

Indian Institute of Technology Madras
Presents

NPTEL
NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING

Aerospace Propulsion
Cycle Analysis-Turbojet V

Lecture 15

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In the last class we had derived expressions for ISP and non dimensional thrust for a case with the after burner switched on and for the flow through the nozzle we had assumed that the flow is optimally expanded through the nozzle.

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for choked flow through the nozzle $\eta = 1$.

$$\frac{F}{m_0} = \left[\sqrt{\frac{T_7}{T_0}} - M_0 \right] + \frac{P_7 A_7}{m_0 a_0} \left(\frac{P_7}{P_0} - 1 \right)$$

for after burner switched on

$$\frac{T_7}{T_0} = \frac{Q_{ab}}{h c_p m_0} \quad M_7 = 1$$

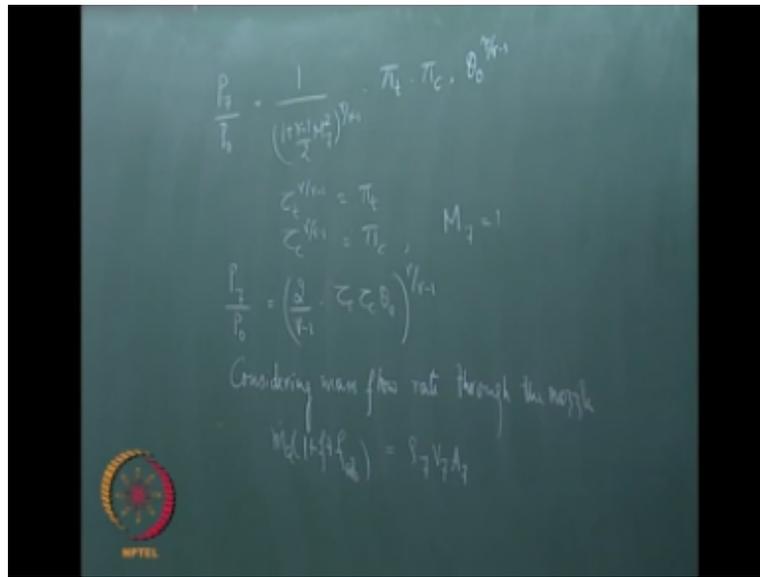
The image shows a chalkboard with handwritten mathematical equations. The top part discusses choked flow through a nozzle with efficiency $\eta = 1$. The main equation is $\frac{F}{m_0} = \left[\sqrt{\frac{T_7}{T_0}} - M_0 \right] + \frac{P_7 A_7}{m_0 a_0} \left(\frac{P_7}{P_0} - 1 \right)$. Below this, it says 'for after burner switched on' and provides $\frac{T_7}{T_0} = \frac{Q_{ab}}{h c_p m_0}$ and $M_7 = 1$. There is a small NPTEL logo in the bottom left corner of the chalkboard image.

Now let us take the other case where in the flow through the nozzle is a choked flow okay and efficiencies are all one and we will see how this case looks like so we had done a very similar analysis without the afterburner being switched on and we had got this expression for non

dimensional thrust as $f / m \cdot A_E$ not this was the expression that we had for the non dimensional thrust for a choked flow.

