Aircraft Dynamic Stability & Design of Stability Augmentation System

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Module 1

Lecture No 07

Aircraft Rigid Body Equation of Motion

Good morning friends. If I recapitulate whatever we are discussing, our primary focus is on understanding dynamic stability of an aircraft. And how did we start? We said, when I try to understand dynamic stability of an aircraft, we need to first identify the equilibrium about which we will be introducing a small perturbation and then we will write equations of motion using that perturbed variable. And then we will find the characteristic equation.

And typically, we have second order system and we try to interpret this transient response or free response through dynamic variables edit. We want to understand the dynamics of mass spring damper system through damping ratio and natural frequency.

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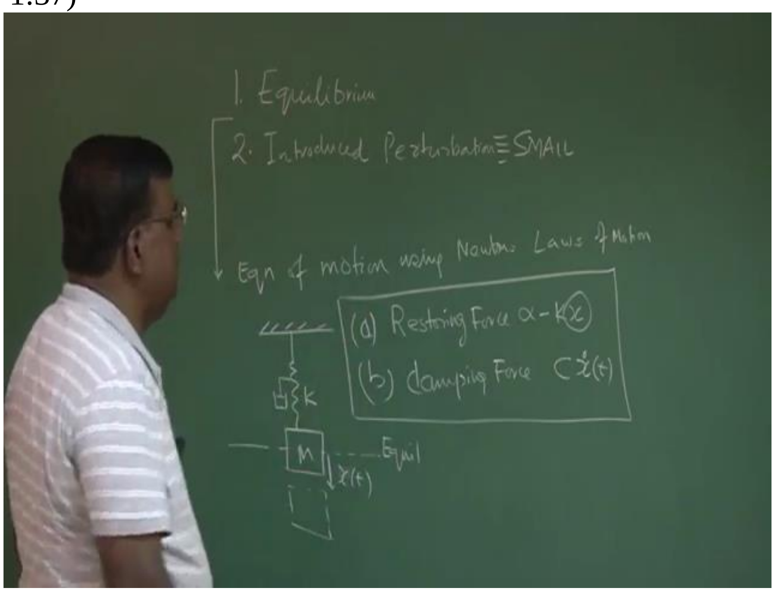

If I write down what we did, number 1, we identified the equilibrium. Then introduced perturbation, of course small perturbation. Before we introduced small perturbation, you should also notice that we have also developed equation of motion using Newton’s laws of motion. These are the primary steps. If I try to recall, this is the spring, this is the mass and here, there is a damper. Then we identify first, this is the equilibrium. That is net force acting on this mass is 0.

Now we introduce small perturbation and this X or X of T which is measured from the equilibrium. This is a one-dimensional motion. So we are talking about X axis. And this X is the perturbed motion variable. Please understand, because of this disturbance, X has been created and then we release it and we try to see what is happening to this X. Also, once I draw a mass spring damper system, there are salient points to be noticed.

One is, there is a restoring force which is typically KX where X is the perturbed variable and also, there is a damping force which is CX dot which is proportional to the rate of change of perturbed quantity, X. And of course, you could understand that we have assumed linear damping. In practice, there could be huge non-linear damping. But here, since we are giving small perturbation and for most of the cases, this approximation is not very far off. So this thing, these 2 things should be in your mind when we are talking about mass spring damper system.

