unsteady state diffusion equation, and which is the Fick's second law of diffusion. Let us consider the control volume and elemental volume, and the balance equations are in plus generation is equal to out plus accumulation. So, mass flow rate of component A mass flow rate of component A in at location m we can write M A into N A x at x delta y delta z plus N A y at A x delta x delta z plus N A z at x delta x delta y. So, this is the mass flow rate of component A in at location x. So, N A x flux in the x direction, and N A x at x is equal to value of flux at location x, and M A molecular weight molecular weight of component A. Now, if we do the rate of generation or production of component A by chemical reaction.

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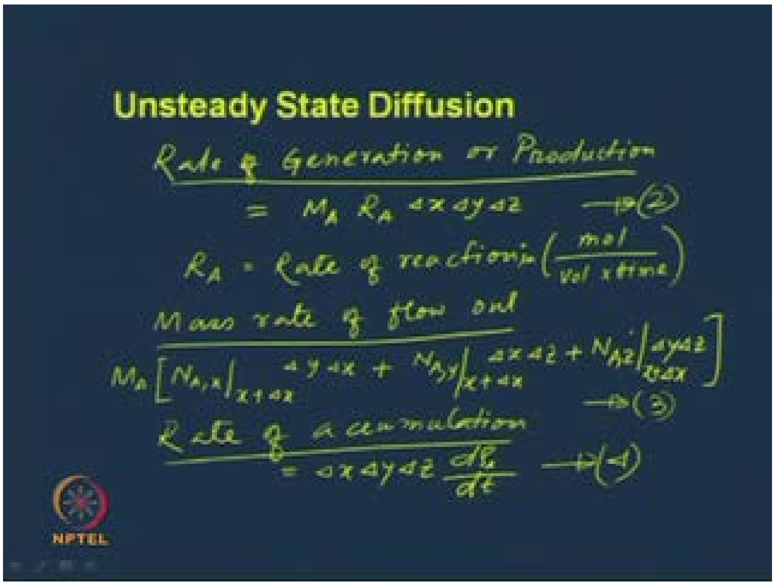

We can write rate of generation or production, this will be equal to molecular weight into rate of reaction into the control volume delta x delta y delta z. So, R A is equal to rate of reaction and it is the unit will be mole per volume into time, and the rate of mass flow rate out we can write mass rate of flow out. So, let us say out will be... So, at location N, what is the mass flow rate out, that is at distance x plus delta x y plus delta y and z plus delta z. Similarly, the equations remain similar, but at location N. So, for this control volume the mass rate out, we can write m A into N A x at x plus delta x delta y delta z plus N A y x plus delta x will be delta x delta z plus N A z at x plus delta x will be delta y delta x.

So, this is the mass flow rate out, and then another term is the accumulation. So, rate of accumulation, rate of accumulation we can write will be equal to delta x delta y delta z; this is the volume into d rho A dt. So, now if we just write the equation number equation number 1, so equation number 2, this is 3, and this is 4.

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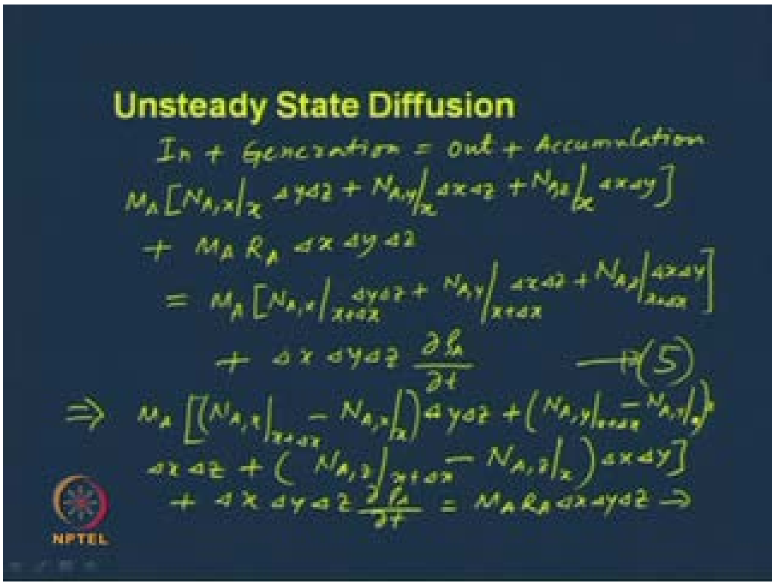