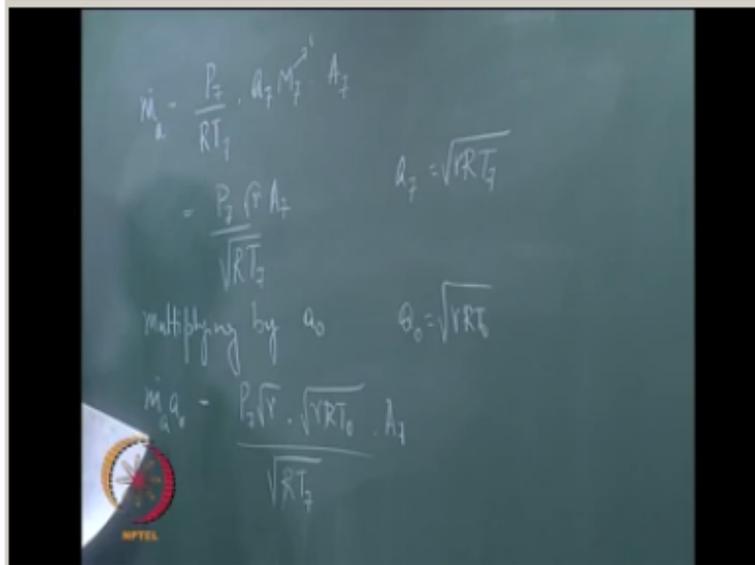
Through the nozzle with all efficiencies being one and this is the same expression that we will have with the afterburner switched on also only these quantities change okay so for after burner switched on we need T_7 / T_0 and in the previous class we have seen that this is nothing but $\eta_a \frac{\gamma - 1}{2} M_7^2$ right we know that for a choked flow through the nozzle what is it that we know $M_7 = 1$ so substituting it here we will get that it this is the expression for T_7 / T_0 and similarly we need expression for P_7 / P_0 .

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When we cascade we will get this one $\frac{\gamma - 1}{2} M_7^2$ right into πT I see okay this is the expression that we get now I also know that I can write πt as τT to the power of γ by right and I will substitute this I because all of them will then be raised to the same power so it will be easier for me to handle it I will get and I also know that $M_7 = 1$. So substituting this I will get symmetry so we have got pressure ratios also then for getting we have got this ratio we need to get this quantity just like in the previous case we will get this by looking at mass flow rate through the nozzle considering mass flow rate through the nozzle we get that $M \cdot \rho \cdot A \cdot V$ in this case must be equal to $\rho_7 V_7$ now again we can use what we had done earlier that $f + f a b$ is very much less than 1.

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Then I will get in dot $e = \rho_7$ I can write it as $e_7 / RT_7 \times V_7$ is a seven in time seven this I know is one okay for the tube nozzle so I get a seven if I write it as γRT_7 seven I will get e_7 into under root γe_7 now again we multiply both sides by a_0 so I get what I was looking for $m \dot{a}_0 = p_7$ under root γ you notice again γRT not so I will use that right so I can cancel out R&R here.

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dividing by $P_0 A_0$

$$\frac{\dot{m}}{P_0 A_0} = \frac{\gamma P_0}{P_0} \sqrt{\frac{T_0}{T}}$$

$$= \frac{\gamma \sqrt{\gamma}}{\sqrt{2\gamma}} \left(\frac{P_0}{P} \right)^{1/2} \left(\frac{T_0}{T} \right)^{1/2}$$

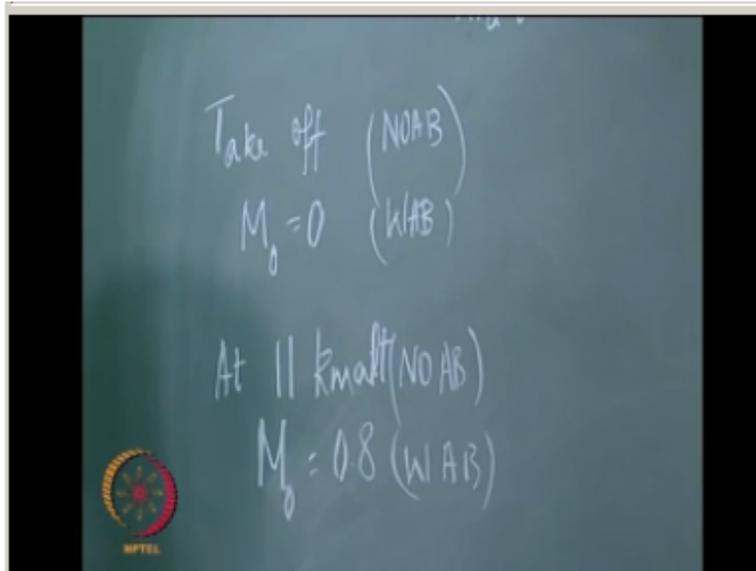
$$\frac{F}{\dot{m}} = \left[\sqrt{\frac{P_0}{P}} - M \right] + \frac{1}{\gamma} \sqrt{\frac{P_0}{P}} \left(\frac{P_0}{P} \right)^{1/2} \left[\sqrt{\frac{P_0}{P}} - 1 \right]$$