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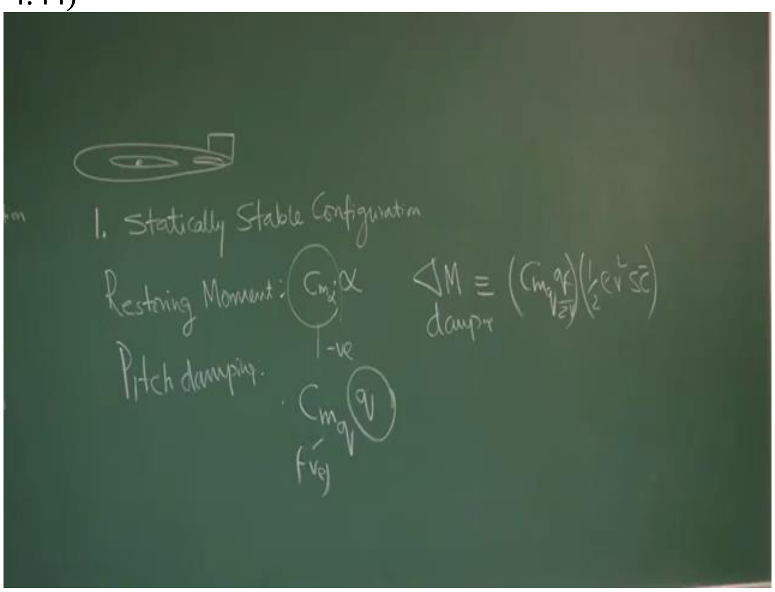

When you try to model aircraft motion, pitching motion, one-dimensional pitching motion. That is only pitching is allowed. Here also, we have noticed that we are considering statically stable configuration. Why I am stressing this? Because here in this case, it was a linear motion and this case, angular motion. And here, the moment I say statically stable means the restoring moment instead of force was also proportional to alpha.

And we say CM alpha into alpha and of course CM alpha is negative. So that is why, it is restoring. So similar to KX, this one, restoring force. And second thing, we also talk about damping or pitch damping. In a non-dimensional form, pitch damping, we modelled through

CMQ into Q and noticed that Q is the rate D Theta by DT, rate at which the altitude is changing. And CMQ is also negative. It is a damping derivative.

So this is like the CX dot. But here, please understand, C is not negative. But here, CMQ is negative so that your net moment is modelled because of damping, it is modelled as CMQ into Q into half rho V square SC bar, of course this is QC by 2V. So this is the net moment. M CMQ is negative. So for a positive Q, there is a negative moment. Again, restoring.

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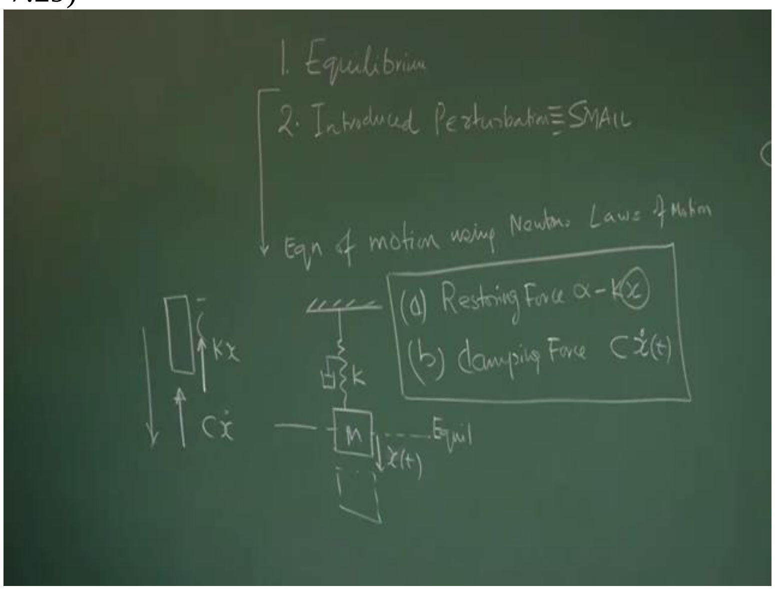

For a mass spring system, remember we model it like this. This is KX and we put a - sign here. And this is CX dot. But this is again, the force is acting in the opposite direction of motion, disturbance. So this clarity must be there in your mind. That is why we try to draw a similarity between a mass spring damper system and angular motion, dynamics of an airplane. This was our understanding.