If we substitute this 4 equations in the balance equations original balance equation. So we can write the original balance equation was in plus generation is equal to out plus accumulation, now if we substitute it will be M A at N A x at x delta y delta z plus N A y at x delta x delta z plus N A z at x delta x delta y, this is mass flow rate in plus generation is M A R A delta x delta y delta z. So, this is generation and then out will be M A N A x at x plus delta x delta y delta z plus N A y at x plus delta x delta x delta z plus N A z at x plus delta x delta x delta y. So, this is the mass flow rate out plus accumulation; the accumulation term is delta x delta y delta z into del rho A - rho A is the density of component A del t.

So, this is the equation number 5, and if we just rearrange these equation we can write M A into N A x at x plus delta x minus N A x at x into delta y delta z plus N A y at x plus delta x minus N A y at x into, we can write delta x delta z plus N A z at x plus delta x minus N A z at x multiplied by delta x delta y. So, this is first term and then we can write plus delta x delta y del z del rho A del t is equal to M A R A delta x delta y delta z. So, this is equation number 6.

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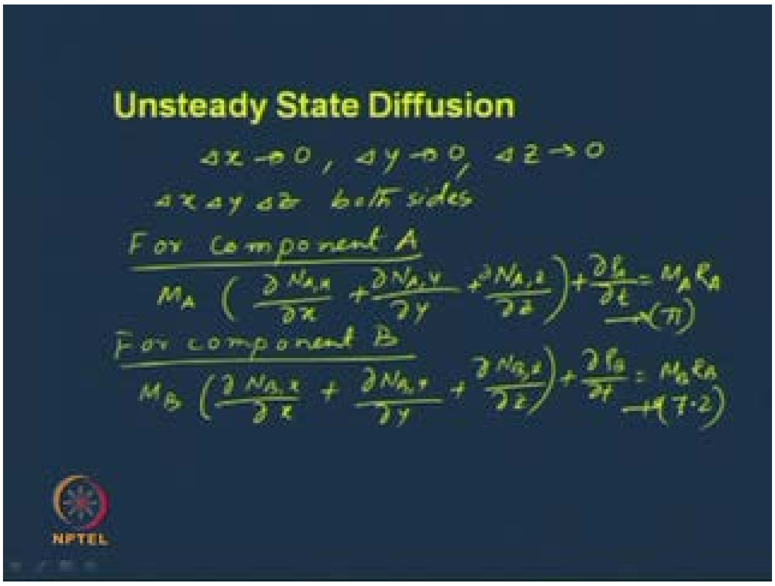

Now, if we take limit delta x tends to 0, delta y tends to 0 and delta z tends to 0, then divide by delta x delta y and delta z; both sides of the previous equation 6 both sides, then for component A - component A we can write M A into del N A x del x plus del N A y del y plus del N A z del z plus del rho A del t is equal to M A R A. So, this is equation number 7.1. So, similarly for component; component B we can write M B del M B x del x plus del N B y del y plus del N B z del z plus del rho B del t is equal to M B R B. So, this is equation 7.2; so if we at equation 7.1 and 7.2.