And I will be left with $\gamma p_0 / p$ now if I divide both sides by p_0 / p you get the expression that we wanted and not a a_0 / P $\gamma \times p_0 / p$ and we know the values of both of them p_0 / p and T_0 / T so if we substitute it will get this is the expression that we get now we know all the three quantities that we were looking for so I can write f by $m \dot{a}$ not as equal to under root t_0 / t which is nothing but $2 \gamma \times \sqrt{p_0 / p} - M + \frac{1}{\gamma} \sqrt{p_0 / p} \left[\sqrt{p_0 / p} - 1 \right]$.

That is the same quantity okay this is the expression let me get what $F / m \dot{a}$ foreign to flow condition with the afterburner switched on you can also similarly get an expression for p only thing that needs to be changed is that f by $m \dot{a}$ knot is this expression right the only other things in the ISP by a knot expression that we derived earlier holds good so you just need to change the f by $m \dot{a}$ not expression so you have now got good things that is for an optimal expanded flow.

And for a choke nozzle choke nozzle is the more general case and optimally expanded flow as I said is a very special case of choke model wherein the exit pressure is equal to the ambient pressure now just like what we had done earlier that is look at typical calculations and see what happens with the afterburner turned on I will use the same set of numbers that I had used for a choked of flow in the nozzle so we will keep the flow in the nozzle as choked and use the same set of parameters and find out what happens with f by $m \dot{a}$ knot and p if the afterburner turned on.

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So first is take off that is $M_0 = 0$ and at 11 kilometer altitude M_0 not being equal to point it so we look at firstly f / \dot{m} not and then we look at P and sfc , sfc is in kg , kg are and we look at two ratios that is and okay and we look at two conditions here one on a bee that is no afterburner and then the other one with after burner again here same thing okay now you are using all these parameters the same.

So I will get f by \dot{m} notes 2.27 and 2.15 similar to the last case that we had looked at now let us look yes bookcases being for the joke nozzle so let us look at what happens now with the afterburner being switched on all this is for a tube nozzle now let us look at what happens to these values when we switch on the after burner with afterburner this goes to a fairly large value of 6.07 sorry 3.67.

And p reduces through 29.5 and this increases to 1.2 and similarly this goes to 3.851.37 and this ratio f by \dot{m} that is the non-dimensional thrust how much do we get an increase from with the after burner being switched on to without it this ratio would be 1.6 to of this case and 1.79 or 1.8 for the second case okay and sfc ratio is what we need to pay to get this kind of ratio if you look at this.

Okay what we see is that we need to really sell out a lot more fuel if we have to get a trust increment by this much if you have to get in this case sixty to eighty percent trust increase we have to spend something like fifty to eighty percent on the sfc right so it is a very large sfc

increase compared to what you get for the thrust you can also put it the other way that if the same thing was done.

Let us say if we had if you remember our discussions wherein we talked about the limitation on turbine Inlet temperatures we said turbine inlet temperatures cannot be raised because of material considerations and therefore we limit it by using excess air now suppose we were free to increase this to a larger value then the same kind of thrust increase remember turbine Inlet temperature is in the hands you can give the more fuel to the main burner or less fuel.

But there is an overall limit right that can be changed within certain limits if we were allowed to increase the turbine Inlet temperature the same kind of what you see as f by $m \cdot a$ ratio could have been obtained with same can be obtained with same ratio can be obtained with thirty-five percent if turbine Inlet temperatures could have been increased okay.

So what we are saying is you are spending eighty percent increase in sfc well the same thrust increment could have been got if you were allowed to increase the turbine Inlet temperature this shows the need for you know better materials where and we can go for a larger turbine Inlet temperature and when you can say one fuel if you want excess thrust you can increase the turbine Inlet temperature.

And get the same excess thrust at a much lower cost why does this happen why do we say that if we increase the turbine Inlet temperature if we are allowed to increase the turbine Inlet temperature the sfc would reduce whereas if you switch on the afterburner you see that sfc increase for the same thrust increment is more why does this happen the those velocities that's the one that gives the higher thrust that is not the reason what I am asking you is.