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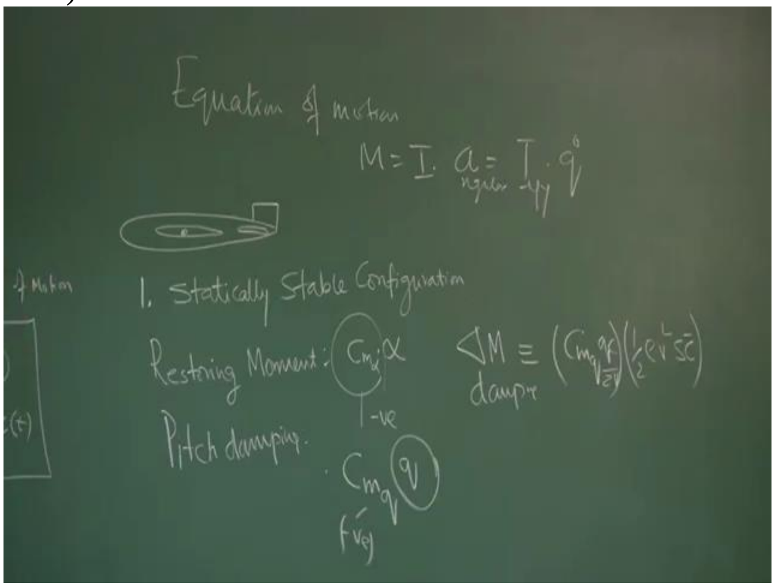

And again, if you see here, when we are developing this dynamics, one-dimensional pitching dynamics, we also wrote equation of motion and we used this concept, M is equal to I into angular acceleration. This is angular acceleration and which is typically written as IYY into Q dot. So that is the equation of motion.

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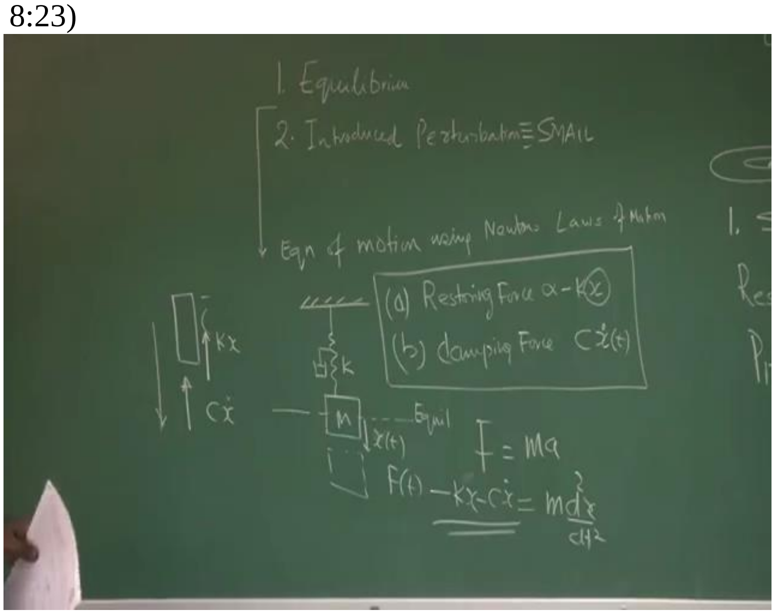