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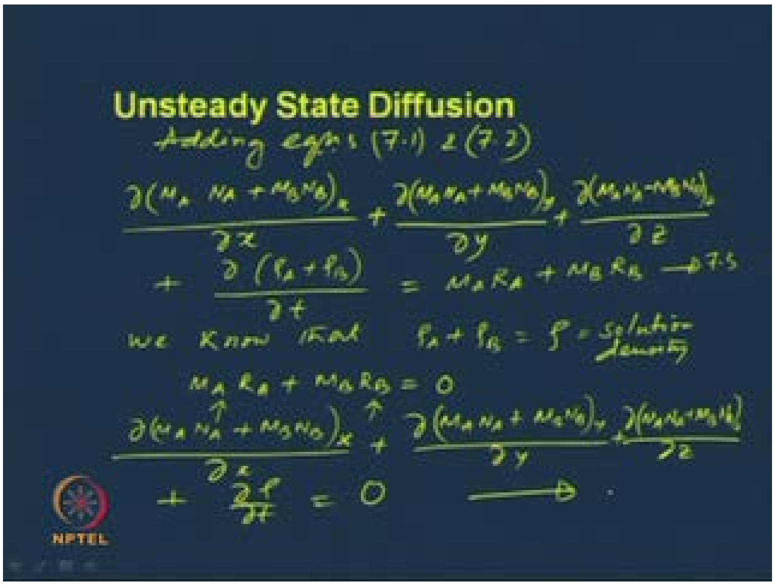

We can write 7.1 and 7.1, we have del M A N A plus M B N B at x will be del into del x plus del M A N A plus M B N B at y del y plus del M A N A plus M B N B at z del z plus del rho A plus rho B del t will be equal to M A R A plus M B R B. We know that rho A plus rho B will be equal to rho a solution density, and M A R A plus M B R B is equal to 0, because the mass rate of generation of A and B, mass B equal to 0. So, in that if we substitute these two in this equation - equation number 7.3, we will have del M A N A plus M B N B at x del x plus del M A N A plus A B N B at y del y plus del M A N A plus M B N B at z del z plus del rho del t is equal to 0. So, this is equation number 8.

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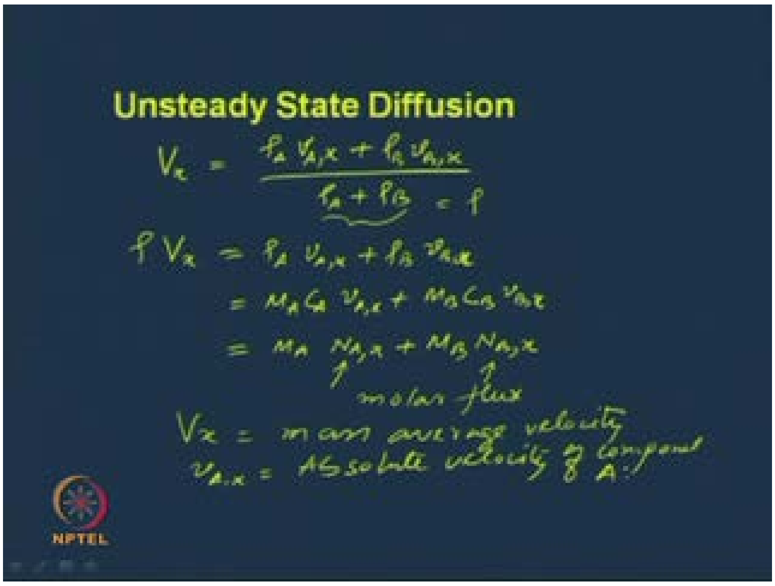

Now, we know that the mass average velocity is V x is rho A v A x plus rho V v B x y rho A rho B. So, we can write rho A and rho B, this is equal to rho, so rho the mass average velocity V x will be equal to rho A v A x plus rho B V B x, and rho A we can write M A C A and then v A x plus M B C B v B x this will be equal to M A N A x plus M B N B x. N A or N B; these are the molar flux, V x is the mass average velocity velocity, and small v A x is the absolute velocity of component A, absolute velocity of component A.