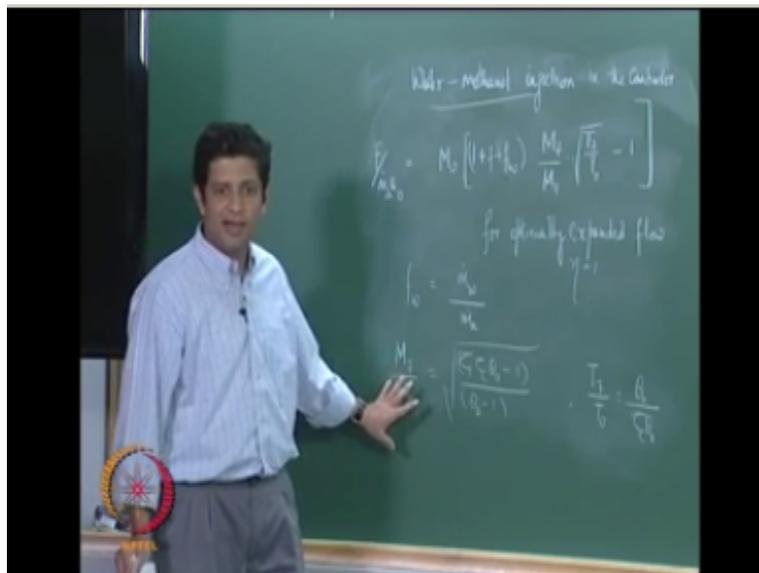
If you look at the two cases sfc increase with the afterburners which dawn is much more for the same thrust increment then the case if we take wherein we are allowed to increase the turbine Inlet temperature the one inlet temperature you could have got the same with a 35% increase whereas you are spending around eighty percent increase in sfc to account for the same thrust increment why should this happen as my question.

Why should burning fuel in the afterburner be more expensive now if you go back to our discussions wherein we talked about the need for the after burner I talked about something known as availability right if you add heat at a very high pressure then you have a opportunity to

expand for more than if you add heat at a lower for a much lower pressure what we are doing in the afterburner is we are we pass the divine where the flow has expanded already.

And the pressures are very low and there we are trying to add heat which means that the opportunity to expand is smaller and therefore we find that the sfc will increase because you are now expanding from a lower pressure to even lower pressure and but you are spending more fuel to increase that temperature okay now let us look at the next method of are getting thrust augmentation that we discussed earlier that is water-methanol injection.

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Now if you remember earlier discussions I said that you can add water in the main combustor and that will because the flow at the exit of the combustor is choked it will act as though it is increasing the pressure inside the combustor okay and we use water for that now water at very low temperatures that are encountered at higher altitudes who tend to freeze therefore we need to add some additives to make sure that it does not freeze that is why methanol is added.

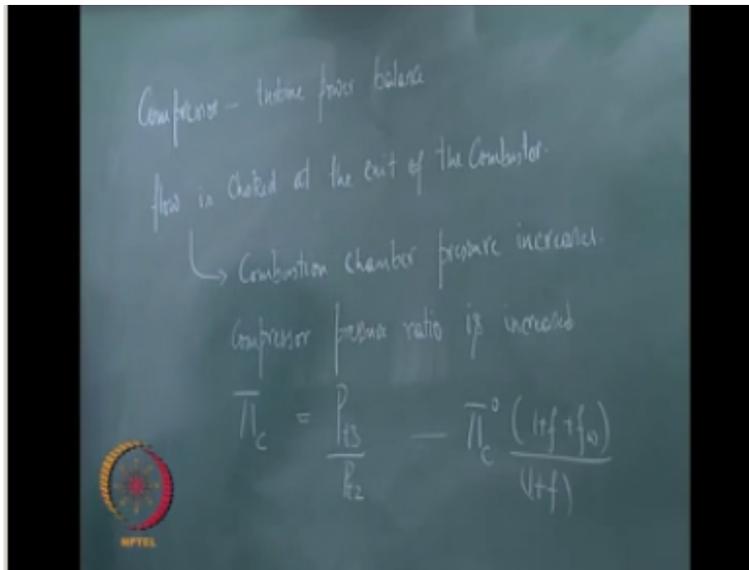
And also methanol provides the additional heat that is required to increase the temperature of water from ambient to the turbine Inlet temperatures okay so let us look at how this system works so we are going to consider the case where water and methanol are injected in the main combustor now what is our non dimensional thrust equation that is for optimally expanded flow and ether being what okay.