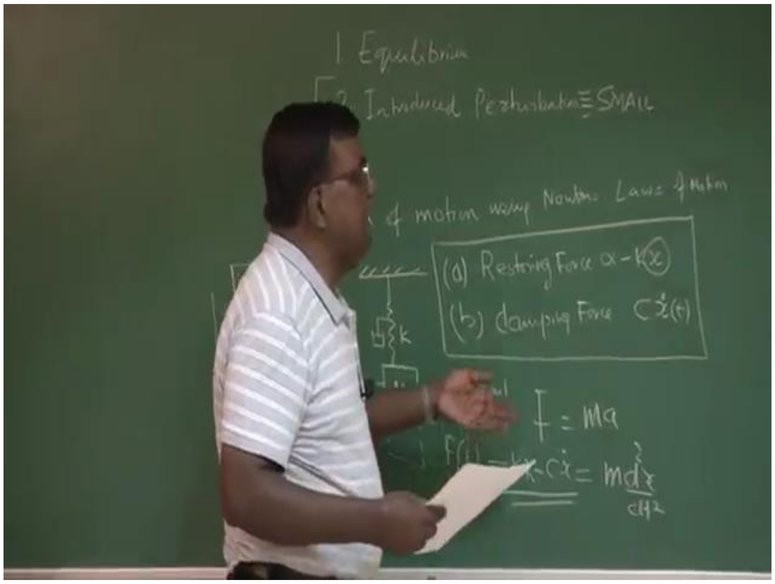

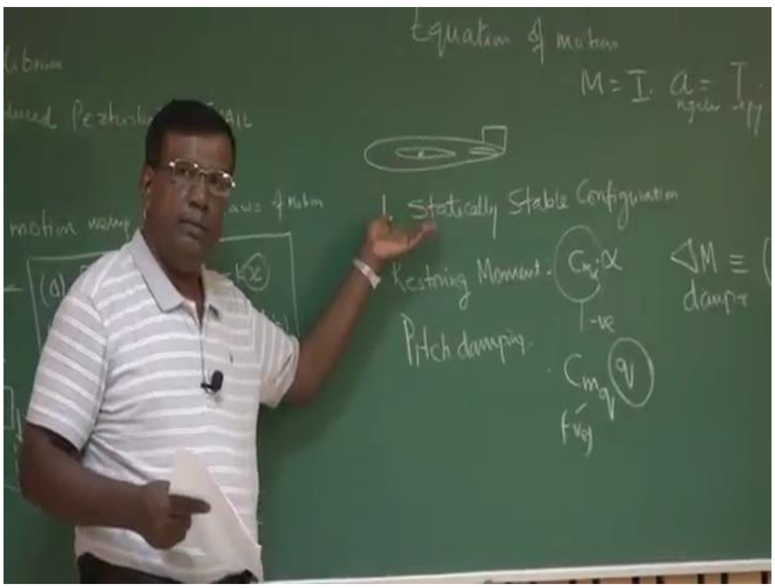

So if I correct, this is F equal to MA and F was - KX dot - CX dot. That was the force and this was equal to M D square X by DT square. For free response if there is a forcing function, then we write FT - this. So this is the equation of motion. This is the force balanced. What are we using? We are using Newton’s laws of motion. That means, we are writing this equation, both here and there with reference to inertial frame of reference.

That is the catch point you must understand. And here, there is a 1 degree freedom and there is also 1 degree freedom. This is angular, that is linear. So we have understood this much. We have also understood. We have also understood how to transform equation from time domain to frequency domain. By using Laplace transform one advantage you get is once you take a Laplace transform those differential equations get converted into algebraic equations.

So this is just to revise whatever we have done in the last class. Why I stress again and again, please understand, we need to understand the basics, the foundation stone, the foundation block before we go into the overall, generalised approach. Otherwise, if these things are not clear in mind, it only looks like some mathematical operations you are doing without having any physical sense and that will be too dangerous for a flight dynamics man to handle real-life problems.

Here also, please understand very clearly. We are talking about statically stable configuration. If the plane is not statically stable, then the spring analogy will not work. Because spring analogy is if there is a disturbance, you try to restore it. Imagine, if this airplane was not statically stable, even if I give a small disturbance in alpha, it will not try to restore.

It will diverge. So then this modelling will not be appropriate. So it is extremely important when you are doing this, I am clear, when I am drawing, I am clear that I am talking about statically stable configuration and for a statically stable configuration, you know, DCM by D alpha should be less than 0. That is the condition for static stability in longitudinal case.