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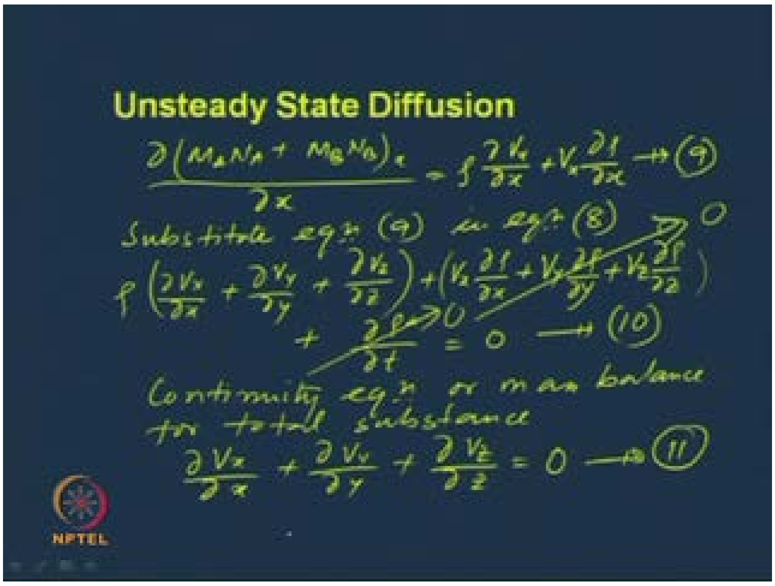

So, then from this we can write del M A N A plus M B N B at x del x will be equal to rho del V x del x plus V x del rho del x. So, this is equation number 9. Now, if we substitute equation 9 in equation 8, the equation rho del V x del x plus del V y del y plus del V z del z plus V x del rho del x plus V y del rho del y plus V z del rho del z plus del rho del t is equal to 0, so equation number 10. So, this is the continuity equation, continuity equation or mass balance, balance for total substance. Now, if the solution density rho is constant if this is constant, so this part will be 0, and this will be 0; in that case, we can write del V x del x plus del V y del y plus del V z del z equal to 0. So, equation number 11.

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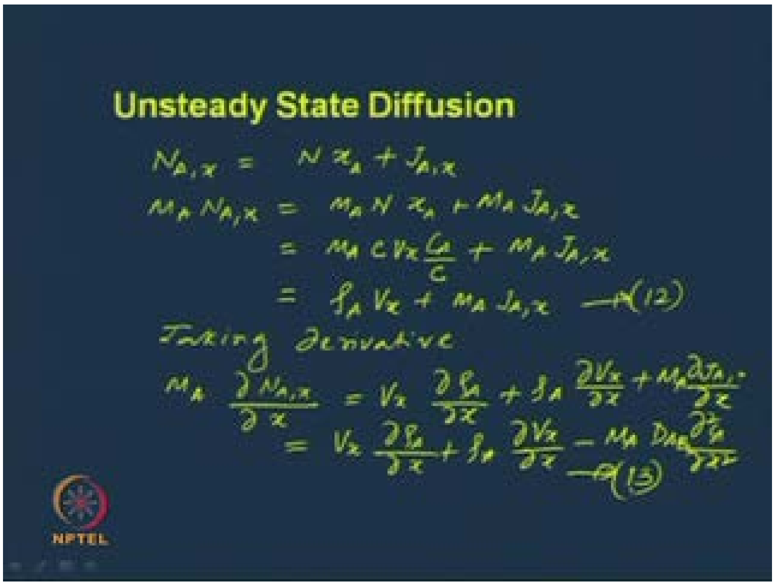

Now, we know that the flux N A x is related with total flux into the mole faction or mass fraction plus J A x. So, if we multiply both sides by molecular weight we can write M A N A x will be equal to M A N x A plus M A J A x, from this we can write M A C into V x into C A by C plus M A J A x. So, this will be equal to rho A V x plus M A J A x, if we take the derivative of this equation. So, suppose this is equation number twelve, we take derivative, so then we can write M A del N A x del x will be equal to V x del rho A del x plus rho A del V x del x plus M A del J A x del x. So, we can write this will be equal to V x del rho A del x plus rho A del V x del x minus M A D A B del 2 C A del x 2, then if we substitute this equation in equation number 7.1.