So this is the expression that we had for non dimensional thrust for these conditions right now if we are injecting water and methanol in the main combustor I need to add a term here okay it is not just $1 + F$ it should be a plus I will call this f_w whereas where f_w is nothing but $m \cdot \text{mass flow rate of water plus methanol}$ divided by mass flow rate of air so this is the expression that we have now here in this case.

I cannot say that combining these two it is much less than 1 because we will see that you can increase this to something like 30% of the overall flow so I cannot neglect this part compared to one dollar to retain it and do all the rest of the algebra again we need expressions $\frac{47}{T_0}$ and m_7 by m_0 okay.

Now in this case if you look at the expressions that we derived for m_7 / m_0 we have got earlier $m_7 / m_0 = \eta R^{-1}$ and $e^{7/D}$ not as equal to $\theta_B / \tau_C \eta$ not right both these expressions do not change the only change in this case comes about when we are looking at the compressor turbine power balance if you are adding water methanol in the combustor all the other expressions these expressions remain the same The only change will come in the compressor turbine power balance we will look at how that happens

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So I will say no changes here to the case without water methanol injection okay so let us look at the compressor turbine power balance okay as I said if you have if you add water in the combustor there are two things that happens one is the flow is choked at the exit of the combustor if the flow is choked then we know that it is only a function of upstream conditions upstream you are introducing the increasing the mass flow rate right you are adding water and methanol you are increasing the mass flow rate which means that.

The pressure upstream will also have to increase so what it essentially does is increases the pressure ratio across the turbine okay it acts as though the compressor pressure ratio is higher right so this combustion chamber pressure increases and therefore we find that it acts as though the compressor pressure ratio is increased now earlier we had a compressor pressure ratio of I see okay now which was nothing.

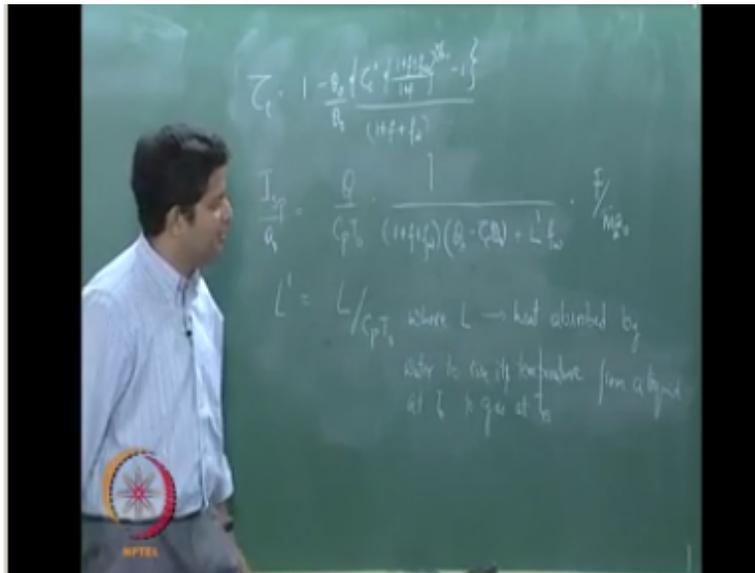
But PT_3 / dt_2 now because of the addition of water and methanol in the main combustor this will get changed to this will become I see not into $1 + F + f_w / 1 + f$ that is this is the new compressor pressure ratio it is the earlier compressed pressure issue excuse me it is the earlier compressor pressure ratio that you have multiplied by the increased mass flow rate divided by the earlier mass flow rate so there is a new compressor pressure ratio that you will get.