Now think of, so far we are talking about motion like this or motion like this which is a pure pitching motion. But for an aircraft, it has more than 1 degree of freedom.

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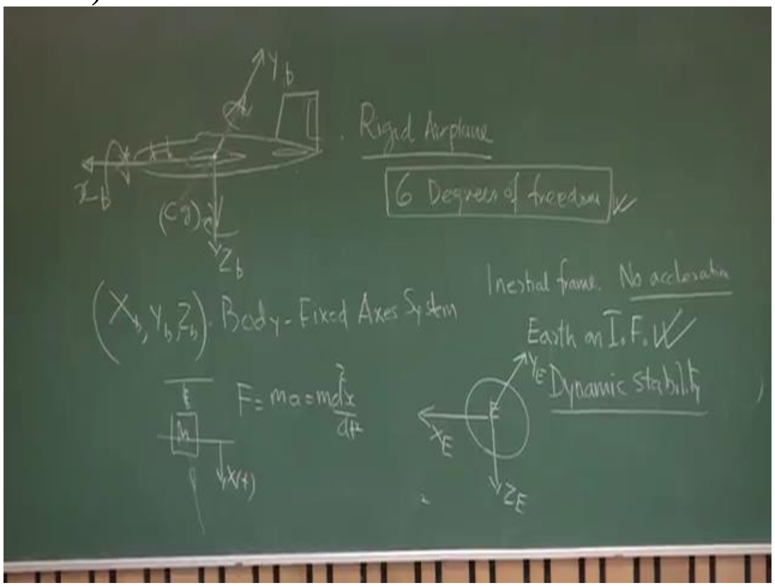

For example, if I take aircraft, let us say this and I fix axis system X, Y and Z and I call it body fixed axis system. That is I write B, B, and B. So this XB, YB, ZB, body fixed axis system. Let us say this is attached at the centre of gravity and also, we have aligned X axis along the line of symmetry or along the chord line or along the line of symmetry or along the line where the gross moment of inertia banishes, so that is in our hand.

Alignment of X axis, X body fixed axis can be different for different types of purposes. So let us say, in this case we are assuming that X is aligned along the fuse lodge reference line. Now if I talk about degrees of freedom, you could easily see, this body which is an air aircraft, it can have motion along x axis, it can have motion, linear motion about y-axis, it can have linear motion along the z axis. So 3, here and here and here. And also, it could have rotation about each axis. So what will happen?

This is roll, this is yawing and this is pitching motion. If I repeat it, if this is the aircraft, it can have one motion like this, one motion like this, one motion like this, along X, Y, Z. One could be rotation about Y, pitching, rotation about Z, yawing and rotation about X, roll motion. So there are 3 translational and 3 rotational degrees of freedom. And that is why for a rigid airplane, we try to model it in 6 degrees of freedom. Okay?

Please understand, I have written the word, rigid airplane. That means, throughout the analysis, the distance between 2 points on this airplane remains constant. So I can think of a not so rigid airplane. If you think there is a high aero elastic behaviour of the airplane, reflexes, it is possible. Or highly manoeuvrable airplane, the airplane is not strictly very rigid. It could be aero elastic in nature.

But we are talking about rigid and for that we remain, if you pick up any 2 points, their distance remains constant. So we now need to develop equation of motion or equations of motion using this 6 degrees of freedom approach. That is, I need to develop an equation of motion which should represent motion along X, motion along Y, motion along Z which is a linear motion, translational motion.

And also rotation about X, rotation about Y, rotation about Z. Think of, in contrast when we are talking about mass spring damper system, ideal mass spring damper system, we are only talking about rotation along X axis. And here, there is 6 degrees of freedom. That is one. Second thing, it is possible that there is a coupling between motion along Y and motion along X or motion along Y and motion along Z which may not be possible for us to separately handle each axis motion.