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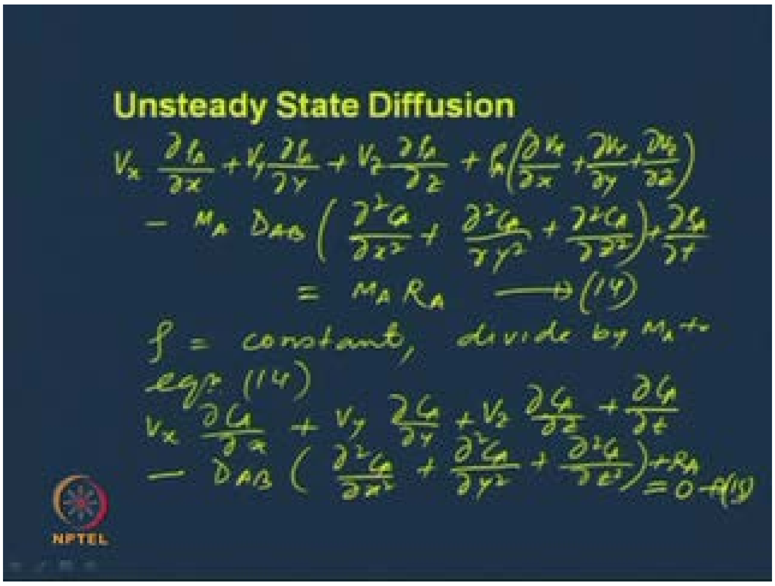

Then we will have the final form V x del rho A del x plus V y del rho A del y plus V z del rho A del z plus rho A del V x del x del V y del y plus V z del z minus M A D A B del 2 C A del x 2 plus del 2 C A del y 2 plus del 2 C A del z 2 plus del rho A del t equal to M A R A, this is equation number 14.

Now, when rho is constant constant, and using equation 11, so in this case, if we divide divide by M A to equation 14, we have V x del C a del x plus V y del C A del y plus V z del C A del z plus del C A del t minus D A B into del 2 C A del x 2 plus del 2 C A del y 2 plus del 2 C A del z 2 plus R A equal to 0. So, this is equation number 15.

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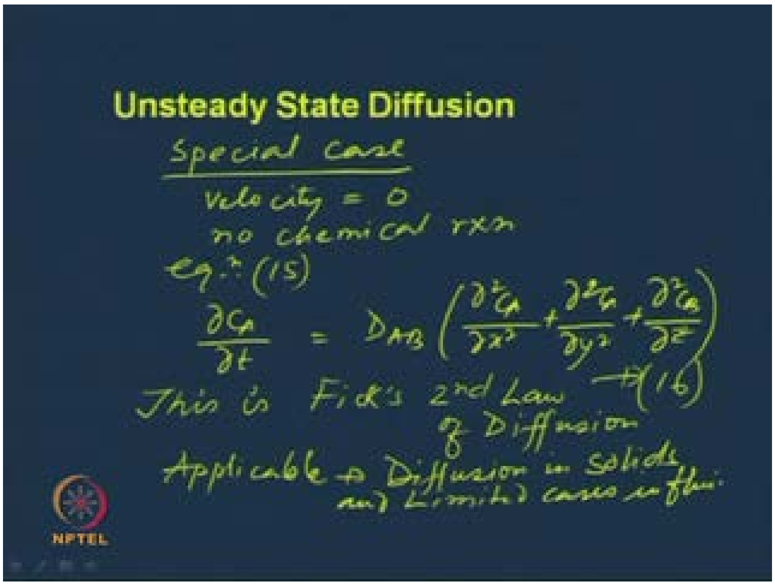

So, if we consider a special case, when velocity is equal to 0 and no chemical reaction. So, in that case the equation 15, we can write del C A del t will be equal to D A B del 2 C A del x 2 plus del 2 C A del y 2 plus del 2 C B del z 2. So, this is equation number 16, and this is Fick’s second law of diffusion, this is Fick’s second law of diffusion. So, this is mainly applicable for diffusion in solids diffusion in solids, and limited cases, cases in fluids. So, this is end of lecture three of module one, and the next lecture, we will start with steady state molecular diffusion.

Thank you.