So if you look at the equation for the turbine compressor power balance what you will get is $\dot{m} \dot{a} CP$ because this pressure ratio across the compressor has changed it acts as though the temp the pressure ratio you will find that you can also Express this as in terms of τ_C not right and if

we do the analysis if we do this analysis what we will get is the expression for τT is equal to earlier equals $1 - \eta_0 / \eta B$ into of $C - 1$ this was the earlier expression.

Now there is an additional mass flow rate that is going through you if you remember we had neglected $1 + F$ which would be in the denominator now there is an additional mass flow rate that is going through and that cannot be neglected so you will have $1 + F + f_w$ here okay and the τC Part I see was earlier different now we knew τC would be $\mu \tau C = \tau C$ naught into $1 + F + f_w / 1 + f$ okay.

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So this is the expression that you will have for τC and if you plug it back in you will get the new expression for τT so you will get finally $\tau T = 1 - \eta_0 / \eta B \times \tau C_0 (1 + F + f_w) / (1 + f)$ so what really happens is the actual value of τT because you increase the mass flow rate and because you increase the pressure upstream of the turbine both the pressure and the temperature at the end of the turbine would be higher than compared to without water methanol injection which means that if you expand.

This flow through the nozzle you have more scope for expansion as well as the temperature at the end of expansion would be much higher therefore you get a larger thrust now obviously you

have to pay for it so ISP has got to be higher in this case why should ISPV higher voltage to it yes water we are injecting as a spray this needs to be water which is at T0 needs to be increased to the turbine Inlet temperature.

So you need to obviously burn fuel to do this so yes but still the latent heat you have to provide for know at least to bring it up to TT three you need to provide for it so this will mean that there is an increase in is p so if you take a look at is p / e 0 expression it will change to you by CPT 0 into earlier we were neglecting this part $1 + f + f_w$ we have to have earlier we did not have this f_w and we said f is very much smaller.

So we neglected it now you have that part into $\eta_b - \tau C \eta_{0+}$ into f_w into F by okay now L' is nothing but L by cpt not where LS heat absorbed by water to rise its temperature from a liquid at T not to gas at TT three then that remaining part is accounted for in this portion okay this is where you are taking it from TT three to a turbine Inlet temperature TT for this part accounts for the latent heat of water that you need to supply for raising .

Its temperature from T not to turbine Inlet temperature okay with Raul SOTA methanol it is yeah I have to say here in this analysis we have looked at only water do not account it for methanol you can account for it by suitably changing the L value okay n value in that case will get reduced so the ISP part will be a little higher okay.

Now if you were to do the same analysis as we had done earlier that is make a table and look at what happens with increasing percentage of methanol injection how much is the thrust increment that we get and what is the cost that we need to pay we will use the same set of values that we are used for the choke nozzle.

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f/w	$C_c (T)$	C_c	F/w_0	$I_{sp} (N/kg)$
0	2.0 (12)	0.8	2.46	41.8
0.2	2.4 (14.5)	0.91	3.28	34
0.3	2.8 (15.3)	0.92	3.7	31.8

That is $\eta_B = 5.55$ this is at eleven kilometer and $M_0 = 0.8$ τ_{C0} would be 2.03 and therefore η_0 would be 1.128 and F let me keep it as point 03e 0 = 294 meters per second if you have this then for varying fractions of f/w we can get τ_C and as I said only things that change in that expression or τ_C or τ_T so you have zero then it is 2.03 and this is 12 and this would be 0.82.46.

Because what we had seen earlier now I can shoot this in kilo Newton's so I will get 41.8 now if we increase it to something like point two percent okay then compressor pressure ratio will rise and correspondingly the temperature ratio will rise it will act as though the compressor is giving out air at 14.5 bar I mean 14.5 pressure ratio whereas the compressor was giving out a 12 it acts as though it is giving out at 14.5 and correspondingly this will also increase.

And you get a higher thrust but your ISP will be reduced and if you further increase it to something like oh point three this goes to 2.18 and the compressor pressure ratio increases this increases also so we see that as the FBM da to a_0 increases we are also getting a decrease in $i_s p$ this is because we need to add more heat here to increase the temperature okay then I will stop here in the next class we will look at what happens if we have efficiencies to deal with in the turbojet okay thank you.

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