That coupling may come because of aerodynamics or because of inertia. For example, you understand, if you remember, if this is the body and spinning about X axis at a high rpm, if I give a disturbance like this, it will also process like this. So there is a coupling because of inertia. So the problem becomes a little more challenging but not complicated because all these things have been solved 50, 60 or 70 years back. So we are not doing anything new. We are not doing any rocket science.

We are doing very simple thing. Only thing, it will be simpler if you follow the steps in a neat and clean manner. Understand each step and go for the next. Learn through small perturbations. Do not jump. Do not assume anything without understanding. This is one difference. Second thing also please understand, when we are doing this mass spring system, we are writing it X of T. So we are actually telling, we are starting the motion along this axis.

So it is extremely important to identify the axis. That is why you have seen there, we have denoted XB, YB, ZB and we have referred to it as body fixed axis system. But what is the problem? Why this additional thing we need to understand? Again you come back here, here also we have used, F equal to MA or MD square X by D T square.. That is force equal to, force is the cause for acceleration and newton's law is mathematically put in this way as long as you remember that mass is not changing and also these are measured with respect to inertial frame.

That is why we say, Newton's law is valid in its expression form, F equal to MA when you are applying it in reference to inertial frame. If the frame is not inertial, you have to give some correction. That is, to incorporate the frame acceleration. So let us not jump now but one thing I understand that I should do all the measurements with respect to inertial frame. What is that inertial frame?

That is, frame has no acceleration. Now see the problem. We are talking about aircraft and we want to measure acceleration. Then try to apply F equal to MA and we need to measure this acceleration with respect to the inertial frame. But where is the inertial frame for an aircraft?

Mostly for the aircraft which are low altitude vehicles, we take Earth as inertial frame although we know, strictly speaking, Earth is not an inertial frame because Earth is rotating but for our analysis, specially for dynamic stability, since the duration of study is very small, we are talking about transient, we will neglect the earth acceleration. We will also neglect the curvature effect of the Earth. We will take it as a flat earth and we will assume for our dynamic stability analysis for an air aircraft, as aiming Earth to be initial frame is good enough.

It will not cause much error. That is one assumption we will be making. Once we make this assumption, then I can implement Newton's laws of motion in a frame which is fixed to Earth. So I say X Earth, Y Earth and Z Earth. So now we have got 2 frames of reference, one is body fixed axis system which is fixed to the body. That means when the body is rotating, this axis will also rotate. But definitely, this is not inertial frame.

We cannot directly apply F equal to MA in the rotating frame unless and until they give appropriate correction. And this is the inertial frame. If we write the equation with respect to inertial frame, we need not give any correction. But we find, there is a demand to operate, write the equation of motion and operate in body fixed axis system rather than inertial frame. What is the reason?

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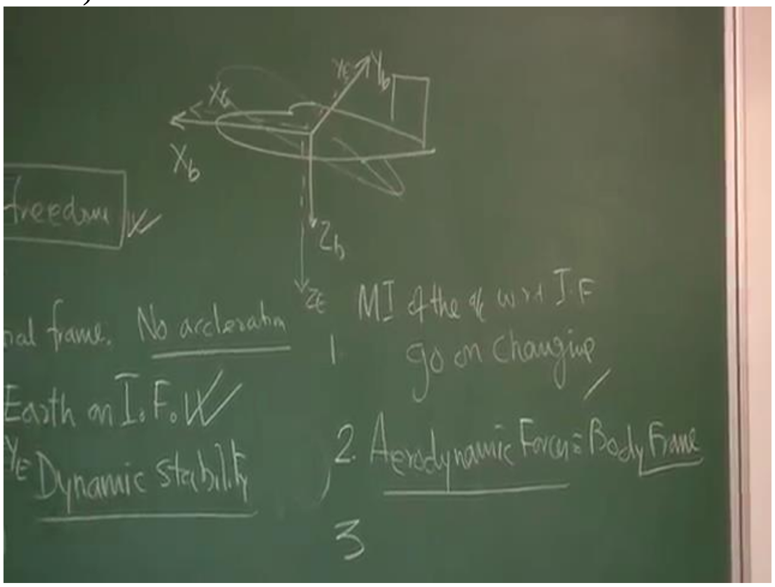

The reason is. Suppose this is the body fixed axis system, XB, YB, ZB and let us say this is X Earth, Y Earth, Z Earth. You could see, I can keep on writing the equation of motion with respect to X Earth, Y Earth, Z Earth and they are Earth fixed axis system. So inertial frame, no problem. But there are complexities. See, in actual practice, the airplane will have motion like this. Motion like yawing motion, rolling motion, and as we are referring to only Earth fixed axis system, then you can understand, the moment of inertia of the airplane with respect to inertial frame will go on changing.

That will almost become function of time depending on the orientation. The moment of inertia will become function of orientation and time. That will create a lot of complexities. So we do not want to work directly in inertial frame. However, we cannot violate because Newton’s law of motion is applicable only in inertial frame. This is one challenge.

Second thing is, the aerodynamic forces depend on the relative airspeed, not on the relative ground speed. For example, airplane is there, with respect to ground it may be 0 but if there is a wind coming like this, it will experience air dynamic force. So it is a relative as speed. That is why, it is advantageous to work in body frame. So that also makes work in body frame.

Third, even if you want it you can manage. Bigger question comes, how do I measure the orientation of the airplane? This is the airplane. If the body rotates, the axis also rotates. How will I measure the orientation? Because with respect to the body axis, the rotation is 0 because the axis is also rotating.

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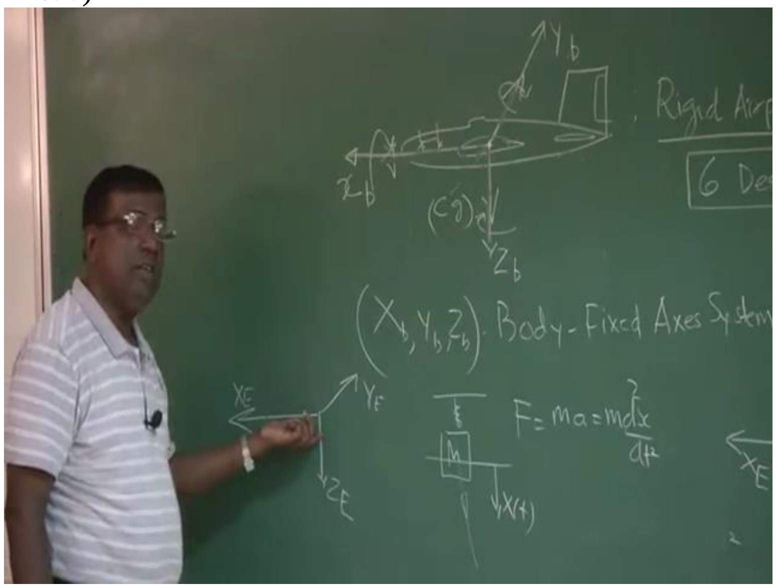

So there, there is a necessity that I measure the orientation of the body with respect to Earth fixed inertial frame. The orientation of the body will be measured with respect to inertial frame. And when we want to work in body frame, you will see that total velocity of the airplane with respect to the inertial frame can be resolved locally into components along body X, body Y and body Z direction.

Similarly also about the angular velocity. Whatever the angular velocity of this frame, with respect to inertial frame is there, that can easily be resolved along XB, YB and ZB axis. So we will be using these concepts but please understand, we need to work in inertial frame because we want to apply Newton's laws of motion. But operational wise, we would like to find out a method so that we can work in a body frame without violating the fundamental law that Newton's law is valid in inertial frame for that some correction we will give. Okay.